HAROLD E. EDGERTON

PAPERS

MC 25

Series III

Laboratory Notebooks

Number 3

Dated Feb. 15, 1930 to June 16, 1931

NOTEBOOK S-3.

Massachusetts Institute of Technology

COMPUTATION BOOK

NAME HAROLD E. EDGERTON

Number 3

Course VI

Used from FEB 15 1930, to JUNE 16 1931.

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

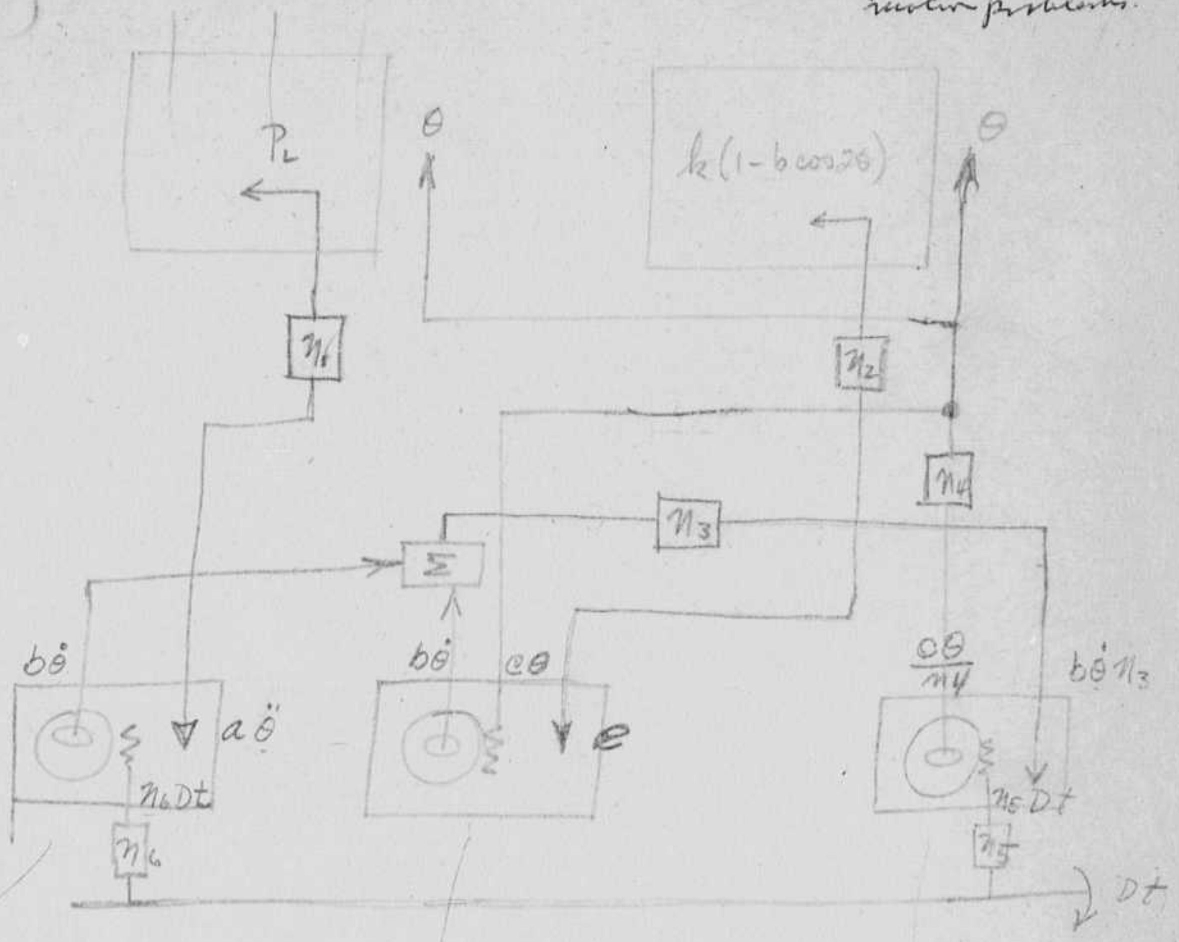
____ negative strip(s)

21 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page ____ and ____.
inside front cover

Item(s) now housed in accompanying folder.

INTEGRAPH Solutions of synchro motor problems.



$a\ddot\theta < 40$ rev. ✓

$e < 40$ rev.

$b\dot\theta n_3 < 40$ rev.

$$b\dot\theta = \frac{1}{32}\int \frac{a\ddot\theta}{b} n_6 D\, dt. \qquad \frac{a n_6 D}{32 b} = 1$$

$$b\dot\theta = \frac{1}{32}\int \frac{e c}{b} \frac{d\theta}{dt} dt. \qquad \frac{e c}{32 b} = 1$$

$$\frac{c\theta}{\cancel{n_4}} = \frac{1}{32} n_4 \int \frac{b\dot\theta}{c} n_3 n_5 D\, dt \qquad \frac{b n_4 n_3 n_5 D}{32 c} = 1$$

$\dot{\theta}_{max} = \underline{\underline{20 \text{ units}}}$ elect degs / x unit.

$b\, n_3\, 20 < 40$ rev.

$b < \frac{40}{n_3\, 20} = \frac{2}{n_3}$.

Let $n_3 = 1/4$

$b < \frac{2}{\frac{1}{4}} = 8$

(1) $\frac{\cancel{a}\, \cancel{n_6}\, \cancel{D}}{32 \times 8} = 1$

Let $n_6 = n_5 = 1$

(2) $\frac{e.c}{32 \times 8} = 1$

(3) $\frac{8\, n_4\, \cancel{n_5}\, D}{32 \times 4\, c} = 1$

a
$\cancel{n_6}$
D
e
c
n_4
$\cancel{n_5}$

7

Let 6" = 1 unit on (P_2 table.)

$a = \underline{120} \text{ rev. } n_1 = 120\, n_1$ rev. / unit

Let $n_1 = 1/4$ so $a = \frac{120}{4} = \underline{\underline{30}}$ rev. / unit

from (1) $D = \frac{32 \times 8}{30 \quad 1} =$

from (3) $c = \frac{8\, n_4}{32 \times 4}\left(\frac{32 \times 8}{30}\right) = \frac{8^2}{\cancel{32} \times \cancel{4} \times 2}\ \frac{\cancel{32} \times 8^2}{30} = \frac{8}{30}$

Let $n_4 = 1/2$

$k = .05$ units

$k = .05 \times 60 \times 32$

$= 120$

from (2) $e = \frac{32 \times \cancel{8}^2\ 30}{\cancel{8}} = \cancel{30} \times 32$ rev/unit

240

3

Let $e = \frac{30 \times \cancel{32}}{\cancel{2}}$

$c = \frac{\quad}{30} =$

$n_4 = \frac{32 \times 4 \times c}{8D} = \frac{32 \times 4 \times 32 \times 8}{8 \times 30 \times D}$

but $D = \frac{32 \times 8}{30}$

$n_4 = \frac{\overset{16}{\cancel{32}} \times \cancel{4} \times \cancel{32} \times \cancel{8} \times \cancel{30}}{\cancel{8}_2 \times \cancel{30} \times \cancel{32} \times \cancel{8}} = 16$

?

should be 8

Let $e = \frac{30 \times 32}{2}$

$c = \frac{\cancel{32} \times 8 \times \cancel{2}}{\underset{15}{\cancel{30}} \times \cancel{32}} \quad \frac{8}{15}$

$n_4 = \frac{32 \times 4 \times 8}{8 \; D \; 15} = \frac{\cancel{32} \times \cancel{4} \times \cancel{8} \quad \overset{2}{\cancel{30}}}{\underset{2}{\cancel{8}} \times \cancel{15} \quad \cancel{32} \times \cancel{8}} = 1$

---

but if $n_5 = 1/2$

$e = 8/15$

$n_4 = \frac{32 \times 4 \times c}{8 \; n_5 \times D} = \frac{\cancel{32} \times \cancel{4} \times \cancel{8} \; \overset{15}{\cancel{30}}}{\cancel{8} \; \cancel{32} \times \underset{2}{\cancel{8}} \times \cancel{15} \; \cancel{n_5} \frac{1}{2}} = 2$

trial II.

Let $n_6 = 1/2$.

$a = 30$

$b = 8$

$$D = \frac{32 \times 8}{a\, n_6} = \frac{32 \times 8 \times 2}{30}$$

Let $$e = \frac{30 \times 32}{2}$$

so $$c = \frac{32 \times 8}{e} = \frac{\cancel{32} \times 8 \ 2}{30 \times \cancel{32}} = \frac{16}{30}$$

$$n_4 = \frac{32 \times 4 \times c}{8\, n_5\, D} = \frac{\cancel{32} \times \cancel{4} \times \cancel{16}\ \cancel{30}}{8 \times \cancel{32} \times \cancel{8} \times \cancel{30} \times 2} = \frac{1}{2}$$

Let $n_5 = 1$

$\frac{16}{30}$ rev/min.

$\frac{16}{2030}$ in/min

$$\frac{\overset{12}{\cancel{360}} \times \overset{4}{\cancel{16}}}{\underset{5}{\cancel{30}} \times \cancel{32}} = \frac{4}{5} =$$

$a = 30$

$b = 8$

$D = \frac{32 \times 8}{15}$

$e = 15 \times 32$

$c = 16/30$

$n_1 = \boxed{1/4}$

$n_2 = 1/4$

$n_3 = 1/4$

$n_4 = 1/2$

$n_5 = 1$

300

Trial II.

4.

~~Let $b = 16$ $\quad \frac{a}{b} = \frac{1}{8}$~~

trial III

Let $c = \frac{32}{30}$

and e must equal $\frac{30 \times 32}{2}$

$$b = \frac{ec}{32} = \frac{32}{\cancel{30}} \frac{\cancel{30} \times \cancel{32}}{2} \frac{1}{\cancel{32}} = 16$$

and a = 30

$$\frac{a\, n_6 D}{32\, b} = 1$$

try $n_4 = 1/2$

$$\frac{e\, c}{32\, b} = 1$$

$$n_3 < \frac{40}{20 b} = \frac{2}{16} = \frac{1}{8}$$

$$\frac{b\, n_4 n_3 n_5 D}{32\, c} = 1$$

Let $n_3 = \frac{1}{8}$

$a = 30$
$b = 16$
$D = \frac{32 \times 32}{30}$
$e = \frac{30 \times 32}{2}$
$c = 32/30$
$n_1 = 1/4$
$n_2 = 1/4$
$n_3 = 1/8$
$n_4 = 1/2$
$n_5 = 1$
$n_6 = 1/2$

