The Place of Relations in Knowledge and Reality.

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In modern philosophical discussion the problems on which the greatest stress is laid are, perhaps, those of the nature and metaphysical significance of knowledge. In most of the schools into which the philosophy of the present day is divided, it is realized that we can get no adequate view of the world through our knowledge until we discover ~~it~~ the relation of the latter to the ~~world~~ former. Epistemology is the keynote of recent philosophical thought, which keeps asking, "What part or phase of experience has an objective, universal value? what phase is merely subjective, meaningless except to the individual to whom it belongs?" Now, it is obvious that one means of attacking the problem is the empirical method, — let us take the experiences of different people whose mental life is normal in every respect, and see what common element runs through these various experiences, absent in those who have some mental or sensory deficiency. We shall try to make use of

this method of procedure in what follows.

Let us begin, then, by contrasting the mental life of a normal person with that of a person suffering from a very common sensory defect; color-blindness. In what way do we learn that the color-blind person sees things in colors different from those which we see? The method usually employed is to give to the suspected person a number of colored skeins of yarn, which he is asked to match with certain test-skeins. If he has that particular form of color-blindness known as red-blindness, he will invariably confuse a dull green with a bright red, and both with a grey. Other deficiencies give other similar results. That is, in certain cases where the normal person discovers a relation of difference, the color-blind person finds a relation of identity. We see, then, that the color-blind person is detected through an anomaly in the relational structure of his experience.

This is an extremely suggestive fact, for it

gives a concrete illustration of the means by which we are able to cross the abyss between our own consciousness and that of another. If we look into any text-book of psychology, if we enter any laboratory where ~~the facts of~~ experience is examined and analyzed, we shall see that some similar process is universally characteristic of the experimental method of investigation. We discover that a man is hard of hearing by the fact that he confuses sounds easily, and cannot tell a slight sound from an absolute silence. Nearsightedness is detected by trying various spectacles on the patient; if he ~~his~~ is nearsighted, his vision will have the greatest distinguishing power in respect to distant objects when he has on a biconcave lens of a certain strength. It is the same with other sensory deficiencies; with deficiencies or abnormalities of a more purely mental nature the case is the same. We call a man feeble-minded if he confuses persistently and to an extreme degree ideas which we are easily able to separate. The lunatic is the man who

lives in a world organized in a manner entirely different from our own, where the experiences which are most indifferent to us are associated with the intensest joy or the deepest sorrow, etc. We penetrate into the conscious life of others by the analysis of the relations which their thoughts bear to one another, putting their relational structure in one-to-one correspondence with that of our own experience, so that term corresponds to term, and order of terms to order of terms.

Let us suppose that we have before us two persons whose experiences, though identical in relational structure, are absolutely different in qualitative nature. Take, for example, a person with a vision like your own, and another, differing from you only in that the lights and shades which ~~he~~ he sees are interchanged. Let us assume further that his lights have all the emotional and other concomitants of your shades, and vice versa. If these two people have seen as they do now from birth up, how will they ever learn of the difference between their ways of see-

ing things? They will have learned the names, 'light' and 'shade', 'black' and 'white', 'pale' and 'dark', in such a manner that the same sensory quality which the one calls 'light', the other will call 'shade'; the appearance which if it occurred in the vision of the one would be named 'white' would be spoken of as 'black' by the other, and so on ad libitum. This is the case because a quality-name must be taught through its denotation before its connotation, which is its definitive meaning, can be grasped. We teach a child the meaning of the name 'red' by denoting to him a certain group of objects which have to us the connotation in common of a certain peculiar sensory quality. Afterwards, it is true, when our quality-names have acquired definite relational contexts, we can indicate grasp the meaning of a quality-name if the connotation of its relational context be indicated. But with these two people, who are trying to give each other a purely qualitative account of their sensations, such means of communication will be impossible. How, then, will they discover that

they see things differently? What is there which the one can predicate of any object of vision which the other can deny? The answer is clear,—there is absolutely no way in which they can learn that their experiences are qualitatively different.

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We can go through our experience from end to end in the same way, and not find a single quality which another person can recognize except through its relational context. But if there is no quality in our experience whose absence we can detect in the experience of another, there is nothing to prove that the qualities which other people feel have anything to do with those of our own mental life. As a matter of fact, the qualitative identity of things which can never be compared is extremely difficult to conceive, for it seems that identity demands at least the idea of a possible comparison between the identical things. Be that as it may, it remains clear that the sole common ground on which the experiences of different people can meet is that of their relational structure.

It might seem that if the knowledge of qualities is limited to the particular individual who perceives them, that science would be rendered impossible. A little consideration of the subject will convince us, however, that science is purely relational in its nature. The law of falling bodies, for instance, states that the distance traversed by a body falling from rest divided by the time of fall varies as the time of fall, and whether the falling body feels hard or soft, whether the time of fall seems pleasant or unpleasant, all these facts of quality are of absolutely no importance to the physicist. As a matter of fact, the scientist is so indifferent to the quality of the objects with which he deals, apart from their relational context, that it is by no means uncommon in a physical laboratory to find a deaf man conducting experiments on the velocity of sound. On the other hand, the similarities of their relations to the substances which form, transmit, and receive, radiant heat and light make the scientist class these together, although they are qualitatively different in nature.

As to the distinctions which the scientist makes between

primary and secondary qualities, more hereafter.

It might be thought, however, that in psychology, at any rate, we were dealing with pure qualities. Is it not important that the psychologist should know the experience-qualities of the subject whom he is observing? Is not introspection a record of the qualities which pass through our minds? If we read the first paragraph of Professor Titchener's, A Text-book of Psychology, we will obtain a clear answer. "A science", he says, "consists of a large body of ~~related~~ observed facts, which are related" to one another, and are arranged under general laws. If, for instance, you open a text-book of physics, you find that it gives the results of numerous observations, or prescribes experiments in which you are to observe for yourself, and you find that the results or experiments are grouped under certain main headings ..... and are made to illustrate certain comprehensive laws ..... All scientific text-books, whether the science is physics or chemistry, biology or psychology, ~~[illegible]~~ philology or economics, are of the same pattern." That is, the function of psychology

" The underlining is my own throughout.

as well as of the other sciences, is to take the individual phenomena of its subject-matter, and show their relational interconnections. What sensation, emotion, or idea feels like, unrelated to other phases of experience, and ~~unanaly~~ unanalyzed into its component elements, does not concern the psychologist at all.

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There is no need, then, for an objective knowledge of pure qualities in any of those sciences whose task it is to know and describe nature. As for the normative sciences, such as ethics, logic, and aesthetics, in so far as they form a description of the ~~at~~ actual ends of humanity, the fall under the same heading as psychology, physics, and the other empirical sciences, - the end must be determined by some relation which it holds to the life of the person who has it, or to the environment wherein he lives, if it is to become a topic for scientific investigation at all. In so far as the normative sciences claim to give objective ideals to humanity, these ideals to be applicable to human conduct, reason, or experience must be of the same nature as those which humanity instinctively form-

lates for itself, and hence must concern the structure of experience, not its quality, for we have no faculty of entering the soul of another, and viewing his purposes from within, painted in the shades and tones of feeling which he sees. We know the purposes and goals of others through the languages of words, gestures, actions, all of which, as we have seen, express relations rather than qualities, and may express different qualities to different people. Surely no sane ethicist would accept an end as the highest good, whose realization would only be possible through some telepathic insight.

