A Theory of Time and Space. A. A. Robb. Cambridge, England: University Press. 1914. Pp. vi + 373.

This book represents an attempt to discuss the theory of relativity from a mathematical standpoint by deriving the formal properties which space and time possess, according to this theory, from a set of postulates concerning the relation of temporal succession. It has been recognized for some years that there is no method by which we can discriminate between the electromagnetic or optical properties of a system at absolute rest and those of a system moving at a constant velocity. The famous experiment of Michelson and Morley destroyed almost the last hope of discovering, by electromagnetic or optical means, the direction and magnitude of the absolute velocity of a point on the surface of the earth, and suggested very strongly that *no difference whatever* could be found between the electromagnetic formulæ of a fixed system and those of one moving without acceleration. This gave Einstein the idea that there might, after all, be no difference between absolute rest and unaccelerated motion, and that what is now regarded as a system of fixed axes of coordinates with reference to which we determine the direction and magnitude of a motion may, from another equally valid standpoint, be regarded as moving with a constant velocity with reference to another set of axes of coordinates which, from this standpoint, are regarded as fixed. These interrelations of the velocity of a system and the position of what we take as our standard axes of coordinates turn out, according to the laws of optics, to be such as can not be expressed except by supposing that space and time are not independent, and that we can not say that two events are simultaneous without involving some reference to the positions of these two events in space, or to some physical magnitude, such as a velocity, which can only be defined in terms both of spatial and of temporal entities. Certain analytical formulæ have been found, which express those essential connections which must subsist between the spatial relations of an event and its temporal relations, unless there is some reason for regarding rest as intrinsically different from unaccelerated motion—and no experiment has been found which enables us to distinguish between these two states. Now, this interdependence of time and space can not be expressed in terms of these two principles, as we ordinarily conceive them, in such a manner that time constitutes a dimension of being independent of the three dimensions of ordinary space. It is consequently necessary to give a new formal analysis of the four-dimensional manifold constituted by time and space together. Two methods of carrying out this analysis have been suggested: one is that of Einstein, while the other is that developed by Robb in this book.

Einstein "made the suggestion that events might be simultaneous to one observer, but not to another,"[1] and developed a theory of the relation between space and time on this basis. Robb rejects this view, since, he claims, it conflicts with the logical fact that "a thing can not both be and not be at the same time." Robb's rejection rests on a misinterpretation of what is meant by this statement. "At the same time" is here simply a metaphor for "taken in the same sense and under the same conditions," and has nothing in particular to do with time. If to say that the event X occurs at the moment indicated by the event Y in the system of time measured with respect to the set of coordinate axes S expresses a different condition concerning X than to say that X is simultaneous with Y with reference to a set of axes T, there is no reason why the maxim cited by Robb should demand that the truth of one of these statements should imply that of the other. If, on the other hand, we accept the truth of the statement that "a thing can not both be and not be at the same time," we must accept this as a physical hypothesis, and not as a logical fact, and it is just as capable of correction and rejection in the course of our further study of physics as any of the other apparent truisms that have been discarded by the upholders of the theory of relativity.

Instead of attempting, like Einstein, to approach the problem of the relation between time and space by allowing the simultaneity of two events to depend upon the set of coordinates chosen as fixed, Robb bases his theory of space and time upon the relation of temporal succession. This, unlike Einstein, he regards as absolute: that is, he holds that if an event in time follows another, it does so without any reference to a set of fixed coordinates. He consequently escapes from the difficulty which he finds in the work of Einstein. To this relation of temporal succession he gives a physical interpretation essentially optical in nature: one instant follows another if a flash of light starting at the second can reach the first either directly or after reflection. An instant in this sense corresponds not to what we should ordinarily regard as an instant, but to an instant *at a particular point in space.* Robb phrases his definition of an instant in a manner somewhat more general than that in which I have just stated it: he says, "If an instant B be distinct from an instant A, then B will be said to be *after* A, if, and only if, it be abstractly possible for a person, at the instant A, to produce an effect at the instant B." (p. 7). This is rather awkwardly put. Entirely apart from the utterly needless introduction of the notion of a "person," it presupposes that we have a fixed and definite notion of what is meant by "causation," not to speak of "the abstract possibility of causation." It is fairly obvious that the notion of causation is at least as obscure as that of time, and that a theory which so radically upsets our established notions of time as the theory of relativity does can not but cause an equally great modification in our views on causality. However, while these alterations in our theory of time have been systematized and organized by the very people who have brought them about, the corresponding work has not been done with the theory of causality. It is indeed much more natural to define causality in terms of time than it is to define time in terms of causality.

What an "abstract possibility of causation" is, I do not know, and I doubt if the phrase has any clear and definite meaning whatever. In any case, Robb defines *ignotum per ignotius.*

While the philosophical basis of Robb's work is rather unsatisfactory, his book has an unquestionably great philosophical significance. That space and time form a system such that neither can be studied without reference to the other, while it is already brought out in the work of Einstein, receives much greater prominence in that of Robb, owing to the fact that he develops a theory of pure mathematics on the basis of a set of postulates which is at once spatial and temporal, which embraces both pure geometry and what may be called rational chronology; in which, however, these two elements can not be separated. Robb has at once made the consideration of space necessary in the discussion of the relation of time to experience, and forged an instrument which enables us to carry out this joint consideration of space and of time. To an even greater extent than Einstein, he has made it obvious that the two problems of Kant's "Transcendental Esthetic" are really but two aspects of a single problem. Furthermore, as a by-product of this philosophical task, he has created a new branch of mathematics of a very considerable intrinsic interest.

Of the technical development of the book one can only speak with the greatest admiration. As has been said, the relation of temporal succession is taken as the primitive idea. This is regarded as asymmetrical and transitive, but not as connected: that is, of two distinct instants, one need not follow the other. However, of a set of instants that represent the successive positions of a particle, one must precede or follow any other, so that the time-path of a particle is serial in character and, in general, has all those formal properties that we normally predicate of time. The relation of temporal succession is closely analogous to that of a cone A, with a vertical axis and a certain given vertical angle, to another such cone B, when the vertex of A lies on or within the upper nappe of B. Most of the postulates in the book apply to this relation among cones as well as to the relation of succession among instants. These postulates are well chosen, and for the most part satisfy the condition of independence. The various forms of the notions which he calls by the names of line, plane, and threefold are developed in terms of the relation of succession among instants, and finally a theory of measurement entirely dependent on the relation of temporal succession, and on that alone, is given, which is such that the fundamental formulæ of the theory of relativity, as developed by Einstein and Minkowski, result solely from Robb's postulates.

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