Let $n_5 = 1$

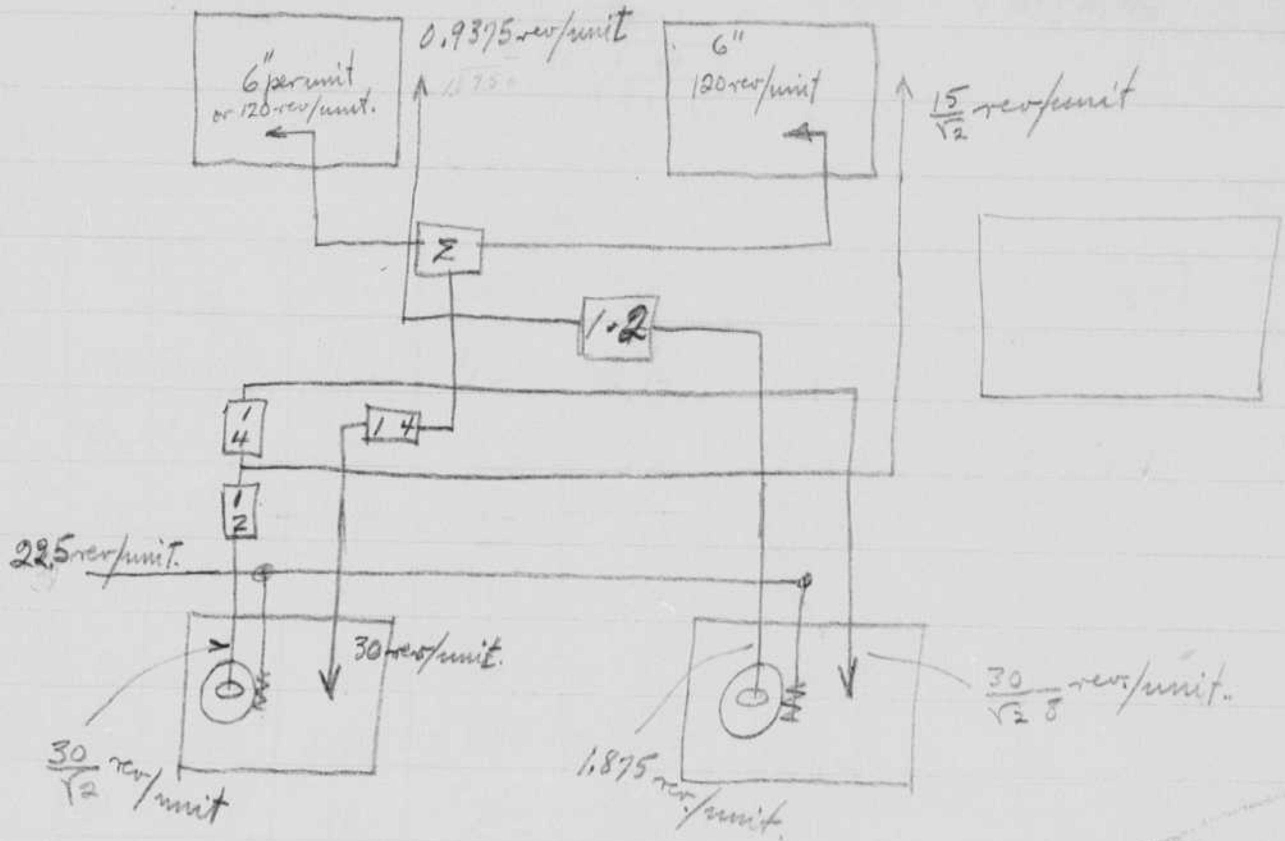
$$D = 32 \frac{32}{30} \frac{\cancel{2}}{\cancel{16}} \frac{\cancel{16}}{\cancel{8}} = \frac{32 \times 32}{30}$$

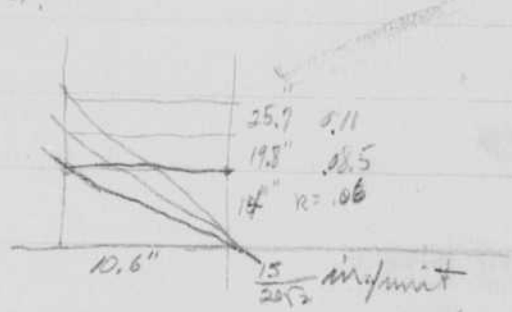
$$n_6 = \frac{32 b}{D a} = \frac{\cancel{32}\ 16\ \cancel{30}}{\cancel{32} \times 32\ \cancel{30}} = \frac{1}{2}$$

When $k = .05$ $\quad k \times \frac{30 \times 32}{2} n_2$

$$.1 \times \frac{30 \times \cancel{32}^{4}}{\cancel{2}} \frac{1}{\cancel{4}} = 12.0 \text{ rev}$$

III.





.53 in. unit

$\frac{P_L}{P_m} = k \frac{d\theta}{dt}$

$14 = .56 \frac{20}{7}$

$$a = \frac{120}{\cancel{NA}} n_6 \qquad c = \sqrt{\frac{ea}{n_1 n_2}}$$

$.9375 \; \frac{9.39}{8.45}$

1.12

$\sqrt{\;} = 1.058$

$$e = \frac{0.9375}{n_5} \qquad c n_1 n_2 = \sqrt{e a n_1 n_2}$$

$$D = 32 \sqrt{\frac{e}{a n_1 n_2}}$$

omit IV

	I	II	III	V	VI	VII	VIII
n_1	1/32	1/2	1/2	1/4	1/4		
n_2		1/4	1/4	1/4	1/4		
n_5	1/8	1/4	1/2	1/2	1/4		
n_6	1/8	1/4	1/4	1/4	1/4		
a	15	30	30	30	30		
e	7.5	3.75	1.875	2.085 ~~1.875~~	3.75		
time of solution D	128	32	$32/\sqrt{2}$	32	$32\sqrt{2}$		
c		30	$30/\sqrt{2}$	30			
Travel of B unit $c n_1 n_2$	1.88	3.75	2.66	1.88	2.66		
Slip Scale in/unit			.53	* 3/8 .375 0.3995			

←2

April 3, 1931

$20 \times \frac{9.39}{180} = 1.043$ rev/unit

$e = \frac{1.043}{1/2} = 2.085$ rev/unit

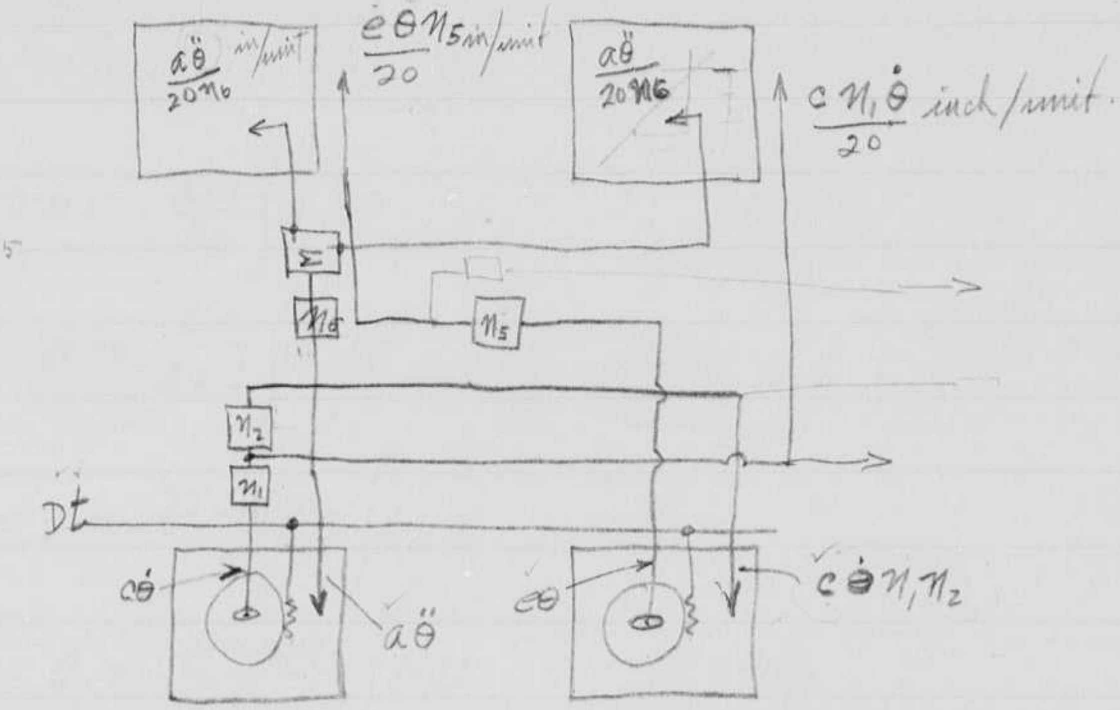
$c = \sqrt{\frac{ea}{n_1 n_2}} = \sqrt{16 \; 2.085 \; 30} = 4\sqrt{62.5}$

$= 4 \sqrt{7.90} = 31.6$ rev/unit

$c \times \frac{1}{4} = 31.6 \frac{1}{4} = 7.9$ rev/unit

$\frac{7.9}{20} = 0.395$ in./unit

$\frac{a}{n_6} = 4 e n_5$



$a\ddot{\theta} \not< 40$ rev.

$c\dot{\theta}\, n_1 n_2 \not< 40$ rev.

$\frac{c n_1 \dot{\theta}}{20} \not< 12'' \pm$

$$\dot{\theta} = \frac{Da}{32e}\int \ddot{\theta}\,dt$$

$$\theta = \frac{c n_1 n_2 D}{32 e}\int \dot{\theta}\,dt$$

$$\theta = \frac{c n_1 n_2 D^2 a}{32^2\, c\, e}\iint \ddot{\theta}\,dt.$$

Conditions for scales is that $\frac{c n_1 n_2 D^2 a}{32^2\, c\, e} = 1.$

Speed of solutions $\quad \ddot{\theta} = \left(\frac{c n_1 n_2 D^2 a}{32^2\, c\, e}\right)\ddot{\theta} = k$

$$\frac{a\ddot{\theta}}{\cancel{20} n_6} = \frac{k\, n_5\, e\theta}{\cancel{20}}$$

$$\ddot{\theta} = n_6 \frac{k\, n_5\, e}{a}\,\theta.$$

$$\ddot{\theta} = \frac{k\, n_5\, e\, n_6}{a}\,\theta.$$

At one point

$$\frac{a\ddot{\theta}}{20\, n_6} = k\, \frac{e\theta n_5}{20}$$

①

$$c\dot{\theta} = \frac{Da}{32}\int \ddot{\theta}\, dt$$

$D = 32\frac{c}{a}$

$$\dot{\theta} = \frac{Da}{32c}\int \ddot{\theta}\, dt \qquad \therefore \quad \frac{Da}{32c} = 1$$

$$e\theta = -\frac{Dc\, n_1 n_2}{32}\int \dot{\theta}\, dt$$

$$\theta = -\frac{Dc\, n_1 n_2}{32e}\int \dot{\theta}\, dt \qquad \therefore \quad \frac{Dc\, n_1 n_2}{32e} = 1$$

$$\theta = -\frac{D^2 a c\, n_1 n_2}{32^2 ce}\iint \ddot{\theta}\, dt^2$$

$$\ddot{\theta} = k\, \frac{e n_5 n_6}{a}\,\theta \qquad k \text{ is geometrical slope}$$

$$\theta = -k\, \frac{D^2 a c\, n_1 n_2}{32^2 ce}\cdot\frac{e n_5 n_6}{a}\iint \theta\, dt^2$$

$$\ddot{\theta} = -k\, \frac{D^2 n_1 n_2 n_5 n_6}{32^2}\,\theta$$

If $\theta = A \sin(\omega t + \alpha)$

$$\omega = \sqrt{k\, \frac{D^2 n_1 n_2 n_5 n_6}{32^2}}$$

T_{sec} for 1 period $= \frac{120\pi}{N} D$

$\frac{60}{N}D$

$T = \frac{2\pi}{\omega}$ in terms of t

T_D in terms of variable (Dt) is D times T

②

$$T_{time} \text{ in seconds} = \frac{\text{sec.}}{\text{rev. of time sh.}} \times \frac{\text{rev}}{\text{unit of } \left.\begin{matrix} Dt \\ t \end{matrix}\right\}} \times \frac{\text{units of } \left.\begin{matrix} Dt \\ t \end{matrix}\right\}}{\text{unit of } \left.\begin{matrix} T_D \\ T \end{matrix}\right\}}$$

In example set up for backlash test

$k = 1 \qquad n_6 = 1 \qquad n_5 = \frac{1}{4}$

$\frac{a}{n_6} = k e n_5 \qquad a = \frac{e}{4}$

$n_1 n_2 = \frac{1}{4}$

$c = \frac{Da}{32} \qquad c = \frac{32 \, e \, 4}{D} = \frac{32 \cdot 16 a}{D}$

$\frac{Da}{32} = \frac{32 \cdot 16 a}{D}$

$D^2 = 32^2 \cdot 16$

$D = 4 \times 32 = 128$

$c = \frac{128 a}{32} = 4a = e$

Time of oscillation in sec. = 88

Speed of time shaft = N rpm $\qquad \frac{\text{sec}}{\text{rev}} = \frac{60}{N}$

$88 = \frac{60}{N} \, 128 \times 2\pi$

$N = \frac{60 \times 128 \times 2\pi}{88} = \frac{60 \times 32\pi}{11} = \frac{1920\pi}{11} = 550$ rpm.