(6) There is, however, a group of ethical theories which seems to make this demand, and to require the direct comparison of the feelings of one person with those of another. Hedonism regards pleasure as the supreme good. While a theory which makes the pleasure of the agent the supreme good may not seem to entail a comparison of the feelings of different people, utilitarianism, which regards the greatest happiness of the greatest number as the end towards which all should

strive, apparently requires that the pleasures of different people should be on a plane of qualitative similarity. Now, from what has been said, it is clear that this demand is essentially impossible to ~~sa~~ satisfy. If a feeling, in quality like what we call pleasure, were associated in ~~an~~ another with the sense-stimuli and motor responses which accompany pleasure in us, and vice versa, we could never know it, for the term 'pleasure' would signify for him the feeling which we would call 'displeasure', but the objective bodily condition which we would associate with our 'pleasure'-feeling; the same holds true, mutatis mutandis, for the term 'displeasure'. Hence, a qualitative utilitarianism, if I may so name the theory which I have just been criticizing, is untenable, for it makes impossible demands on epistemology. The 'pleasure' of the hedonist must be a mode of life, not a feeling; hedonism ~~must~~ must become eudaemonism.

So far we have been discussing the respective place of qualities and relations in the knowledge which is ~~the~~ the common property of humanity, but the same

arguments will show us that the relational aspect of our personal experience is the only aspect to which we can attribute reality. Unless we are to fall into crass solipsism, and regard our own personal experience as the sole arbiter of truth and falsity, we must recognize that, a fortiori, that which cannot be shown to be common to all sound experiences, cannot be shown to belong to the reality ~~approximation~~ to which they approximate as they become more veracious. Whatever objective reality is, whether we regard it from the standpoint of realism or of idealism, we can have certain knowledge of its relations and structure only, not of its qualities.

Whether the objects of experience are part of the experience itself, ideas in the divine mind, or things external to and independent of any subject, something more than the mere fact of its being felt is necessary for a sensation to be regarded as referring to an objective reality. Every item of consciousness, if it is not to be deemed illusory, must fill a place in the organized complex of sensations, feelings, and ideas which we call exper-

ience. That is, our final test of the correctness of a sensation concerns its relational position and relational structure. Of course, all relations must have terms to relate, but what the qualities of these terms are is a matter of utter indifference. Though they may be known to the individual subject by their qualitative connotation, they are known to science by denotation.

The neo-realists and idealists, who regard the idea and its object as in some degree coincident, might object to this, and claim that any theory which separates knowledge and its object renders it impossible to bridge the gulf between experience and reality and bring them into mutual ~~connection~~ organic connection. This difficulty, however, is far less serious than those which lurk in neo-realism or idealism. Most epistemological monists (to use Professor Perry's term) would, I believe, be ready to admit that our experience may at times disagree with reality. For instance, when the color-blind person confuses red and green, he sees red, green, or both in a way different from the

manner in which they appear to us. One way of seeing or the other must be wrong;- if an illusion be simply another, correct way of seeing the real object, reality ceases to be a determinant, and looses all its significance. Every determination is a negation, and an attribute of experience which is common to all experience cannot be used to distinguish a part of it. Besides, there is a very manifest difference between illusion and reality;- reality is the realm in which we can operate. We can walk on a real bridge, but not on a hallucination of a bridge.

It is clear, then, that we can be mistaken about both qualities and relations in experience. What guarantee have we, then, that we ever experience qualities correctly.[2] We have already shown that there is ~~now~~ no guarantee that we perceive ~~sele~~ qualities as they are in reality, while we can obtain a partial guarantee of the reality of the relational phase of our experience by applying to reality the relational structure observed in ~~of~~ our experience, and observing the results, as I

will show in more detail further on. Prediction and verification make possible a knowledge of the relational structure of reality. No such path is open with reference to the qualities of our experience. We can never know a posteriori that we have experienced them correctly, while there is no a priori way open which does not permit at least the possibility of doubt. Our knowledge of the outer world resembles the correspondence between a set of monochrome photographs and the object they represent; any inaccuracy of the form of the photograph, due, perhaps, to the imperfection of the lens, or to other similar causes, may be remedied by the proper use of the camera; spherical and chromatic aberration may be minimized by the addition of proper corrective lenses; the camera may be used to photograph smaller objects than it otherwise would by using it in combination with a microscope, but the color of the photographs will remain absolutely dependent on the material on which they are taken, not on the object photographed. Similarly, however askew our knowledge of the structure of reality may be, it always contains

the germs of its own improvement by further use, whereas however even if ~~similar~~ the qualities which we see may be, similar to those of reality, we cannot know it, and therefore cannot improve our knowledge of them. The relation between experience and reality is of the most intimate sort, and neo-realism has done a great deal of good by pointing this fact out, but it is a relation of correspondence rather than one of partial identity. Perhaps partial formal identity would be a satisfactory name for it, for it concerns the form or structure of reality, not its matter, or qualitative content. The 'how' of existence rather than its 'what' is at once the important and the answerable question.

We have, then, reduced the objective element in experience to its relational structure. We have the problem left on our hands, 'How are we ever to find a starting-point in this indefinitely involved complex of relations.' How are two people to know when they are talking of the same thing?' It would seem at first glance that this problem is absolutely insoluble, for the very act of indicating an object as occupying a certain con-

text demands for its comprehension a knowledge of the relation of indication. Any term is in an infinitude of different relations to an infinitude of different terms — how am I to single out one of these for purposes of comparison? How am I to distinguish the superficial and accidental similarities of two relational complexes from their genuine and real identities.[2]

To a being who should come into this world with sense-organs to record impartially the whole structure of his environment and its changes, a reason to infer the unseen relations from the seen ones, and nothing else, there would actually be no place to begin in the analysis of experience, so that he would remain eternally in suspense, unable to pick out some particular quality that he might relate the other qualities of his sensations and emotions thereto. But with us the case is different. We have with us from birth a fairly complex and stable system of qualities and emotions — our empirical ego — to which we instinctively refer all the subject-matter of our later experiences. For man,

at any rate, man is the measure of all things. Our body is a natural set of standards with which we can compare all the objects which come within the range of our experience. For instance, most of our units of length — the inch, the foot, the hand, the ell, the pace, the mile — are originally derived from parts of the human body. We carry around with us an excellent system of rectangular coordinates, formed by the sagittal plane, the horizontal plane of our eyes, and the plane perpendicular to the two at the level of the eyes. The first spring balances are our muscles, the first chronometers our heart and lungs, and legs; ~~together~~ while it is proverbial that the poor man is his own thermometer. We afterwards improve them greatly, we extend their range of usefulness by means of delicate instruments, and add to their number through acquired scientific interests, and we shift our standpoint to take standards which are of greater aid for the resolution of a particular problem in experience, but they always remain the core of our empirical ego. Within

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the sphere of our moral and esthetic interests the same

1) Not only do we have these natural measures, but we have an instinctive way of using them, and an instinctive tendency to apply them. Space and time are two instinctive ways of using natural standards. Of their peculiar position, more later.