Motor speed = 3.75 × 550

= 2070 r.p.m.

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(3)

Edgerton's problem (New ratios (II))

$$\frac{e\,n_5}{20} = \frac{0.9375}{20} = 0.046875 \text{ in/deg.}$$

$\checkmark\; e = 4 \times 0.9375 = 3.75$ rev/deg @ output of #2 integ.

$\frac{a}{20\,n_6} = 6$ inches per unit torque

$n_6 = \frac{1}{4}$ $\quad\checkmark\; a = \frac{20 \times 6}{4} = 30$ rev of #1 lead sc. /unit torque

$$\frac{D c\, n_1 n_2}{32\, e} = 1 \qquad n_1 n_2 = \frac{1}{8}$$

$$D = \frac{32\, e}{n_1 n_2 c} = \frac{32 \times 3.75 \times 8}{c} = \frac{256 \times 3.75}{c}$$

$$= \frac{960}{c}$$

$$D = \frac{32\, c}{a} = \frac{32}{30} c \qquad c = \frac{30}{32} D$$

$$D = \frac{960}{\frac{30}{32} D} \qquad D^2 = \frac{32 \times 256 \times 3.75}{30}$$

$$D^2 = \frac{2^{13} \times 3.75}{30} = \frac{2^{12} \times 3.75}{15} = 2^{12} \times 0.25$$

$$= 2^{10}$$

$\checkmark\; D = 32$

$\checkmark\; c = 30$

Try in $c\,\dot{\theta}\,n_1 n_2 \leqq 40$ $\qquad \dot{e} = \frac{30}{8}\dot{\theta} = \frac{30 \times 16}{8} = 60$

Edgerton's problem

Take $e = 0.9375 \times 4 = 3.75$ rev./degree of θ on input

corresponds to input table $\angle$ scale 0.1

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New layout of ratios (II)

(4)

$$\omega^2 = k\ \frac{D^2 n_1 n_2 n_5 n_6}{32^2} = 1\ \frac{32^2}{32^2 \times 8 \times 4 \times 4} = \frac{1}{128} = \frac{1}{8^2 \cdot 2}$$

$$T = 2\pi 8\sqrt{2} = 16\sqrt{2}\pi = 71.\ \text{units of } t$$

$= 71 \times 32$ revolutions

$= 2270$

$T = \frac{120\pi}{N} 32$

$\frac{2270}{400} = 5.67$ min.

Old ratios (I) $\quad n_6 = \frac{1}{8} \quad n_5 = \frac{1}{8}$

$n_1 n_2 = \frac{1}{32}$

$$\omega^2 = 1\ \frac{128^2}{32 \times 8 \times 8 \cdot 32^2} = \frac{2^{14}}{2^{15+6}} = 2^{-7} = \frac{1}{128}$$

$$\omega = \frac{1}{8\sqrt{2}}$$

$$T = 2\pi 8\sqrt{2} = \underset{71}{\underline{16\sqrt{2}\pi}}\ \text{units of } t$$

$= 71 \times 128$ revolutions of t shaft

Time in ~~min.~~ $= \frac{71 \times 128}{400} = \frac{71 \times 32}{100} = 22.7$ min.

$$T = \frac{120\pi}{N} 128$$

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Scales III $\left(c_{III} = \frac{c_{II}}{\sqrt{2}}\right)$ (5)

$$\frac{Da}{32c} = 1 \qquad \frac{32 \times 30}{32 \times 30} = 1$$

$$\frac{D c n_1 n_2}{32 e} = 1 \qquad e n_5 = 0.9375$$

$$e = \frac{0.9375}{n_5}$$

✓ Check on new values

$$\frac{D c n_1 n_2 n_5}{32 \times 0.9375} = 1 \qquad \frac{\cancel{32} \times 30}{\cancel{32} \times 0.9375 \times 8 \times 4} = 1$$

$$D = \frac{32c}{a}$$

$$n_1 = \frac{1}{2}$$

$$\frac{32c}{a} \cdot \frac{c n_1 n_2 n_5}{32 \times 0.9375} = 1 \qquad n_2 = 1/4$$

Take $c = \frac{30}{\sqrt{2}}$ find n_5

$$\frac{32}{30} \frac{30^2}{2} \frac{1}{8} \frac{n_5}{32 \times 0.9375} = 1$$

$$n_5 = \frac{\cancel{30} \times 2 \times 8 \times \cancel{32} \times 0.9375}{\cancel{32} \times \cancel{30^2}} = \frac{1}{2}$$

$$\omega^2 = k \frac{D^2 n_1 n_2 n_5 n_6}{32^2} = k \frac{\cancel{32^2} c^2 \cdot n_1 n_2 n_5 n_6}{\cancel{32^2} a^2}$$

$$D = \frac{32 \cdot 30}{30 \cdot \sqrt{2}} = \frac{32}{\sqrt{2}}$$

$$T = \frac{120\pi}{N} \frac{32}{\sqrt{2}}$$

$$\frac{22.5 \;\; 30\sqrt{2}}{32 \;\; 30} = 1 \;✓$$

$$\frac{22.5 \;\; \cancel{30}}{32 \;\; \sqrt{2}} \frac{1}{8} \frac{1}{1.875} = \frac{673.}{670} = 1.0 \;✓$$

$c n_1 n_2 \dot{\theta} < 40$ rev.

$$\frac{30}{\sqrt{2}} \; \frac{1}{8} \; 17$$

6

$$\frac{32 \cdot 30^2}{32 \times 8 \times 4 \times 0.9375} \approx 1$$

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(6)

Scales IV

Take $n_1 n_2 = \frac{1}{16}$ $\qquad c = 30\sqrt{2}$

$$\frac{32c}{a} \cdot \frac{c n_1 n_2 n_5}{32 \times 0.9375} = 1$$

$$\frac{32 \cdot 30^2 \cdot 2 \qquad n_5}{30 \quad \times 32 \times 0.9375 \times 16} = 1$$

$$n_5 = \frac{\cancel{30} \times \cancel{32} \times \cancel{16}^{8} \times 0.9375}{\cancel{32} \times 30^2 \times \cancel{2}} = \frac{7.5}{30} = \frac{1}{4}$$

$$\omega^2 = k \frac{c^2 n_1 n_2 n_5 n_6}{a^2} \qquad D = \frac{32c}{a} = \frac{32 \cdot 30\sqrt{2}}{30} = 32\sqrt{2}$$

$$T_{sec} = \frac{60}{N} D \, 2\pi = \frac{120\pi D}{N}$$

$$= \frac{120\pi}{N} 32\sqrt{2}$$

$c n_1 n_2 6$

$\frac{30\sqrt{2}}{1} \frac{1}{16} 17 = 45$

$\frac{d\theta}{dt}\,dt.$

$$\theta = \int \left[\int P_1\,dt - \int k(1 - b\cos 2\theta)\,\dot{\theta}\,dt \right] dt.$$

$\frac{d\theta}{dt}$

θ

$\dot{\theta}$

$\int P_1\,dt$

$k(1 - b\cos 2\theta)$

$\int k(1 - b\cos 2\theta)\,\dot{\theta}\,dt$

$\Sigma \quad \int P_1\,dt - \int k(1 - b\cos 2\theta)\,\dot{\theta}\,dt$

θ

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Apr 5
1.

$\ddot{\theta}$ max is about 2 or less.

so $a < \frac{40}{2} = 20$ $\therefore$ let $a = \underline{16}$

Max for e units = 0.2

$e \times 0.2 < 40$ $e < \frac{40}{0.2} = 200.$ Let $e = \underline{128}$

$\dot{\theta}$ max is about 20

$b\dot{\theta} < 40$ $b < \frac{40}{20} = 2.$ let $b = 2.$

1 $1 = \frac{n_3 n_6 a D}{32 b} = \frac{n_3 n_6 16 D}{32 \times 2} = \boxed{\frac{n_3 n_6 D}{4}}$

2. $1 = \frac{n_7 e c}{b\, 32} = \frac{n_7 128 c}{2\ 32} = \boxed{2 n_7 c.}$

3. $1 = \frac{n_4 n_5 b D}{32 e} = \frac{n_4 n_5\, 2 D}{32\ e} = \boxed{\frac{n_4 n_5}{16} \frac{D}{c}}$

n_3
n_6
n_7
n_4
n_5
D
c

I Try $n_5 = 1$ $n_6 = 1$ $n_3 = 1/4$ $n_7 = 1/4$

1. $1 = \frac{n_3 n_6 D}{4} = \frac{1}{4}\frac{1 D}{4}$ $D = 16.$

$n_5 = \frac{16 c}{D n_4}$

$= \frac{16 \cdot 2 \cdot 4}{64}$ 2

2. $1 = 2 n_7 c = 2 \frac{1}{4} c$ $c = 2.$

3. $1 = \frac{n_4 n_5}{16}\frac{D}{c} = \frac{n_4}{16}\frac{1\ 16}{2}$ $n_4 = 2.$ but $n_4 \leqq \frac{1}{2}.$

II try $n_4 = 1/2$ which gives from 3 $\frac{D}{c} = \frac{16 \times 2}{n_5}$

Let $n_3 = n_7 = 1/4$ (2) $c = \frac{1}{2 n_7}$

$n_6 = 1/4$ $D = \frac{4}{n_3 n_6}$

$D = \frac{4}{\frac{1}{4}\frac{1}{4}} = 16 \times 4 = 64$

$c = \frac{1}{2 \cdot 1/4} = 2.$

$n_5 = \frac{c}{D}\frac{16 \times 2}{} = \frac{2}{64} 16 \times 2 = 1$ 2.

D
c
n_5
n_6
n_7 —
n_3 — 3

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III try $n_4 = 1/2$ (same as II)

Let $n_3 = n_7 = 1/2$

$n_6 = 1/2$

$c = \frac{1}{2 \frac{1}{2}} = 1$

$D = \frac{4}{\frac{1}{2}\frac{1}{2}} = 16.$

$n_5 = \frac{32}{D} c = \frac{32 \cdot 1}{16} = 2.$

NG

IV try $n_4 = 1/\cancel{4}$

try $n_3 = n_7 = 1/4$

$n_6 = 1$

$\frac{n_3 n_6 D}{4} = 1$

$2 n_7 c = 1$

$\frac{1 \; n_5}{4 \times 16} \frac{D}{c} = 1$

$D = \frac{4}{\frac{1}{4} 1} = 16 \quad c = \frac{1}{2 \frac{1}{4}} = 2$

$n_5 = 64 \frac{c}{D} = 64 \frac{2}{16} = 8.$ NG

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check on case II

n_1

n_2

n_3 1/4

n_4 1/4

n_5 1

n_6 1/4

n_7 1/4

a 16

b 2

c 2

e 128

D 64

1. $1 = \frac{n_3 n_6 a D}{32 b} = \frac{1/4 \; 1/4 \; 16 \; 64}{32 \times 2} = 1$ ✓

2. $1 = \frac{n_7 e c}{b \, 32} = \frac{1/4 \; 128 \times 2}{2 \times 32} = 1$ ✓

3. $1 = \frac{n_4 n_5 b D}{32 c} = \frac{1/4 \; 1 \; 2 \; 64}{32 \times 2} = 2$ ✗

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$\frac{400}{20}$ = 20 in min rev/min. 3

1.043

~~$\frac{c}{2}$ = 1.043 rev/unit~~

~~c = $\frac{1.043}{2}$ = .5~~

V

$n_4 = \frac{1}{2}$ 1.056

$\frac{c}{n_4} = c \times 2 = \underline{\underline{2.086}}$ rev/unit.

c = 1.043 rev./unit

Quantities determined

c = 1.043 rev/unit.

a = 16 rev

e = 128 rev

b = 2 rev.

1. $1. = \frac{n_3 n_6 aD}{32b} = \frac{n_3 n_6 16 D}{32 \cdot 2} = \frac{n_3 n_6 D}{4}$

2 $1. = \frac{n_7 e c}{b 32} = \frac{n_7 128 \, 1.043}{32 b} = \frac{4 n_7 1.043}{2.}$

$n_7 = \frac{2}{4 \; 1.043} = \frac{1}{2} \frac{1}{1.043}$

Replot chart so that c = 1.0

Let $n_8 = 1$ c = 1.0 rev per unit. (elect deg)

or $\frac{1.0 \times 18\pi}{2\pi}$ = 9 inches for 180 elect degrees.

Sent to Hayden Lab

4.

VI Quantities determined

$c = 1.0$

$a = 16$

$e = 128$

$b = 2$

1. $1 = \frac{n_3 n_6 aD}{32b} = \frac{n_3 n_6 \cancel{16} D}{32_2 \; 2} = \frac{n_3 n_6 D}{4}$

2. $1 = \frac{n_7 ec}{b32} = \frac{n_7 \, 128 \times 1.}{\cancel{2} \; \cancel{32}} = 2n_7.$

$$\boxed{n_7 = 1/2}$$

3. $1 = \frac{n_4 n_5 bD}{32c} = \frac{n_4 n_5 2D}{32 \times 1} = \frac{n_4 n_5 D}{16}$

(5 unknowns) $n_3 \; n_4 \; n_5 \; n_6 \; D$ (two equa)

Let $n_3 = 1/2$. $n_4 = 1/2$ $n_5 = 1$

from 3 $1 = \frac{1/2 \; 1}{16} D$ $D = 16 \times 2 = 32.$

from 1 $1 = \frac{\frac{1}{2} n_6 \, 32}{4} = 4n_6$

$n_6 = 1/4.$

Retabulation

n_1	1/8
n_2	1/4
n_3	1/2
n_4	1/2
n_5	1
n_6	1/4
n_7	1/2
a	16
b	2
c	1
D	32
e	128

for $\frac{a\theta}{20n_1} = \frac{16 \times 8}{20} = 6.4$ in per unit

$\frac{e}{n_2} = \frac{128}{20 \times 1/8} = \frac{128 \times 8}{20} = 51.2$ in/unit.

VII For cases where $k = 0.05$ as max.

Then $e = 256$.

a remains 16

b 2

c 1.

Unit equations $1 = \frac{n_3 n_6 a D}{32 b} = \frac{n_3 n_6 \cancel{16} D}{\cancel{32}_2 \times 2} = \frac{n_3 n_6 D}{4}$

$n_7 = 1/4$. $1 = \frac{n_7 e c}{b\, 32} = \frac{n_7 \overset{\cancel{128}\,4}{\cancel{256}}\, 1}{\cancel{2} \times \cancel{32}} = n_7 4$

$1 = \frac{n_4 n_5 b D}{32 c} = \frac{n_5 n_4 2 D}{32 \times 1} = \frac{n_5 n_4 D}{16}$

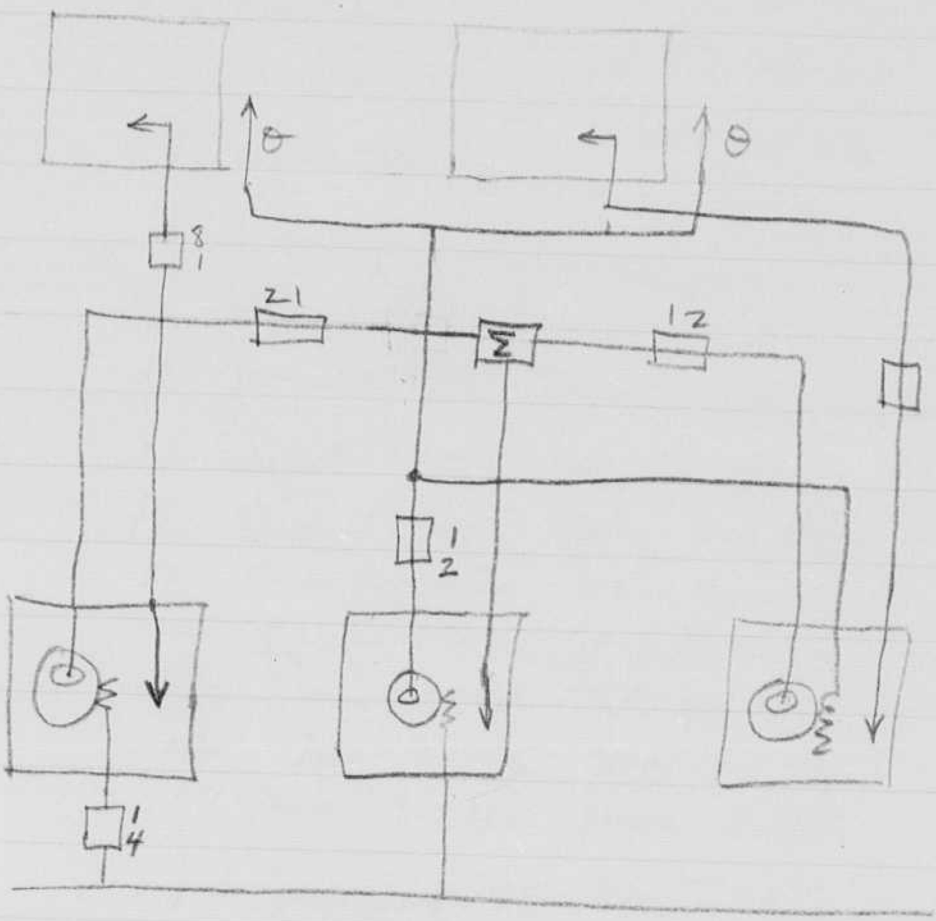
$\cancel{P_j}\frac{ds}{dt} + \frac{\cancel{P_j}P_d}{P_j}(1 - b\cos 2\theta) s = \frac{P_L}{P_j'}$

$s = \cancel{f\theta\,dt}\ \frac{d\theta}{dt}$

$\frac{d^2\theta}{dt^2} + \frac{P_d}{P_j}(1 - b\cos 2\theta)\frac{d\theta}{dt} = \frac{P_L}{P_j'}$

$A\frac{d^2q}{dt^2} \cancel{+} + \frac{R}{L}\frac{dq}{dt} = \frac{E}{L}$

Sent to Hayden Lab



Sent to Hayden Lab

Computation

Apr 27 1981

#3

N.B. 3 Feb 15 30 June 16 31

T-II July 20 31 Jan 12 32

T-5 Oct 27 34 Aug 27 35

T-6 Aug 27 35 - Apr 25 36

7 Apr 28 36 - May 27 37

8 June 1 37 April 16 37

9 Apr 18 38 June 12 39

10 June 13 39 Sept 17 40

11 Sept 17 40 Dec 3 41

12 Dec 4 41 Aug 24 42

13 Aug 24 42 Mar 31 43

14 Apr 1 43 Jan 30 44

15 Jan 30 44 Feb 16 45

16 Feb 17 45 Mar 30 46

17 Mar 30 46 June 18 48

1

19 June 18 48 Feb 7 1950

EG&G Dec 8 48 April 8, 1951

Underwater Photography Experiments U.P. Aug 4 52 Oct 19 1952

20 Feb 7 50 Dec 27 51

21 Dec 27 51 Jan 9 54

22 Jan 9 54 Apr 19 55

23 Apr 19 55 Dec 19 56

24 Dec 19 56 Apr 29 58

25 Apr 29 58 May 14 60

26 May 14 60 Jan 18 62

27 Jan 18 62 Nov 18 63

28 Nov 18 63 May 30 65

29 May 30 65 July 11 69

30 Oct 2 69 Jan 11 73

31 Jan 11 73 Aug 17 75

Ken Germeshausen Book 1 10-15-31, Jan-14-31

(small, red composition Book) over.

H. Edgerton Jan 21 1967 (white comp'n book)

H. Edgerton June 29 1947 H Bond unit

small composition book (B.W.)

HAROLD E. EDGERTON
52 MASS. AVE.
CAMBRIDGE MASS.
M.I.T. ROOM 4-210
FEB. 15, 1930.

SELF-HUNTING OF SYNCHRONOUS MACHINES.

P102-105 Strobs.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

COMPUTATION BOOK

GENERAL INSTRUCTIONS

In all work in which *accuracy* and *ease of reference* are important, much depends upon carrying out the computation in a systematic manner. The following instructions, taken from the *Engineering Department Figuring Book of the Allis-Chalmers Co., serve as a guide in this matter.*

"All computations, of whatever kind, are to be made in these books, except in cases where special blanks may be provided for specific kinds of computation. Computations may be made in ink or pencil, whichever may be more convenient. Pencil figuring should be done with a soft pencil. All the work of computation should be done in these books, including all detail figuring."

"Each subject should begin on a new page, no matter how much space may be left on the previous page. The subject, with the date of beginning it, should be plainly written at the top of the first page of the subject."

"Work should be done systematically, and as neatly as consistent with rapidity. The books are, however, intended for convenience, and no unnecessary work should be done for sake of appearance only. Errors should be crossed off instead of erased, except where the latter will facilitate the work. Work should not be crowded. Paper costs less than the time which would be expended in attempting to economize space in making erasures."

"Where curves drawn on section paper (or sketches) are necessary parts of a computation, they should be pasted in the book, except where specifically otherwise provided for."

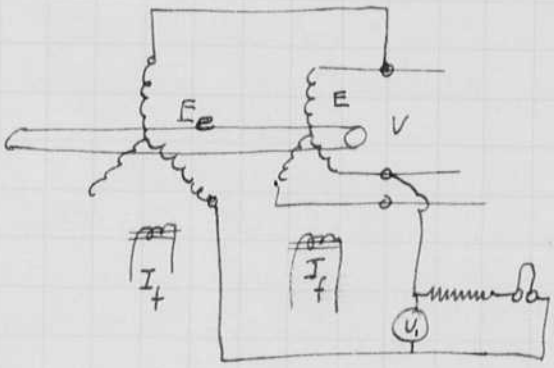
"Computations should be indexed, in the back of the book, by the person using the book."
* * * * * * *"

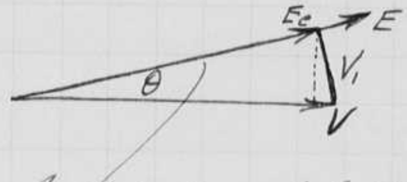
Harold E. Edgerton

Feb. 15, 1930
H. E. Edgerton

Oscillographic Measurement of the Angular Displacement in a Syn. Machine.

An extra synchronous machine on the shaft of a machine being tested gives a voltage which ~~is in~~ has a definite relationship with the induced emf. E of that machine ~~being tested since~~ since the field pole have a rigid relationship. The open-circuit voltage from this extra machine is combined vectorially with the terminal~~al~~ voltage of the machine under test and a trigonometric 3 voltage problem gives the angular displacement.