holds true; we have a crude, imperfect, instinctive norm of virtue or of beauty, which we refine and elaborate as the range of our experience widens. A being without interests, no matter how long he might live, would never develop them a priori, and would remain in a permanent state of mental inaction, unable to find (if I may be pardoned the use of a homely metaphor) any loose ends to unravel the world by. These 'loose ends' our inborn propensities furnish us, not our reason, for reason unaided tends to see things as a whole, and hence to sink individual differences in a chaos of unity, - it can work out determinations, but cannot form them

This train of reasoning which we have been ~~pursuing~~ pursuing bears both a similarity and a difference to Descartes' 'Cogito, ergo sum.' There is a similarity in that the possibility of thought, which is assumed in both, is declared in each to be dependent on the existence of a percipient subject, but whereas Descartes demands that there shall be a transcendent ego, the self which my argument leads to is thoroughly empirical

in nature. Descartes requires a substance out of experience, to which in some mysterious way experience belongs; my self is the very warp and woof of experience, its internal center of organization. To Descartes' self belong the powers of conscious will and ratiocination, while the ego with which my argument is concerned is precisely the instinctive, non-rational factor in our mental life, without which we would be mere calculating machines, with nobody to turn the cranks. The self is the phase of our soul which enables us to blunder, for it is only through learning the impermanence and superficiality of certain relational conjunctions in our experience that we can ever discover the depth and significance of others. Computing-engines and formulae can do the work of the <u>purely</u> rational part of our nature, while we can get no substitute for the mass of normas and standards with which we are born which is not derived from them. Without them we would be like an untuned wireless receiving-station, hearing confused and chaotic fragments of the messages intelligible <u>per se</u> of a multitude of

variously tuned sending-stations, unable to attend to anyone message and comprehend it. Our intuitive nature is a sort of selective tuning.

The fact that our intuitional nature plays such an important part in our scientific knowledge of the relations of the outer world has led many philosophers to think that science is only a construction of our own, independent of any external reality over and above ourselves. This, however, is false. We cannot interpret our experience in any way we wish. No matter how much we will that the Ptolemaic system of astronomy be true, the Copernican system still continues to give a more adequate account of the facts of the case. "Eppur' si muove." No amount of volition will make the velocity of all falling bodies constant, if we retain the old meaning of the term 'velocity', which as we have seen, must be determined in some way other than by reference to its total context, and which is a relation, as it is a proportion. Some relational structures may be affirmed of the world, while others may be denied of it. Whether the

world to which science refers be in mind or out of it, that some such world exists independent of whether we will it, offering a sphere for our activity, is certain. Reason or instinct could never order chaos, for there would be no point in the chaos from which to start its arrangement. Knowledge implies not one order but two,- the order in experience and the order of reality, and (V9) a mediator between these, the empirical ego, whereby to mould the former after the latter. Reverting to our figure of the wireless stations, experience, as we have seen, corresponds to the receiving station, reality to the various sending-stations, and the empirical ego to the tuner which enables us to hear one message — that is, one aspect of the order of nature — without disturbance from the rest. We are as incapable of forming a science without the presence of an objective order, an experience, and an empirical ego, as we are of receiving an intelligible wireless message without sending-stations[1], receiving station, and tuning-coil. Science is the ordering of experience in such a

[1] We are supposing that the various sending-stations are all continually active (that the various phases of reality are all simultaneously and perpetually true).

way that to every item of experience there corresponds an item of reality. It is the richness rather than the poverty of reality which makes selection necessary, and prevents us from being able to add that to each item of reality there should correspond an item of experience. It is our instinctive and acquired interests that enable us to pick certain terms out and follow them through their mutual relationships.

However, we are not yet through with our difficulties. On page 3 we defined the identity of relational structures as the one-to-one correspondence of the terms and orders to be found in them. We have also seen that our way of identifying a term throughout its various relations is through the fixed place it holds with reference to some one or more of our interests. We would seem, then, to be perilously near the admission that our interests determine reality, for a little reflection will show us that a finite system of terms can be artificially arranged in correspondence with another system with no other law of order than

the order itself. We can put one system of three terms in one-~~two~~ to-one correspondence with ~~another~~ another system of three terms in six ways (if we call the terms of one system a, b, and c, and of the other α, β, and γ, we have the following ~~as~~ correspondences possible: $a\alpha, b\beta, c\gamma$; $a\alpha, b\gamma, c\beta$; $a\beta, b\gamma, c\alpha$; $a\beta, b\alpha, c\gamma$; $a\gamma, b\alpha, c\beta$; $a\gamma, b\beta, c\alpha$). What meaning, then, has the identification of the ideal relational structure of experience with the relational structure of a part of reality.[2]

This question must be answered in some such way as the following:[3] although experience is finite, it is growing, and therefore potentially infinite, in that from the present we can set no definite limit to its future growth. Now, owing to the fact that it is not possible to go through the terms of an infinite set one by one, the position of a term in any ordering of an infinite set must be determined by some law other than the ~~order~~ position itself. Therefore, experience cannot be made to correspond to reality except according to some law. What this law is we must determine

by arranging its experience tentatively, applying this tentative arrangement to nature, and observing whether it fits reality, or whether there are predicted terms which do not make their appearance, or unpredicted terms which appear. By eliminating all hypothetical arrangements which must in the course of time show some disagreement with reality, or revising them until they fit their subject-matter, by trying new hypotheses until we find one in which we are unable to pick flaws, as far as we can see, we discover the laws of nature. This process is the inductive method. The hypotheses which the scientist tries so earnestly to establish or to discredit are formed first by instinct, then by the conscious act of the imagination, working on the basis of previously determined laws. They are subject to a progressive self-correction, as new facts are discovered and old hypotheses rejected. Induction would be an impossibility to a finite being whose experience should be given to him complete once for all, for he could order it with no other law than the order itself, so that he would be unable to formu-

late any hypotheses which would be more than convenient summaries of what he had seen. Science would seem no more real to him than the faces which we see in clouds. It is due to the fact that our experience is never definitely complete that science is both needed and possible, for it demands that its laws shall embrace more than the facts on which they are built; this is really the principle of the uniformity of nature. This is not the place to discuss the technique of the proper formation of hypotheses and the methods of securing one-one correspondence between experience and reality, which was first worked out, though in a very imperfect and inadequate form, by John Stuart Mill.

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There is left on our hands, however, the problem whether the other fundamental process of reason and science, deduction, can be explained in terms of relation. The most perfect use of deduction is in mathematics, and the simplest branch of mathematics is the algebra of logic, so it is to the algebra

of logic that we would naturally look for the solution of this problem. But given a transitive, asymmetrical relation (that is such a one that if a is in the relation to b, and b to c, a will be in the relation to c, but b need not be in the relation to a) conditioned in one way of which I shall speak later, the whole algebra of logic can be obtained. Let us denote the relation by the symbol $\prec$, so that 'a is in the assy asymmetrical transitive relation to b' will be denoted by the formula $a \prec b$. Let the state of affairs when $a \prec b$ and $b \prec a$ be represented by $a = b$ (or $b = a$, which is the same thing). Let there be a term 1, so that $a \prec 1$, whatever a is, and a term 0, so that $0 \prec a$, whatever a is. Let $a + b$ (or $b + a$) represent a term such that $a \prec (a + b)$, $b \prec (a + b)$, and if $a \prec x$, $b \prec x$, $(a + b) \prec x$. Let ~~ab~~ $\alpha\beta$ be a term such that ~~$ab \prec a$, $ab \prec b$~~ $\alpha\beta \prec \alpha$, $\alpha\beta \prec \beta$, and if ~~$x \prec a$, $x \prec b$~~ $x \prec \alpha$, $x \prec \beta$, whatever x is, ~~$x \prec ab$~~ $x \prec \alpha\beta$. Let $\bar{a}$ represent a term such that $a\bar{a} = 0$, while $a + \bar{a} = 1$. Let us make the single assumption that $a(b + c) \prec ab + ac$. From these conditions, with no other law than that a term or operation shall retain ~~its~~ the meaning throughout a theorem into which it enters which has

been assigned to it at the beginning. To give an instance of the method used, let us give a theorem.