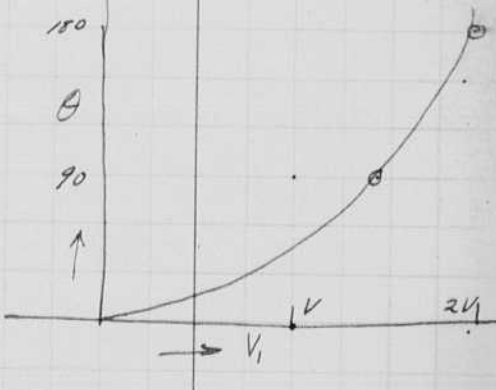


field poles of both machines in line.

If E_e and V are not equal then $\theta = \cos^{-1} \frac{V^2 + E_e^2 - V_1^2}{2 E_e V}$

When E_e and V are equal then

$$\theta = \cos^{-1} \frac{2V^2 - V_1}{2V^2} = \underline{\cos^{-1} 1 - \frac{V_1^2}{2V^2}}$$



This method has been used in the past with great success.

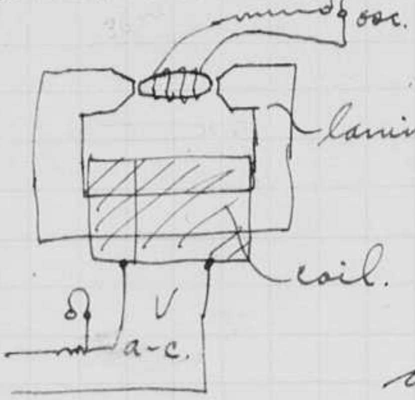
When the rotor is slipping, for instance during a transient following a sudden load, then the length of E_e depends directly upon the speed and some small errors are introduced. The expression for angle is then

$\theta = \cos^{-1} \frac{V^2 + (1-s)^2 E_e^2 - V_1^2}{2 E_e V (1-s)}$ where s is the slip. The slip is usually less than 2% so this is ordinarily neglected.

Feb 15 1930
H. Edgerton

Angle Measurement.

I had Mr. Kershaw build me a device which I hoped would aid in measuring angle with the oscillograph. This was to be a variable reluctance motor, its stator supplied by a.c. from the terminals, and its rotor circuit to be connected to an oscillograph vibrator. A sketch is shown of the machine. It was driven at 3600 r.p.m. by a 2:1 gear on a 1800 r.p.m. set in the dynamo laboratory (Sine wave generator set 99 a.b.c.).



Needless to say the output voltage of the rotor was of a very peculiar shape. The current input was also interesting and offers some possibility of future development. The current to the exciting a.c. coil has a nick in it every time that the eccentric rotor lines up with the pole faces. If the speed is constant this will occur at the <u>same place</u> on the current wave. Otherwise it will occur at different places and its position will depend upon the relative positions of the rotor and the terminal voltage.

Trouble was experienced with the copper brushes that were used and finally caused the experiment to be discontinued after four osc. were taken.

Data for osc.

Osc. 1. Chattering brushes on rotor gave poor record. Film also too slow.

Osc. 2. Poor Focus. Wave interfere. Too slow drum speed.

Osc. 3. O.K. on page 5.

Osc. 4. Focus poor.

Angle voltage V, as explained on page 3 is shown on all osc.

Notebook # 3

Filming and Separation Record

1 unmounted photograph(s)

___ negative strip(s)

___ unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 4 and 5.

Item(s) now housed in accompanying folder.

Angle Measurement.

Feb 15 1930
H. Edgerton

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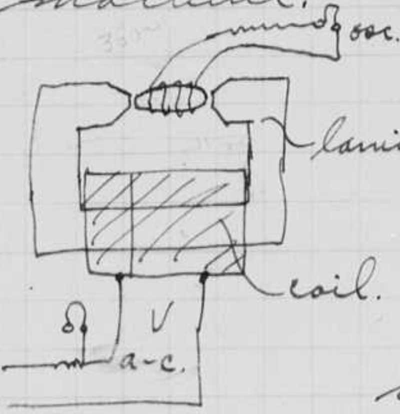

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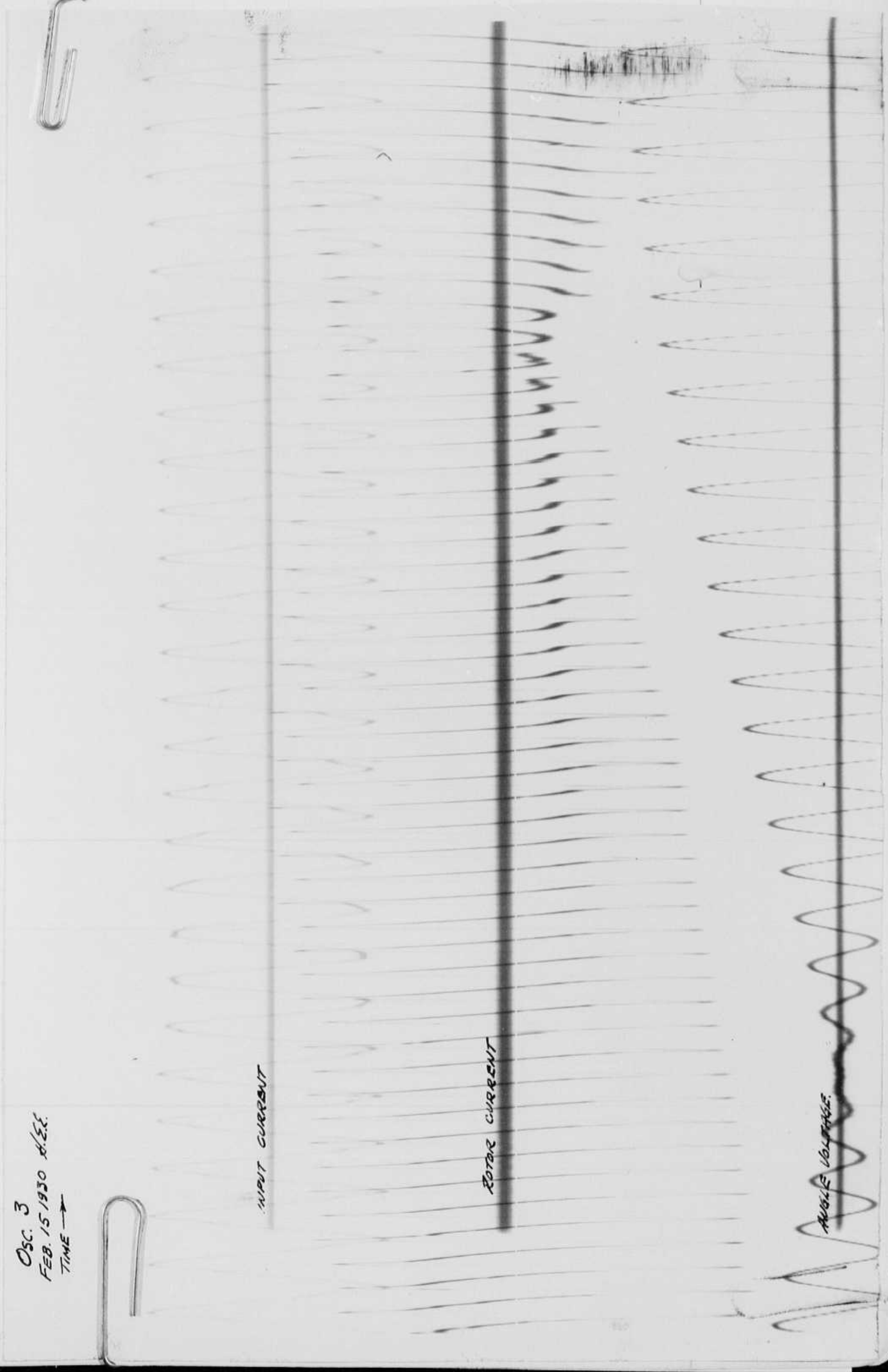
Angle voltage V, as explained on page 3 is shown on all osc.


Osc. 3
FEB. 15 1930 H.E.E.
TIME →
INPUT CURRENT
ROTOR CURRENT
ANGLE VOLTAGE

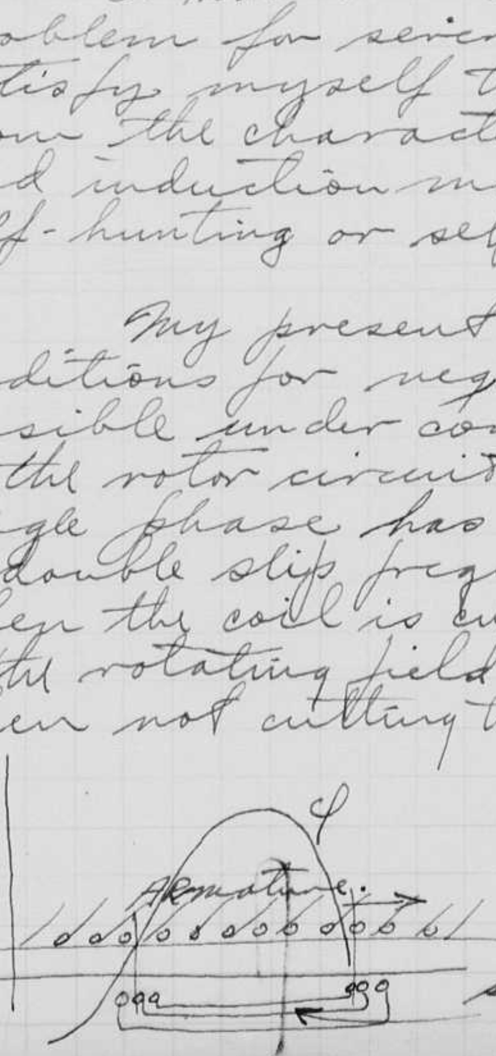
Feb. 21, 1930
H. E. Edgerton.

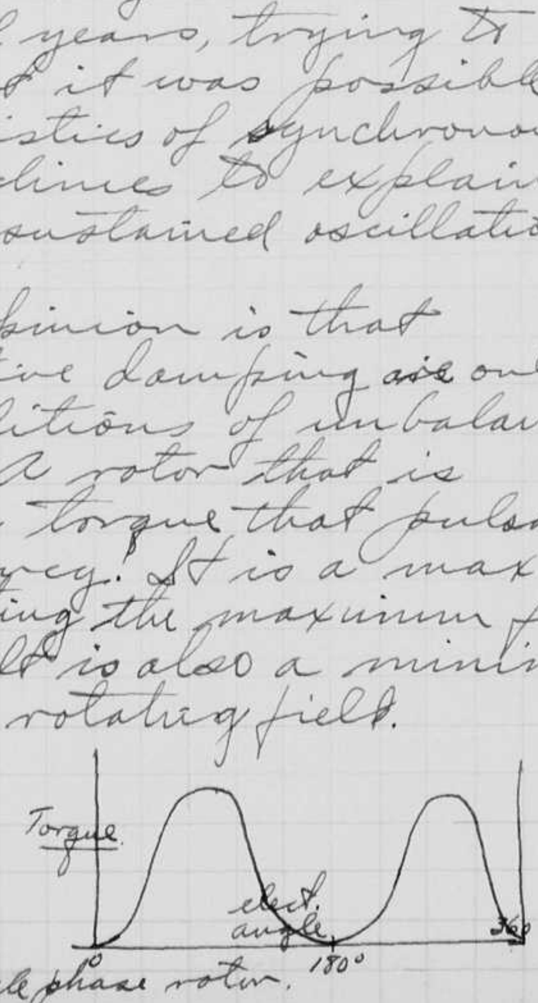
Source of Energy for Self Oscillations of Synchronous Machines.

A recent paper by Nickle and Pierce in the A. I. E. E. blames self hunting ~~on~~ armature resistance. The unbalance of the rotor circuit is also considered and it is noted that pole face damping windings in the quadrature axis suppresses the possibility of self-oscillation. No physical picture was given in this paper of the mechanism whereby ~~a~~ a synchronous machine should hunt accumulatively. Our common reasoning powers tell that if a winding on the rotor is oscillated back and forth during an oscillation, the induced currents from Len's law always oppose any change of position. Such an action would tend to reduce oscillations.

I have been thinking about this problem for several years, trying to satisfy myself that it was possible from the characteristics of synchronous and induction machines to explain self-hunting or self sustained oscillations.

My present opinion is that conditions for negative damping are only possible under conditions of unbalance on the rotor circuit. A rotor that is single phase has a torque that pulsates at double slip frequency. It is a maximum when the coil is cutting the maximum flux of the rotating field. It is also a minimum when not cutting the rotating field.





single phase rotor.

As shown in the sketch on the preceding page this torque due to a single phase winding is a minimum at zero angular displacement. If the currents in the single phase rotor effect the rotating field, ϕ, then this field can be represented by two oppositely rotating fields. This is necessary since the flux of the rotating field is pulsating.

The positively rotating field reacts with the field winding and gives positive damping acording to some law as the $\sin^2\theta$ as a function of angle and directly proportional to the slip for small values.

The negative field however is in the opposite sense always to the positive and due to ~~#~~ circuit requirements does not have its maximum at the same time the positive field lines up with the coil. This being true the negative field at small angles is in such a direction as to give torque which appears to push instead of pull as a function of slip. This causes oscillations to build up. They will continue to increase in magnitude until large enough angles are reached in the course of the swinging where the positive damping again occurs.

Mechanical-Electrical Torque Equation.

Feb 23 1930
H. E. Edgerton

The swinging of a synchronous machine is always in accord with the differential equation that states the sum of the various mechanical and electrical torques that exist during the disturbance. This equation is of the form:

$$P_j \frac{d^2\theta}{dt^2} + f(\theta)\cdot F\left(\frac{d\theta}{dt}\right) + f_a(\theta) = \text{load torque.}$$

In an ideal machine with negligible armature resistance, a smooth rotor, and for small values of slip, this reduces to approximately the following:

$$P_j \frac{d^2\theta}{dt^2} + P_d \frac{d\theta}{dt} + P_m \sin\theta = \text{load torque}.$$

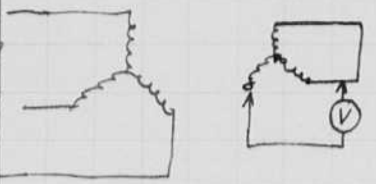
The term $P_d \frac{d\theta}{dt}$ is such that it always opposes any force that tries to change the speed of the machine from synchronism. This effect then in a machine with a balanced rotor circuit always tends to damp out oscillations. Such damping is termed by definatin as being positive.

In order to get self-oscillations this force must have a negative sense some of the time since there can be no source of oscillation in the synchronizing or inertia terms of the differential equations. Dreyfus points out this fact. A rotor with a single phase winding will give this condition for small angular displacements.

Induction Motor with a Single-Phase Rotor.

Feb 23 1930
H. E. Edgerton

Balanced 3φ voltage supply V volts/ph.



From the open phase of the rotor to the short circuited pair there will be a voltage of slip frequency. This may be split up into its positive and negative sequence components by Fortescue's method of symmetrical phase components.

Since the current in the open circuited phase is zero the sum of the positive and negative sequence currents are zero in this phase and thus equal to each other in magnitude but 180° in phase

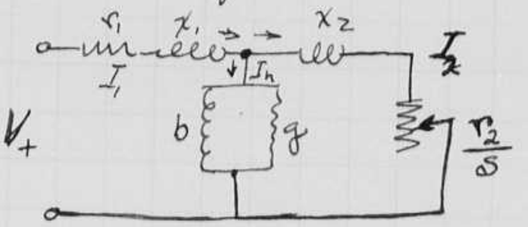
$I = I_{2+} + I_{2-} = 0$ where the 2 refers to the rotor circuit.

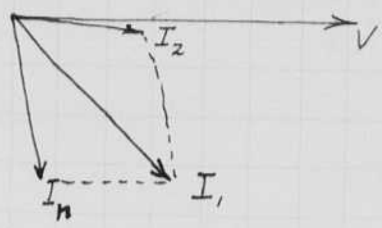
I think that this problem can be best analysed by considering that the stator has two components of voltage applied to it. Each will be polyphase and balanced. One will be the impressed emf. V. the other will depend upon the slip at ~~to~~ which the rotor is considered. It will be of such a value that the rotor current in one phase is zero. Such a requirement states that this fictitional voltage shall have a freq. of $(1-2S)f$ where S is the slip.

The stator current will be the sum of the two current components and since they are at different frequencies the total stator current will contain beats which occur in the different phases in rotation. I know this to be the case since I have observed it in the laboratory, both with meters and with the oscillograph.

Feb 23 1930
H. E. Edgerton

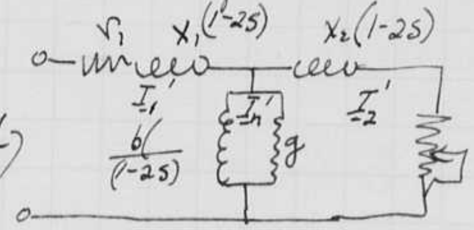
Equivalent Circuit for impressed voltage V.





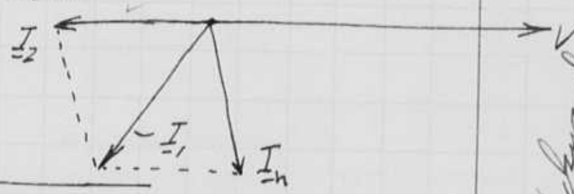
$$I_{+2} = \frac{(V_+ - I_n Z_+)}{r_1 + \frac{r_2}{s} + j(x_2 + x_1)}$$

Equivalent circuit for 1-2s voltage



Generator action for slips from 0 to .5. Motor then on down to slip of 1. or stand still.

frequency of (1-2s) times that of V_+



$$I_{-2}' = \frac{V_- - I_{-n} Z_-}{r_1 - \frac{r_2 s}{1-2s} + j\left[x_2(1-2s) + x_1(1-2s)\right]}$$

These two currents in the rotor winding are to be equal and opposite in one phase that is open.

$I_{-2} = I_{+2}$ from the above equations we can find V_- in terms of V_+ and the slip.

$$V_- - I_{-n} Z_- = \frac{Z_{-T}(V_+ - I_{+n} Z_+)}{Z_{+T}} = (V_+ - I_{+n} Z_+) \frac{\overbrace{\left(r_1 - \frac{r_2 s}{1-2s}\right) + j(1-2s)(x_1 + x_2)}^{= Z_{-T}}}{\underbrace{\left(r_1 + \frac{r_2}{s}\right) + j(x_1 + x_2)}_{= Z_{+T}}}$$

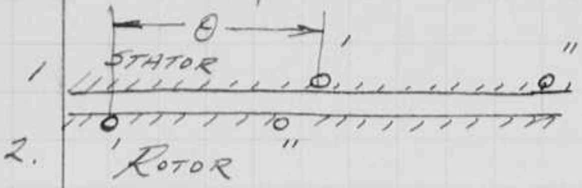
$$\underline{\underline{V_- = (V_+ - I_{+n} Z_+) \frac{Z_{-T}}{Z_{+T}} + I_{-n} Z_-}}$$

$Z_+ = r_1 + j x_1$
$Z_- = r_1 + j x_1(1-2s)$

This scheme is not so good. The voltage for the backward flux component should be found in the rotor. Feb. 24, 1930.

Differential equation of a two phase machine and the reduction of these equations to a simple form.

Feb 24 1930
H. E. Edgerton



Sinusoidal flux is assumed in this treatment. The air gap is also uniform and the iron has a constant permeability.

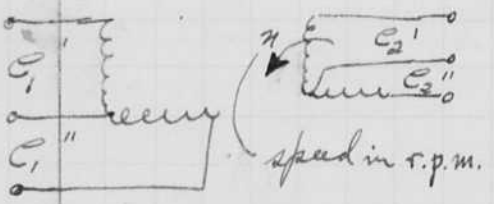
Voltage drops around phase one of the stator.

$$e_1' = r_1 i_1' + L_1 p i_1'' + M p\, i_2' \cos\theta + M p\, i_2'' \cos\theta + \tfrac{\pi}{2} \qquad (1)$$

phase 2. $$e_1'' = r_1 i_1'' + L_1 p\, i_1'' + M p\, i_2'' \cos\theta + M p\, i_2' \cos\theta - \tfrac{\pi}{2} \qquad (2)$$

Rotor ph. 1. $$e_2' = r_2 i_2' + L_2 p\, i_2' + M p\, i_1' M\cos\theta + M p\, i_1'' M\cos\theta - \pi/2 \qquad (3)$$

ph 2. $$e_2'' = r_2 i_2'' + L_2 p\, i_2'' + M p\, i_1'' M\cos\theta + M p\, i_1' M\cos\theta + \pi/2 \qquad (4)$$



These may be reduced from four equations and four unknowns to two equations, two unknowns by splitting the currents into components.

In order to do this let:

$$i_1' = i_{+1} + i_{-1} \qquad i_2' = i_{+2} + i_{-2} \qquad e_1' = e_{+1} + e_{-1}$$

and $$i_1'' = -j\, i_{+1} + j\, i_{-1} \qquad i_2'' = -j\, i_{+2} + j\, i_{-2}. \qquad e_1'' = -j e_{+1} + j e_{-1}$$

$$e_2' = e_{+2} + e_{-2}$$
$$e_2'' = -j e_{+2} + j e_{-2}$$

This change of variables gives the differential equations a new form wherein there are only ~~...~~ two variables and two equations

$$e_{+1} = (r_1 + L_1 p)\, i_{+1}' + M p\, i_{+2}'\, \varepsilon^{j\theta}$$
$$e_{-1} = (r_1 + L_1 p)\, i_{-1}' + M p\, i_{-2}'\, \varepsilon^{-j\theta}$$

$$e_{+2} = (r_2 + L_2 p)\, i_{+2}' + M p\, i_{+1}'\, \varepsilon^{-j\theta}$$

$$e_{-2} = (r_2 + L_2 p)\, i_{-2}' + M p\, i_{-1}'\, \varepsilon^{j\theta}$$

Notice the change of sign for the stator and rotor equations is only in the exponent.

Preliminary proof Subject to revision

EDGERTON

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

____ negative strip(s)

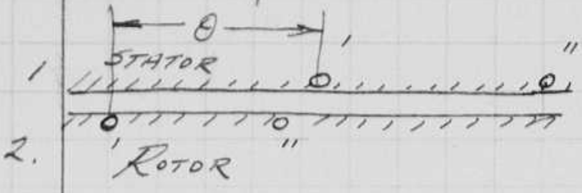
13 unmounted page(s) 12 page reprint with one page
(notes, drawings, letters, etc.) of notes inserted

was/were filmed where originally located between page 12 and 13.

Item(s) now housed in accompanying folder.

Differential equation of a two phase machine and the reduction of these equations to a simple form.