Let $ac + b\bar{c} = 0$

Then $ac + b\bar{c} \prec 0$. By definition of $=$.

$ac \prec (ac + b\bar{c})$ and $b\bar{c} \prec (ac + b\bar{c})$ By definition of $+$.

$0 \prec ac$ and $0 \prec b\bar{c}$ By definition of 0.

$\therefore$ $ac = 0$ and $b\bar{c} = 0$ By definition of $=$. I

$(c + \bar{c})a \prec (ca + \bar{c}a)$ By assumption. II

$ca \prec c$ and $ca \prec a$. $\bar{c}a \prec \bar{c}$ and $\prec a$ By definition of ab.

$\therefore (ca + \bar{c}a) \prec c + \bar{c}$ and $\prec a$ By definition of $+$.

$\therefore (ca + \bar{c}a) \prec (c + \bar{c})a$ By definition of $\alpha\beta$. III

$\therefore ca + \bar{c}a = (c + \bar{c})a$ From II, III, and definition of $=$.

$ac = 0$. $ca + \bar{c}a = \bar{c}a + 0 = \bar{c}a$ From I, and definition of 0.

$\bar{c}a = ca + \bar{c}a = (c + \bar{c})a = 1a = a$ By definition of $\bar{\alpha}$

$a \prec \bar{c}a \prec \bar{c}$ $\therefore a \prec \bar{c}$ By definitions of $+$ and $\alpha\beta$ and $\prec$.

Similarly, $b \prec c$

$ab \prec c$ and $ab \prec \bar{c}$ By definition of $\alpha\beta$.

$\therefore ab \prec c\bar{c} \prec 0$ By definitions of 0 and $\alpha\beta$.

$\therefore ab = 0$ By definitions of $=$ and 0.

This is a typical mathematical theorem. The pro-

cess of reasoning has been to lay out the outlines of a certain conceptual system through certain definitions and axioms. From these, and other limitations a more detailed account of the structure of the system is next sought after. That is, the task of mathematical deduction, and for that matter of all deduction, is to find the implicit structure of a system when its explicit structure has been given. Relations are all that count; what the quality of the subject-matter is is utterly indifferent; induction and mathematical deduction agree in being based on system.

To give an exposition of the nature of mathematical concepts would be beyond the scope of this paper, but it can be shown that by limiting the sphere of the algebra of logic in ways which bear no reference to the particular senses or feelings, and hence, which make no demands on the quality of the subject-matter, all mathematical concepts, even those of geometry may be obtained. It becomes possible, therefore, to give a minute analysis of the relational nature of the space and the time with which the mathematician

deals. As the mathematical notions of space and time have been taken over bodily by the physicist, it is very evident that physical science need attribute to its dimensions no particular qualitative nature. ~~For~~ Both the inductive and the deductive sides of natural science make no demands on the quality of the subject-matter.

It has been shown, then, that a knowledge of the structure of reality is all that we can aspire to, and that it furnishes a perfectly adequate and sufficient basis for science. We must now inquire why it is that this limitation of our knowledge to relations has not been generally recognized. Why is it that the qualities of our sensations have been considered as giving us true information of the nature of reality? Several causes combine to produce this result. In the first place, although to accept the qualities of our experience as if they originated from reality gives us no trustworthy information, and is not true, it does the scientist no real harm, for it is not really false, but does not belong to the universe of discourse to

which the terms 'truth' and 'falsity' apply. Such a statement as 'Red is true', or 'Green is false' is meaningless, and so is not liable to lead one into error if it is not ontologized. In the second place, the physicist really does see a difference between quality and relation, for the distinction between primary and secondary qualities is simply a somewhat garbled form of this difference. As we have seen, the empirical ego is, as it were, the measure of the relational nature of our experience, and since self-consciousness is normally only an occasional state of mind, explicit reference to the self is liable to be suppressed. Therefore, those relations which concern at once the self and some external object often seem to refer to the external object alone, and hence have the specious appearance of being pure qualities. In this way a number of relational complexes, and among them more especially space and time, which, as we have seen, have no particular qualitative character whatever, seem to be of a qualitative nature. These relations, especially such of them as are more rich and complex in their struc-

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ture, form what are called the primary qualities, whereas the true qualities which enter into these relations are regarded by the scientist as unreal and illusory, and are called the secondary qualities. It may be objected to this ~~that~~ exposition of the distinction between primary and secondary qualities that certain relations, such as 'brighter than', 'louder than', etc. are regarded as belonging rather to the secondary than to the primary qualities; however, in the first place, these relations are less rich in structure than those which belong to the primary qualities, and therefore are less fitted for purposes of scientific explanation, and in the second place, they are not referred so directly to the empirical ego (a fact which concerns our instinctive nature rather than the relations themselves) and therefore appear in their true form rather than as qualities.

The knowledge of relations, then, is the beginning, the end, and the whole of true knowledge. It is all that we have, and it gives us all that we need for science. Our knowledge is an imperf

I think this is an interesting and well-sustained defense of your thesis; but, since I am opposed wholly to the thesis, I am not able to say that the defense holds. The following refer to the figures marked in pencil on the margin:

1) Is epistemology prior to the other philosophical studies? If you say, for example, that epistemology presupposes psychology I should say that it is just as true that epistemology presupposes psychology as that conversely.

2) Is failure to perceive difference the same thing as the perception of identity? Is failure to perceive red and green as two the same as perceiving them alike? I think that nothing is perceived here. This is a very important point in your relational theory, because you cannot have a relation between something (in the normal persons) and nothing (in the color-blind).

3) Your major-fallacy is here. You speak as if the only order of experience were the time-order, whereas, as even Mill admits, this order is rearranged into a scientific order. That which is first in actual experience is not necessarily first in the causal succession. Hence, for any subject experience is made into a system. Now, apart from such system, it might easily be that what is light for you might be shade for me etc. and we should never discover the qualitative difference. But light and shade have a relation, say, to ocular comfort. Too much light hurts my eyes. Let us say that for you too much shade does this. In that case your eye would have to show a totally opposite structure from mine--or, in short, your whole world would have to be mine turned inside out. Then the question is whether you could conceive it at all. Let it be a genealogical series which is inverted in the two cases. I say that Abraham was an old man when Isaac was born. For you, since the time-relation holds and only the terms are transposed, this means that the son was an old man when the father was born. No quality can be defined apart from its relations to others. Present quality stands for a certain relation to past quality. Reverse the qualities and leave the relations the same--you then have an unintelligible order in which no quality could be defined. If you reverse the relations also, then you have only a different language. In conceiving such a system do you not think mainly of colrs etc. where the quality is very loosely joined to other qualities? I can easily enough think of an experience in which apples grow green (from red) as they ripen, because I have no idea of connection between redness and ripeness. But all experience is not equally disconnected.