Feb 24 1930
J. E. Edgerton



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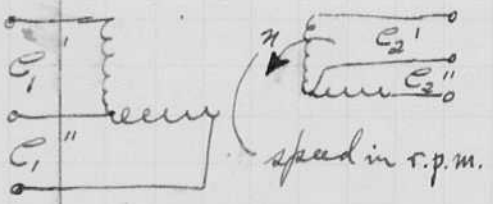
ph. 2. $$e_2'' = r_2 i_2'' + L_2 p i_2'' + M p\, i_1'' M\cos\theta + M p\, i_1' M\cos\theta + \pi/2. \qquad (4)$$



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This change of variables gives the differential equations a new form wherein there are only ~~two~~ two variables and two equations

$$e_{+1} = (r_1 + L_1 p) i_{+1}' + M p\, i_{+2}' \varepsilon^{j\theta}$$
$$e_{-1} = (r_1 + L_1 p) i_{-1}' + M p\, i_{-2}' \varepsilon^{-j\theta}$$

$$e_{+2} = (r_2 + L_2 p) i_{+2}' + M p\, i_{+1}' \varepsilon^{-j\theta}$$

$$e_{-2} = (r_2 + L_2 p) i_{-2}' + M p\, i_{-1}' \varepsilon^{j\theta}$$

Notice the change of sign for the stator and rotor equations is only in the exponent.

EDGERTON

Preliminary proof   Subject to revision

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

____ negative strip(s)

13 unmounted page(s) 12 page reprint with one page
(notes, drawings, letters, etc.) of notes inserted

was/were filmed where originally located between page 12 and 13.

Item(s) now housed in accompanying folder.

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Preliminary proof. Subject to revision.

Two-Reaction Theory of Synchronous Machines

Generalized Method of Analysis—Part I

BY R. H. PARK*

Associate, A. I. E. E.

***Synopsis.**—Starting with the basic assumption of no saturation or hysteresis, and with distribution of armature phase m. m. f. effectively sinusoidal as far as regards phenomena dependent upon rotor position, general formulas are developed for current, voltage, power, and torque under steady and transient load conditions. Special detailed formulas are also developed which permit the determination of current and torque on three-phase short circuit, during starting, and when only small deviations from an average operating angle are involved.*

In addition, new and more accurate equivalent circuits are developed for synchronous and asynchronous machines operating in parallel, and the domain of validity of such circuits is established.

Throughout, the treatment has been generalized to include salient poles and an arbitrary number of rotor circuits. The analysis is thus adapted to machines equipped with field pole collars, or with amortisseur windings of any arbitrary construction.

It is proposed to continue the analysis in a subsequent paper.

* * * * *

THIS paper presents a generalization and extension of the work of Blondel, Dreyfus, and Doherty and Nickle, and establishes new and general methods of calculating current power and torque in salient and non-salient pole synchronous machines, under both transient and steady load conditions.

Attention is restricted to symmetrical three-phase† machines with field structure symmetrical about the axes of the field winding and interpolar space, but salient poles and an arbitrary number of rotor‡ circuits are considered.

Idealization is resorted to, to the extent that saturation and hysteresis in every magnetic circuit and eddy

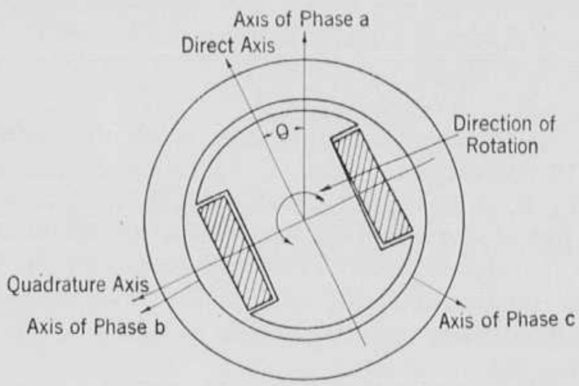

FIG. 1

currents in the armature iron are neglected, and in the assumption that, as far as concerns effects depending on the position of the rotor, each armature winding may be regarded as, in effect, sinusoidally distributed.[3]

A. Fundamental Circuit Equations

Consider the ideal synchronous machine of Fig. 1, and let

i_a, i_b, i_c = per unit instantaneous phase currents

e_a, e_b, e_c = per unit instantaneous phase voltages

ψ_a, ψ_b, ψ_c = per unit instantaneous phase linkages

t = time in electrical radians

$$p = \frac{d}{d\,t}$$

Then there is

$$\begin{aligned} e_a &= p\,\psi_a - r\,i_a \\ e_b &= p\,\psi_b - r\,i_b \\ e_c &= p\,\psi_c - r\,i_c \end{aligned} \qquad (1)$$

It has been shown previously[3] that

$$\begin{aligned} \psi_a = {} & I_d \cos\theta - I_q \sin\theta \\ & - \frac{x_0}{3}\left[i_a + i_b + i_c\right] - \frac{x_d + x_q}{3}\left[i_a - \frac{i_b + i_c}{2}\right] \\ & - \frac{x_d - x_q}{3}\left[i_a \cos 2\,\theta + i_b \cos(2\,\theta - 120)\right. \\ & \left. + i_c \cos(2\,\theta + 120)\right] \\ \psi_b = {} & I_d \cos(\theta - 120) - I_q \sin(\theta - 120) \\ & - x_0 \frac{i_a + i_b + i_c}{3} - \frac{x_d + x_q}{3}\left[i_b - \frac{i_c + i_a}{2}\right] \\ & - \frac{x_d - x_q}{3}\left[i_a \cos(2\,\theta - 120) + i_b \cos(2\,\theta + 120)\right. \\ & \left. + i_c \cos 2\,\theta\right] \end{aligned} \qquad (2)$$

$$\begin{aligned} \psi_c = {} & I_d \cos(\theta + 120) \\ & - I_q \sin(\theta + 120) - x_0 \frac{i_a + i_b + i_c}{3} \\ & - \frac{x_d + x_q}{3}\left[i_c - \frac{i_a + i_b}{2}\right] \\ & - \frac{x_d - x_q}{3}\left[i_a \cos(2\,\theta + 120) + i_b \cos 2\,\theta\right. \\ & \left. + i_c \cos(2\,\theta - 120)\right] \end{aligned}$$

where,

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†Single-phase machines may be regarded as three-phase machines with one phase open circuited.

‡Stator for a machine with stationary field structure.

[3]For numbered references see Bibliography.

Presented at the Winter Convention of the A. I. E. E., New York, N. Y., Jan. 28-Feb. 1, 1929.

$x_0 = ?$

Page 2 - ?
7-5 ?

I_d = per-unit excitation in direct axis
I_q = per-unit excitation in quadrature axis
x_d = direct synchronous reactance
x_q = quadrature synchronous reactance
x_0 = zero phase-sequence reactance

As shown in the Appendix, if normal linkages in the field circuit are defined as those obtaining at no load* there is in the case of no rotor circuits in the direct axis in addition to the field,

Φ = per-unit instantaneous field linkages
$= I - (x_d - x_d')\, i_d$

where,

I = per-unit instantaneous field current

$$i_d = \frac{2}{3}\left\{i_a \cos\theta + i_b \cos(\theta - 120) + i_c \cos(\theta + 120)\right\} \tag{3}$$

On the other hand, if n additional rotor circuits exist in the direct axis there is,

$$\Phi = I + X_{f1d} I_{1d} + X_{f2d} I_{2d} + \ldots + X_{fnd} I_{nd} - (x_d - x_d')\, i_d$$

where,

I_{1d}, I_{2d}, . . etc., are the per-unit instantaneous currents in circuits 1, 2, etc., of the direct axis, X_{f1d}, X_{f2d}, . . . etc., are per-unit mutual coefficients between the field and circuits 1, 2, etc., of the direct axis.

Similar relations exist for the linkages in each of the additional rotor circuits except $x_d - x_d'$ is to be replaced by a term x_m. However, since all of these additional circuits are closed, it follows that there is an operational result

$$I_d = I + I_{1d} + I_{2d} + \ldots + I_{nd} = G(p)\,E + H(p)\, i_d \tag{4}$$

where E is the per-unit value of the instantaneous field voltage, and $G(p)$ and $H(p)$ are operators such that

$$G(o) = 1 \qquad G(\infty) = 0$$
$$H(o) = 0 \qquad H(\infty) = x_d - x_d''$$

x_d'' = the subtransient reactance[2]

It will be convenient to write $H(p) = x_d - x_d(p)$ and to rewrite (**4**) in the form,

$$I_d = G(p)\,E + [x_d - x_d(p)]\, i_d \tag{4a}$$

If there are no additional rotor circuits, there is, as shown in Appendix I,

$$\Psi = I - (x_d - x_d')\, i_d$$
$$E = T_0\, p\Psi + I$$

where T_0 is the open circuit time constant of the field in radians.

There is then,

$$G(p) = \frac{1}{T_0 p + 1}$$

$$x_d(p) = \frac{x_d' T_0 p + x_d}{T_0 p + 1}$$

*This definition is somewhat different from that given in reference 2.

If there is one additional rotor circuit in the direct axis there is,

$$\Psi = I + X_{f1d} I_{1d} - (x_d - x_d')\, i_d = \frac{E - I}{T_0 p}$$

$$\Psi_{1d} = X_{11d} I_{1d} + X_{f1d} I - x_{m1}d\, i_d = \frac{-I_{1d}}{T_{01d}\, p}$$

which gives,

$$G(p) = \frac{[X_{11d} - X_{f1d}]\, T_{01d}\, p + 1}{A(p)}$$

$$x_d(p) = x_d - \frac{T_0 T_{01d}[X_{11d}(x_d - x_d') - X_{f1d} x_{m1d}]\, p^2 + [(x_d - x'_d)\, T_{01d} + x_{m1d} T_0]\, p}{A(p)}$$

where,

$$A(p) = [X_{11d} - X_{f1d}^2]\, T_0 T_{01d}\, p^2 + [X_{11d} T_0 + T_{01d}]\, p + 1$$

If there is more than one additional rotor circuit the operators $G(p)$ and $x_d(p)$ will be more complicated but may be found in the same way. The effects of external field resistance may be found by changing the term I in the field voltage equation to RI. Open circuited field corresponds to R equal to infinity.

Similarly, there will be

$$I_q = [x_q - x_q(p)]\, i_q \tag{5}$$

where,

$$i_q = -\frac{2}{3}\{i_a \sin\theta + i_b \sin(\theta - 120) + i_c \sin(\theta + 120)\} \tag{3a}$$

$$x_q(o) = x_q,\; x_q(\infty) = x_q''$$

So far, 10 equations have been established relating the 15 quantities e_a, e_b, e_c, i_a, i_b, i_c, ψ_a, ψ_b, ψ_c, i_d, i_q, I_d, I_q, E, θ in a general way. It follows that when any five of the quantities are known the remaining 10 may be determined. Their determination is very much facilitated, however, by the introduction of certain auxiliary quantities e_d, e_q, e_0, i_0, ψ_d, ψ_q, ψ_0.

Thus, let

$$i_0 = \frac{1}{3}\{i_a + i_b + i_c\} \tag{3b}$$

$$e_d = \frac{2}{3}\{e_a \cos\theta + e_b \cos(\theta - 120) + e_c \cos(\theta + 120)\}$$

$$e_q = -\frac{2}{3}\{e_a \sin\theta + e_b \sin(\theta - 120) + e_c \sin(\theta + 120)\} \tag{6}$$

$$e_0 = \frac{1}{3}\{e_a + e_b + e_c\}$$

$$\psi_d = \frac{2}{3}\{\psi_a \cos\theta + \psi_b \cos(\theta - 120) + \psi_c \cos(\theta + 120)\}$$

$$\psi_q = -\frac{2}{3}\{\psi_a \sin\theta + \psi_b \sin(\theta - 120) + \psi_c \sin(\theta + 120)\} \tag{7}$$

Nov. 26, 1930.