4) Is he dealing with <u>sound</u> or with air-vibrations? - This makes a difference.

5) If, however, an idea involves in itself a relation of many in one (as I should hold) then the relation is important--and if not what is the difference between an idea and that which is not an idea--say a picture?

6) And yet this is just what many ethicists and most art-critics are saying.

7) Why object to solipsism; have you not begun there?

8) I seem to see a vicious circle here. What is a sound experience but that which, as you say, shows identities with

others?

9)There are no differences of color in the photo--in the ordinary sense which calls the black to white series shades. Hence,the figure is bad. The photo simply fails to take any cognizance of the colors in the object. If it did the question would arise whether its rearrangement of those colors could be made intelligible when color were once connected with the other qualities of the object. See note 3

10)On what ground would you affirm that your own foot remains a constant? Do you not see that this is tied up with the world that is measured by it?

11)I am sure that no present analyst of reason would agree with this. Reason is quite as much a matter of differentiation as of identification.

12)"More adequate"from what standpoint? Are you not begging the question here? What you mean is that,when you set up a given standard of adequacy, experience replies to the question raised by it with Yes or No,the answer not being presupposed in the question.

13)Do you have the two orders of experience? In fact,from what standpoint could this relational theory ever be asserted except from that which somehow claimed to look into two minds at once?

14) But does just this differentiate science from fiction? Might not one argue that the fact that a man can assert a law which is not already offered by experience proves that there is no essential difference? I mean,just as far as this aspect is concerned.

15)My objection,as stated,to the symbolic notion of an entity is that under this apparently non-committal term it introduces the conception of a mechanical or even atomic entity. If you were to allow conscious entities to come in here,it would be necessary for the entities,to be conscious at all,to know each other,and this would make hash of the original relations.

16) I have denied this in the Individualism,p.70-71.

THE PLACE OF RELATIONS IN KNOWLEDGE AND REALITY.

A thesis offered in competition for the Bowdoin Prize for dissertations in English, written by graduate students of Harvard University.

By

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THE PLACE OF RELATIONS IN KNOWLEDGE AND REALITY.

In modern philosophical discussion the problems on which the greatest stress is laid are, perhaps, those of the nature and metaphysical significance of knowledge. In most of the schools into which the philosophy of the present day is divided, it is realized that we can get no adequate view of the world through our knowledge until we discover the relation of the latter to the former. Epistemology is the keynote of recent philosophical thought, which keeps asking, "What part or phase of experience has an objective, universal value? What phase is merely subjective, meaningless except to the individual to whom it belongs?" Now, it is obvious that one means of attacking the problem is the empirical method, - let us take the experiences of different people whose mental life is normal in every respect, and see what common element runs through these various experiences, absent in those who have some mental or sensory deficiency. We shall try to make use of this method of proceedure in what follows.

Let us begin, then, by contrasting the mental life of a normal person with that of a person suffering from a very common sensory defect, - color-blindness. In what way do we learn that the color-blind person sees things in colors different from those which we see? The method usually employed is to give to the suspected person a number of colored skeins of yarn, which he is asked

to match with certain test-skeins. If he has that particular form of color-blindness known as red-blindness, he will invariably confuse a dull green with a bright red, and both with a grey. Other deficiencies give other similar results. That is, in certain cases where the normal person discovers a relation of difference, the color-blind person finds a relation of identity. We see, then, that the color-blind person is detected through an anomaly in the relational structure of his experience.

This is an extremely suggestive fact, for it gives a concrete illustration of the means by which we are able to cross the abyss between our own consciousness and that of another. If we look into any text-book of psychology, if we enter any laboratory where experience is examined and analyzed, we shall see that some similar process is universally characteristic of the experimental method of investigation. We discover that a man is hard of hearing by the fact that he confuses sounds easily, and cannot tell a slight sound from an absolute silence. Nearsightedness is detected by trying various spectacles on the patient, - if he is nearsighted, his vision will have the greatest distinguishing power in respect to distant objects when he has on abiconcave lens of a certain strength. It is the same with other sensory deficiencies; with deficiencies or abnormalities of a more purely mental nature the case is the same. We call a man feeble-minded if he

confuses persistently and to an extreme degree ideas which we are easily able to separate. The lunatic is the man who lives in a world organized in a manner entirely different from our own, where the experiences which are most indifferent to us are associated with the intensest joy or the deepest sorrow, etc. We penetrate into the conscious life of others by the analysis of the relations which their thoughts bear to one another, putting their relational structure in one-to-one correspondence with that of our own experience, so that term corresponds to term, and order of terms to order of terms.

Let us suppose that we have before us two persons whose experiences, though identical in relational structure, are absolutely different in qualitative nature. Take, for example, a person with a vision like your own, and another, differing from you only in that the lights and shades which he sees are interchanged. Let us assume further that his lights have all the emotional and other concomitants of your shades, and vice versa. If these two people have seen as they do now from birth up, how will they ever learn of the difference between their ways of seeing things? They will have learned the names, 'light' and 'shade', 'black' and 'white', 'pale' and 'dark', in such a manner that the same sensory quality which the one calls 'light', the other will call 'shade'; the appearance which if it occurred in the vision of the one would be named 'white' would be spoken of

as 'black' by the other, and so on ad libitum. This is the case because a quality-name must be taught through its denotation before its connotation, which is its definitive meaning, can be grasped. We teach a child the meaning of the name 'red' by denoting to him a certain group of objects which have to us the connotation in common of a certain peculiar sensory quality. Afterwards, it is true, when our quality-names have acquired definite relational contexts, we can grasp the meaning of a quality-name if the connotation of its relational context be indicated. But with these two people, who are trying to give each other a purely qualitative account of their sensations, such means of communication will be impossible. How, then, will they discover that they see things differently? What is there which the one can predicate of any object of vision which the other can deny? The answer is clear, -- there is absolutely no way in which they can learn that their experiences are qualitatively different.

We can go through our experience from end to end in the same way, and not find a single quality which another person can recognize except through its relational context. But if there is no quality in our experience whose absence we can detect in the experience of another, there is nothing to prove that the qualities which other people feel have anything to do with those of our own mental life. As a matter of fact, the qualitative identity of

things which can never be compared is extremely difficult to conceive, for it seems that identity demands at least the idea of a possible comparison between the identical things. Be that as it may, it remains clear that the sole common ground on which the experiences of different people can meet is that of their relational structure.

It might seem that if the knowledge of qualities is limited to the particular individual who perceives them, that science would be rendered impossible. A little consideration of the subject will convince us, however, that science is purely relational in its nature. The law of falling bodies, for instance, states that the distance traversed by a body falling from rest divided by the time of fall varies as the time of fall, and whether the falling body feels hard or soft, whether the time of fall seems pleasant or unpleasant, -- all these facts of quality are of absolutely no importance to the physicist. As a matter of fact, the scientist is so indifferent to the quality of the objects with which he deals, apart from their relational context, that it is by no means uncommon in a physical laboratory to find a deaf man conducting experiments on the velocity of sound. On the other hand, the similarities of their relations to the substances which form, transmit, and receive radiant heat and light make the scientist class these together, although they are qualitatively different in nature. As to the distinctions

which the scientist makes between primary and secondary qualities, more hereafter.

It might be thought, however, that in psychology, at any rate, we were dealing with pure qualities. Is it not important that the psychologist should know the experience-qualities of the subject whom he is observing? Is not introspection a record of the qualities which pass through our minds? If we read the first paragraph of Professor Titchener's A Text-book of Psychology, we will obtain a clear answer. "A science," he says, "consists of a large body of observed facts, which are related" to one another, and are arranged under general laws. If, for instance, you open a text-book of physics, you find that it gives the results of numerous observations, or prescribes experiments in which you are to observe for yourself, and you find that the results or experiments are grouped under certain main headings.....and are made to illustrate certain comprehensive laws.....All scientific text-books, whether the science is physics or chemistry, biology or psychology, philology or economics, are of the same pattern." That is, the function of psychology as well as of the other sciences, is to take the individual phenomena of its subject-matter, and show their relational interconnections. What sensation, emotion, or idea feels like, unrelated to other phases of experience, and un-

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(1) The underlining is my own throughout.

analyzed into its component elements, does not concern the psychologist at all.

There is no need, then, for an objective knowledge of pure qualities in any of those sciences whose task it is to know and describe nature. As for the normative sciences, such as ethics, logic, and aesthetics, in so far as they form a description of the actual ends of humanity, they fall under the same heading as psychology, physics, and the other empirical sciences, - the end must be determined by some relation which it holds to the life of the person who has it, or to the environment wherein he lives, if it is to become a topic for scientific investigation at all. In so far as the normative sciences claim to give objective ideals to humanity, these ideals, to be applicable to human conduct, reason, or experience, must be of the same nature as those which humanity instinctively formulates for itself, and hence must concern the structure of experience, not its quality, for we have no faculty of entering the soul of another, and viewing his purposes from within, painted in the shades and tones of feeling which he sees. We know the purposes and goals of others through the languages of words, gestures, actions, all of which, as we have seen, express relations rather than qualities, and may express different qualities to different people. Surely no sane ethicist would accept an end as the highest

good, whose communication would only be possible through some telepathic insight.

There is, however, a group of ethical theories which seems to make this demand, and to require the direct comparison of the feelings of one person with those of another. Hedonism regards pleasure as the supreme good. While a theory which makes the pleasure of the agent the supreme good may not seem to entail a comparison of the feelings of different people, utilitarianism, which regards the greatest happiness of the greatest number as the end towards which all should strive, apparently requires that the pleasures of different people should be on a plane of qualitative similarity. Now, from what has been said, it is clear that this demand is essentially impossible to satisfy. If a feeling, in quality like what we call pleasure, were associated in another with the sense-stimuli and motor responses which accompany dis-pleasure in us, and vice versa, we could never know it, for the term 'pleasure' would signify for him the <u>feeling</u> which we would call 'displeasure', but the <u>objective bodily condition</u> which we would associate with our 'pleasure'-feeling; the same holds true, <u>mutatis mutandis</u>, for the term 'displeasure.' Hence, a <u>qualitative</u> utilitarianism, if I may so name the theory which I have just been criticizing, is untenable, for it makes impossible demands on epistemology. The 'pleasure' of the

hedonist must be a mode of life, not a feeling; hedonism must become eudaemonism.

So far we have been discussing the respective place of qualities and relations in the knowledge which is the common property of humanity, but the same arguments will show us that the relational aspect of our personal experience is the only aspect to which we can attribute reality. Unless we are to fall into crass solipsism, and regard our own personal experience as the sole arbiter of truth and falsity, we must recognize that, a fortiori, that which cannot be shown to be common to all sound experiences, cannot be shown to belong to the reality to which they approximate as they become more veracious. Whatever objective reality is, whether we regard it from the standpoint of realism or of idealism, we can have certain knowledge of its qualities.

Whether the objects of experience are part of the experience itself, ideas in the divine mind, or things external to and independent of any subject, something more than the mere fact of its being felt is necessary for a sensation to be regarded as referring to an objective reality. Every item of consciousness, if it is not to be deemed illusory, must fill a place in the organized complex of sensations, feelings, and ideas which we call experience. That is, our final test of the correctness of a sensation concerns its relational position and relational

structure. Of course, all relations must have terms to relate, but what the qualities of these terms are is a matter of utter indifference. Though they may be known to the individual subject by their qualitative connotation, they are known to science by denotation.

The neo-realists and idealists, who regard the idea and its object as in some degree coincident, might object to this, ~~and claim that any theory which object to this,~~ and claim that any theory which separates knowledge and its object renders it impossible to bridge the gulf between experience and reality and bring them into mutual organic connection. This difficulty, however, is far less serious than those which lurk in neo-realism or idealism. Most epistemological monists (to use Professor Perry's term) would, I believe, be ready to admit that our experience may at times disagree with reality. For instance, when the color-blind person confuses red and green, he sees red, green, or both in a way different from the manner in which they appear to us. One way of seeing or the other must be wrong, - if an illusion be simply another correct way of seeing the real object, reality ceases to be a determinant, and loses all its significance. "Every determination is a negation", and an attribute of experience which is common to all experience cannot be used to distinguish a part of it. Besides, there is a very manifest difference between illusion and

reality, - reality is the realm in which we can operate. We can walk on a real bridge, but not on a hallucination of a bridge.

It is clear, then, that we can be mistaken about both qualities and relations in experience. What guarantee have we, then, that we ever experience qualities correctly? We have already shown that there is no guarantee that we perceive qualities as they are in reality, while we can obtain a partial guarantee of the reality of the relational phase of our experience by applying to reality the relational structure observed in our experience, and observing the results, as I will show in more detail further on. Prediction and verification make possible a knowledge of the relational structure of reality. No such path is open with reference to the qualities of our experience. We can never know a posteriori that we have experienced them correctly, while there is no a priori way open which does not permit at least the possibility of doubt. Our knowledge of the outer world resembles the correspondence between a set of monochrome photographs and the object they all represent, - any inaccuracy of the form of the photograph, due, perhaps, to the imperfection of the lens, or to other similar causes, may be remedied by the proper use of the camera; spherical and chromatic aberration may be minimized by the addition of proper corrective lenses, the camera may be used to photograph smaller

objects than it otherwise could by using it in combination with a microscope, but the color of the photographs will remain absolutely dependent on the material on which they are taken, not on the object photographed. Similarly, however askew our knowledge of the structure of reality may be, it always contains the germs of its own improvement by further use, whereas even if the qualities which we see be similar to those of reality, we cannot know it, and therefore cannot improve our knowledge of them. The relation between experience and reality is of the most intimate sort, and neo-realism has done a great deal of good by pointing this fact out, but it is a relation of correspondence rather than one of partial identity. Perhaps partial formal identity would be a satisfactory name for it, for it concerns the form or structure of reality, not its matter, or qualitative content. The 'how' of existence rather than its 'what' is at once the important and the answerable question.

We have, then, reduced the objective element in experience to its relational structure. We have the problem left on our hands, 'How are we ever to find a starting-point in this indefinitely involved complex of relations? How are two people to know when they are talking of the same thing?' It would seem at first glance that this problem is absolutely insoluble, for the very act of indicating an object as occupying a certain context demands for its comprehension a knowledge of the relation of indication.

Any term is in an infinitude of different relations to an infinitude of different terms -- how am I to single out one of these for purposes of comparison? How am I to distinguish the superficial and accidental similarities of two relational complexes from their genuine and real identities?

To a being who should come into this world with sense-organs to record impartially the whole structure of his environment and its changes, a reason to infer the unseen relations from the seen ones, and nothing else, there would actually be no place to begin in the analysis of experience, so that he would remain eternally in suspense, unable to pick out some particular quality that he might relate the other qualities of his sensations and emotions thereto. But with us the case is different. We have with us from birth a fairly complex and stable system of qualities and emotions -- our empirical ego -- to which we instinctively refer all the subject-matter of our later experiences. For man, at any rate, man is the measure of all things. Our body is a natural set of standards with which we can compare all the objects which come within the range of our experience. For instance, most of our units of length -- the inch, the foot, the hand, the ell, the pace, the mile -- are originally derived from parts of the human body. We carry around with us an excellent system of rectangular

coordinates, formed by the sagittal plane, the horizontal plane of our eyes, and the plane perpendicular to the two at the level of the eyes. The first spring balances are our muscles, the first chronometers our heart, lungs, and legs,(1) while it is proverbial that the poor man is his own thermometer. We afterwards improve them greatly, we extend their range of usefulness by means of delicate instruments, and add to their number through acquired scientific interests, and we shift our standpoint to take standards which are of greater aid for the resolution of a particular problem in experience, but they always remain the core of our empirical ego. Within the sphere of our moral and esthetic interests the same holds true; we have a crude, imperfect, instinctive norm of virtue or of beauty, which we refine and elaborate as the range of our experience widens. A being without interests, no matter how long he might live, would never develop them a priori, and would remain in a permanent state of mental inaction, unable to find (if I may be pardoned the use of a homely metaphor) any loose ends to unravel the world by. These 'loose ends' our inborn propensities furnish us, not our reason, for reason unaided tends to see things as a whole, and hence to sink individual differences in a

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(1) Not only do we have these natural measures, but we have an instinctive way of using them, and an instinctive tendency to apply them. Space and time are two instinctive ways of using natural standards. Of their peculiar position, more later.

chaos of unity, - it can work out determinations, but cannot form them.

This train of reasoning which we have been pursuing bears both a similarity and a difference to Descartes' 'Cogito, ergo sum?' There is a similarity in that the possibility of thought, which is assumed in both, is declared in each to be dependent on the existence of a percipient subject, but whereas Descartes demands that there shall be a transcendent ego, the self which my argument leads to is thoroughly empirical in nature. Descartes requires a substance out of experience, to which in some mysterious way experience belongs; my self is the very warp and woof of experience, its internal center of organization. To Descartes' self belong the powers of conscious will and ratiocination, while the ego with which my argument is concerned is precisely the instinctive, non-rational factor in our mental life, without which we would be mere calculating machines, with nobody to turn the cranks. The self is the phase of our soul which enables us to blunder, for it is only through learning the impermanence and superficiality of certain relational conjunctions in our experience that we can ever discover the depth and significance of others. Computing-engines and formulae can do the work of the <u>purely</u> rational part of our nature, while we can get no substitute for the mass of normas and standards with which we are born

which is not derived from them. Without them we would be like an untuned wireless receiving-station, hearing confused and chaotic fragments of the messages intelligible per se of a multitude of variously tuned sending-stations, unable to attend to any one message and comprehend it. Our instinctive nature is a sort of selective tuning.

The fact that our intuitional nature plays such an important part in our scientific knowledge of the relations of the outer world has led many philosophers to think that science is only a construction of our own, independent of any external reality over and above ourselves. This, however, is false. We cannot interpret our experience in any way we wish. No matter how much we will that the Ptolamaic system of astronomy be true, the Copanican system still continues to give a more adequate account of the facts of the case. "Eppur si muove." No amount of volition will make the velocity of all falling bodies constant, if we retain the old meaning of the term 'velcity', which, as we have seen, must be determined in some way other than by reference to its total context, and which is a relation, as it is a proportion. Some relational structures may be affirmed of the world, while others may be denied of it. Whether the ~~world~~ universe to which science refers be in mind or out of it, that some such world exists independent

of whether we will it, offering a sphere for our activity, is certain. Reason or instinct could never order chaos, for there would be no point in the chaos from which to start its arrangement. Knowledge implies not one order but two, -- the order in experience and the order of reality, and a mediator between these, the empirical ego, whereby to mould the former after the latter. Reverting to our figure of the wireless stations, experience, as we have seen, corresponds to the receiving-station, reality to the various sending-stations, and the empirical ego to the tuner which enables us to hear one message - that is, one aspect of the order of nature -- without disturbance from the rest. We are as incapable of forming a science without the presence of an objective order, an experience, and an empirigal ego, as we are of receiving an intelligible wireless message without sending-stations,(1) receiving station, and tuning-coil. Science is the ordering of experience in such a way that to every item of experience there corresponds an item of reality. It is the richness rather than the poverty of reality which makes selection necessary, and prevents us from being able to add that to each item of reality there should correspond an item of experience. ~~It is~~ Our instinctive and acquired interests ~~that~~ enable us to pick

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(1) We are supposing that the various sending-stations are all continually active (that the various phases of reality are all simultaneously and perpetually true.)

certain terms out and follow them through their mutual relationships.

However, we are not yet through with our difficulties. On page 3 we defined the identity of relational structures as the one-to-one correspondence of the terms and orders to be found in them. We have also seen that our way of identifying a term throughout its various relations is through the fixed place it holds with reference to some one or more of our interests. We would seem, then, to be perilously near the admission that our interests determine reality, for a little reflection will show us that a finite system of terms can be artificially arranged in correspondence with another system with no other law of order than the order itself. We can put one system of three terms in one-to-one correspondence with another system of three terms in six ways (if we call the terms of one system a, b, and c, and of the other α, β, and γ, we have the following correspondences possible: $a\alpha, b\beta, c\gamma$; $a\alpha, b\gamma, c\beta$; $a\beta, b\gamma, c\alpha$; $a\beta, b\alpha, c\gamma$; $a\gamma, b\alpha, c\beta$; $a\gamma, b\beta, c\alpha$.) What meaning, then, has the identification of the ideal relational structure of experience with the relational structure of a part of reality?

This question must be answered in some such way as the following, -- although experience is finite, it is growing, and therefore potentially infinite, in that from the present we can set no definite limit to its

future growth. Now, owing to the fact that it is not possible to go through the terms of an infinite set one by one, the position of a term in any ordering of an infinite set must be determined by some law other than the position itself. Therefore, experience cannot be made to correspond to reality except according to some law. What this law is we must determine by arranging experience tentatively, applying this tentative arrangement to nature, and observing whether it fits reality, or whether there are predicted terms which do not make their appearance, or unpredicted terms which appear. By eliminating all hypothetical arrangements which in the course of time show some disagreement with reality, or revising them until they fit their subject-matter, by trying new hypotheses until we find one in which we are unable tp pick flaws, as far as we can see, we discover the laws of nature. This process is the inductive method. The hypothesis which the scientist tries so earnestly to establish or to discredit are formed first by instinct, then by the conscious act of the imagination, working on the basis of previously determined laws. They are subject to a progressive self-correction, as new facts are discovered and old hypotheses rejected. Induction would be an impossibility to a finite being whose experience should be given to him complete once for all, for he could order it with no other law than the order itself, so that he would be unable to formulate any

hypotheses which would be more than convenient summaries of what he had seen. Science would seem no more real to him than the faces which we see in clouds. It is due to the fact that our experience is never definitely complete that science is both needed and possible, for it demands that its laws shall embrace more than the facts on which they are built; this is really the principle of the uniformity of nature. This is not the place to discuss the technique of the proper formation of hypotheses and the methods of securing one-one correspondence between experience and reality, which was first worked out, though in a very imperfect and inadequate form, by John Stuart Mill.

There is left on our hands, however, the problem whether the other fundamental process of reason and science, deduction, can be explained in terms of relation. The most perfect use of deduction is in mathematics, and the simplest brand of mathematics is the algebra of logic, so it is to the algebra of logic that we would naturally look for the solution of this problem. But given a transitive, asymmetrical relation (that is such a one that if a is in the relation to b, and b to c, a will be in the relation to c, but b need not be in the relation to a) conditioned in one way of which I shall speak later, the whole algebra of logic can be obtained. Let us denote the relation by the symbol $\prec$, so that 'a is in the asymmetrical transitive relation to b' will be

denoted by the formula $a \prec b$. Let the state of affairs when $a \prec b$ and $b \prec a$ be represented by $a = b$ (or $b = a$, which is the same thing.) Let there be a term 1, so that $a \prec 1$, whatever a is, and a term o, so that $o \prec a$, whatever a is. Let $a + b$ (or $b + a$) represent a term such that $a \prec (a + b)$, $b \prec (a + b)$, and if $a \prec x$, $b \prec x$, $(a + b) \prec x$. Let $\alpha\beta$ be a term such that $\alpha\beta \prec \alpha$, $\alpha\beta \prec \beta$, and if $x \prec \alpha$, $x \prec \beta$, whatever x is, $x \prec \alpha\beta$. Let $\bar{a}$ represent a term such that $a\bar{a} = o$, while $a + \bar{a} = 1$. Let us make the single assumption that $a(b + c) \prec ab + ac$. From these conditions, with no other law than that a term or operation shall retain the meaning throughout a theorem into which it enters which has been assigned to it at the beginning. To give an instance of the method used, let us give a theorem.

Let $ac + b\bar{c} = o$

Then $ac + b\bar{c} \prec o$ By definition of =.

$ac \prec (ac+b\bar{c}) \prec o$ and $b\bar{c} \prec (ac+b\bar{c}) \prec o$ By definition of +.

$o \prec ac$ and $o \prec b\bar{c}$ By definition of o.

$\therefore ac = o$ and $b\bar{c} = o$ By definition of =. I

$a(c+\bar{c}) \prec (ca+\bar{c}a)$ By assumption. II

$ca \prec c$ and $ca \prec a$. $\bar{c}a \prec c$ and $\prec a$ By definition of $\alpha\beta$.

$\therefore (ca + \bar{c}a) \prec c + \bar{c}$ and $\prec a$ By definition of +.

$\therefore (ca + \bar{c}a) \prec (c + \bar{c})\, a$ By definition of $\alpha\beta$ III.

$\therefore ca + \bar{c}a = (c + \bar{c})\, a$ From II, III and definition of =.

$ac = o$. $ca + \bar{c}a = \bar{c}a + o = \bar{c}a$ From I, and definition of o

$\bar{c}a = ca + \bar{c}a = (c + \bar{c})a = 1a = a$ By definition of $\bar{a}$

$a \prec \bar{c}a \prec \bar{c}$ $\therefore a \prec \bar{c}$. By definion of + and $\prec$ and $\alpha\beta$.

Similarly, $b \prec c$

$ab \prec c$ and $ab \prec \bar{c}$ By definition of $\alpha\beta$.

$\therefore ab \prec c\bar{c} \prec o$. By definitions of o and $\alpha\beta$.

$\therefore ab = o$ By definitions of = and o.

This is a typical mathematical theorem. The process of reasoning has been to ~~lay~~ lay out the outlines of a certain conceptual system through certain definitions and axioms. From these and other limitations a more detailed account of the structure of the system is next sought after. That is, the task of mathematical deduction, and for that matter of all deduction, is to find the implicit structure of a system when its ex-

$\alpha\beta = \beta\alpha$ by definition.

plicit structure has been given. Relations are all that count; what the quality of the subject-matter is, is utterly indifferent; induction and mathematical deduction agree in being based on system.

To give an exposition of the nature of mathematical concepts would be beyond the scope of this paper, but it can be shown that by limiting the sphere of the algebra of logic in ways which bear no reference to the particular senses or feelings, and hence, which make no demands on the quality of the subject-matter, all mathematical concepts, even those of geometry, may be obtained. It becomes possible, therefore, to give a minute analysis of the relational nature of the space and the time with which the mathematician deals. As the mathematical notions of space and time have been taken over bodily by the physicist, it is very evident that physical science need attribute to its dimensions no particular qualitative nature. Both the inductive and the deductive sides of natural science make no demands on the quality of the subject-matter.

It has been shown, then, that a knowledge of the structure of reality is all that we can aspire to, and that it furnishes a perfectly adequate and sufficient basis for science. We must now inquire why it is that this limitation of our knowledge to relations has not been generally recognized. Why is it that the qualities of

our sensations have been considered as giving us true information of the nature of reality? Several causes combine to produce this result. In the first place, although to accept the qualities of our experience as if they originated from reality gives us no trustworthy information, and is not true, it does the scientist no real harm, for it is not really false, but does not belong to the universe of discourse to which the terms,'truth' and 'falsity' apply. Such a statement as 'Red is true', or 'Green is false', is meaningless, and so is not liable to lead one into error if it is not ontologized. In the second place, the physicist really does see a difference between quality and relation, for the distinction between primary and secondary qualities is simply a somewhat garbled form of this difference. As we have seen, the empirical ego is, as it were, the measure of the relational nature of our experience, and since self-consciousness is normally only an occasional state of mind, explicit reference to the self is liable to be suppressed. Therefore, those relations which concern at once the self and one external object often seem to refer to the external object alone, and hence have the specious appearance of being pure qualities. In this way, a number of relational complexes, and among them more especially space and time, which, as we have seen, have no particular qualitative character whatever, seem to be

of a qualitative nature. These relations, especially such of them as are more rich and complex in their structure, form what are called the primary qualities, whereas the true qualities which enter into these relations are regarded by the scientist as unreal and illusory, and are called the secondary qualities. It may be objected to this exposition of the distinction between primary and secondary qualities that certain relations, such as 'brighter than', 'louder than', etc. are regarded as belonging rather to the secondary than to the primary qualities; however, in the first place, these relations are less rich in structure than those which belong to the primary qualities, and therefore are less fitted for purposes of scientific explanation, and in the second place, they are not referred so directly to the empirical ego (a fact which concerns our instinctive nature rather than the relations themselves) and therefore appear in their true form rather than as qualities.

The knowledge of relations, then, is the beginning, the end, and the whole of true knowledge. It is all that we have, and it gives us all that we need for science. Our knowledge is an imperfect and incomplete map of reality, drawn to scale, which can be improved and corrected as time goes on, though the material on which it is drawn always remains a matter of indifference. The task of science is to explore the unknown parts of existence, and to survey and plot its known parts.