Round Rotor
no excitation in quad axis.
no ~~?~~ neut current
$x_d = x_q$

$$\psi_a = I_d \cos\theta - \cancel{I_q \sin\theta} - \frac{x_0}{3}\cancel{\left(i_a + i_b + i_c\right)} - \frac{x_d + x_q}{3}\left[i_a - \frac{i_b + i_c}{2}\right].$$

$$\psi_a = I_d \cos\theta - I_q \sin\theta - \frac{2x_d}{3}\left[i_a - \frac{i_b + i_c}{2}\right].$$

$$\psi_b = I_d \cos(\theta - 120) - I_q \sin(\theta - 120) - \frac{2x_d}{3}\left[i_b - \frac{i_c + i_a}{2}\right].$$

$$\psi_c = I_d \cos(\theta + 120) - I_q \sin(\theta + 120) - \frac{2x_d}{3}\left[i_c - \frac{i_a + i_b}{2}\right]$$

$6F = f(P)$

$$\psi_0 = \frac{1}{3}\,\{\psi_a + \psi_b + \psi_c\}$$

then from Equation (1) there is

$$e_d = \frac{2}{3}\{\cos\theta\, p\,\psi_a + \cos(\theta - 120)\, p\,\psi_b + \cos(\theta + 120)\, p\,\psi_c\} - r\,i_d$$

$$e_q = -\frac{2}{3}\,\{\sin\theta\, p\,\psi_a + \sin(\theta - 120)\, p\,\psi_b + \sin(\theta + 120)\, p\,\psi_c\} - r\,i_q$$

$$e_0 = p\,\psi_0 - r\,i_0$$

but,

$$p\,\psi_d = \frac{2}{3}\,\{\cos\theta\, p\,\psi_a + \cos(\theta - 120)\, p\,\psi_b + \cos(\theta + 120)\, p\,\psi_c\}$$

$$-\frac{2}{3}\{\sin\theta\,\psi_a + \sin(\theta - 120)\, p\,\psi_b + \sin(\theta + 120)\, p\,\psi_c\}\, p\,\theta$$

$$= e_d + r\,i_d + \psi_q\, p\,\theta$$

$$p\,\psi_q = -\frac{2}{3}\,\{\sin\theta\, p\,\psi_a + \sin(\theta - 120)\, p\,\psi_b + \sin(\theta + 120)\, p\,\psi_c\}$$

$$-\frac{2}{3}\{\cos\theta\,\psi_a + \cos(\theta - 120)\,\psi_b + \cos(\theta + 120)\,\psi_c\}\, p\,\theta$$

$$= e_q + r\,i_q - \psi_d\, p\,\theta$$

hence there is

$$e_d = p\,\psi_d - r\,i_d - \psi_q\, p\,\theta \qquad (8)$$

$$e_q = p\,\psi_q - r\,i_q + \psi_d\, p\,\theta \qquad (9)$$

$$e_0 = p\,\psi_0 - r\,i_0 \qquad (10)$$

Also it may be readily verified that

$$\psi_d = I_d - x_d\, i_d = G(p)\,E - x_d(p)\, i_d \qquad (11)$$

$$\psi_q = I_q - x_q\, i_q = -\,x_q(p)\, i_q \qquad (12)$$

$$\psi_0 = -\,x_0\, i_0 \qquad (13)$$

Equations (8) to (13) establish six relatively simple relations between the 11 quantities e_d, e_q, e_0, i_d, i_q, i_0, ψ_d, ψ_q, ψ_0, E, θ. In practise it is usually possible to determine five of these quantities directly from the terminal conditions, after which the remaining six may be calculated with relative simplicity, After the direct, quadrature, and zero quantities are known the phase quantities may be determined from the identical relations

$$i_a = i_d \cos\theta - i_q \sin\theta + i_0$$

$$i_b = i_d \cos(\theta - 120) - i_q \sin(\theta - 120) + i_0 \qquad (14)$$

$$i_c = i_d \cos(\theta + 120) - i_q \sin(\theta + 120) + i_0$$

$$\psi_a = \psi_d \cos\theta - \psi_q \sin\theta + \psi_0$$

$$\psi_b = \psi_d \cos(\theta - 120) - \psi_q \sin(\theta - 120) + \psi_0 \qquad (15)$$

$$\psi_c = \psi_d \cos(\theta + 120) - \psi_q \sin(\theta + 120) + \psi_0$$

$$e_a = e_d \cos\theta - e_q \sin\theta + e_0$$

$$e_b = e_d \cos(\theta - 120) - e_q \sin(\theta - 120) + e_0 \qquad (16)$$

$$e_c = e_d \cos(\theta + 120) - e_q \sin(\theta + 120) + e_0$$

Referring to Fig. 2, it may be seen that when there are no zero quantities, that is, when $e_0 = \psi_0 = i_0 = 0$, the phase quantities may be regarded as the projection of vectors $\bar{e}$, $\bar{\psi}$, and $\bar{i}$ on axes lagging the direct axis by

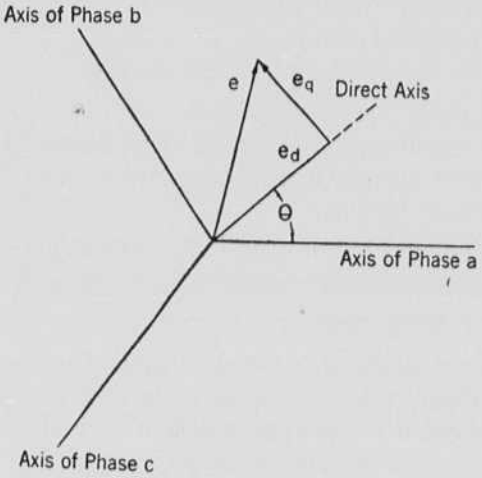


FIG. 2

angles θ, $\theta - 120$ and $\theta + 120$, where taking the direct axis as the axis of reals,

$$\bar{e} = e_d + j\,e_q$$

$$\bar{\psi} = \psi_d + j\,\psi_q$$

$$\bar{i} = i_d + j\,i_q$$

If we introduce in addition the vector quantity,

$$\bar{I} = I_d + j\,I_q$$

the circuit equations previously obtained may be

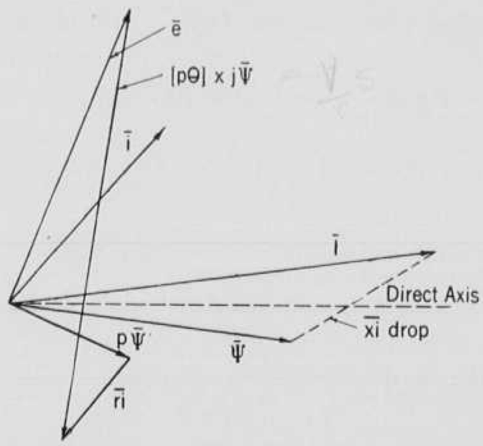


FIG. 3

transferred into the corresponding vector forms,

$$\bar{e} = p\,\bar{\psi} - \overline{ri} + [p\,\theta]\,j\,\bar{\psi}$$

$$\bar{\psi} = \bar{I} - \overline{xi}$$

where, $\quad \overline{xi} = x_d\, i_d + j\,x_q\, i_q$

Fig. 3 shows these relations graphically.

B. Armature Power Output

The per-unit instantaneous power output from the armature is necessarily proportional to the sum

$e_a i_a + e_b i_b + e_c i_c$. By consideration of any instant during normal operation at unity power factor it may be seen that the factor of proportionality must be 2/3. That is,

$$P = \text{per-unit instantaneous power output}$$
$$= 2/3 \{ e_a i_a + e_b i_b + e_c i_c \}$$

Substituting from Equations (**14**) and (**16**) there results the useful relation,

$$P = e_d i_d + e_q i_q + e_0 i_0 \qquad (\mathbf{17})$$

C. Electrical Torque on Rotor

It is possible to determine the electrical torque on the rotor directly from the general relation,

{Total power output} =

{mechanical power transferred across gap}

+ {rate of decrease of total stored magnetic energy}

− {total ohmic losses} (**18**)

However, since this torque depends uniquely only on the magnitudes of the currents in every circuit of the machine, it follows that a general formula for torque may be derived by considering any special case in which arbitrary conditions are imposed as to the way in which these currents are changing as the rotor moves.

The simplest conditions to impose are that I_d, I_q, i_d, i_q, and i_0 remain constant as the rotor moves. In this case there will be no change in the stored magnetic energy of the machine as the rotor moves, and the power output of the rotor will be just equal in magnitude and opposite in sign to the rotor losses. It follows that under the special conditions assumed, Equation (**18**) becomes simply,

{armature power output} =

{mechanical power across gap} − {armature losses}

$$\text{or, } P = T p \theta - \frac{2r}{3} \{ i_a^2 + i_b^2 + i_c^2 \}$$
$$= T p \theta - r \{ i_d^2 + i_q^2 + i_0^2 \}$$

Then,

$$T = \text{per-unit instantaneous electrical torque}$$
$$= \frac{e_d i_d + e_q i_q + e_0 i_0 + r \{ i_d^2 + i_q^2 + i_0^2 \}}{p \theta}$$

but subject to the conditions imposed,

$$e_d = - \psi_q p \theta - r i_d$$
$$e_q = \psi_d p \theta - r i_q$$
$$e_0 = - r i_0$$

It therefore follows that,

$$T = i_q \psi_d - i_d \psi_q \qquad (\mathbf{19})$$
$$= \text{vector product of } \bar{\psi} \text{ and } \bar{i}$$
$$= \bar{\psi} \times \bar{i} \qquad (\mathbf{19a})$$

a result which could have been established directly by physical reasoning. Formula (**19**) is employed by Dreyfus in his treatment of self-excited oscillations of synchronous machines.[14]

D. Constant Rotor Speed

Suppose that the constant slip of the rotor is s.

Then there is,

$$e_d = p \psi_d - r i_d - (1 - s) \psi_q$$
$$e_q = p \psi_q - r i_q + (1 - s) \psi_d$$

but,

$$\psi_d = G(p) E - x_d(p) i_d$$
$$\psi_q = - x_q(p) i_q$$

Putting

$$p x_d(p) + r = z_d(p)$$
$$p x_q(p) + r = z_q(p)$$

there is

$$e_d = p G(p) E - z_d(p) i_d + (1 - s) x_q(p) i_q \qquad (\mathbf{20})$$
$$e_q = (1 - s) [G(p) E - x_d(p) i_d] - z_q(p) i_q \qquad (\mathbf{21})$$

Solving gives,

$$i_d = \{ [p z_q(p) + (1 - s)^2 x_q(p)] G(p) E - z_q(p) e_d - (1 - s) x_q(p) e_q \} \div D(p) \qquad (\mathbf{22})$$

$$i_q = \frac{(1 - s) r G(p) E - z_d(p) e_q + (1 - s) x_d(p) e_d}{D(p)} \qquad (\mathbf{23})$$

where, $D(p) = z_d(p) z_q(p) + (1 - s)^2 x_d(p) x_q(p)$

E. Two Machines Connected Together

Suppose that two machines which we will designate respectively by the subscripts g and h, are connected together, but not to any other machines or circuits, and assume in addition that there are no zero quantities. In this case the voltages of each machine will be equal

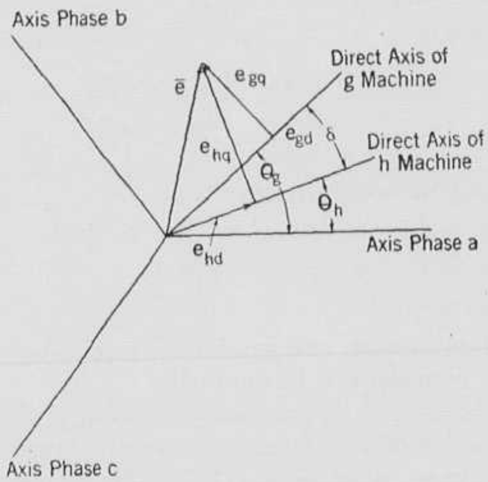
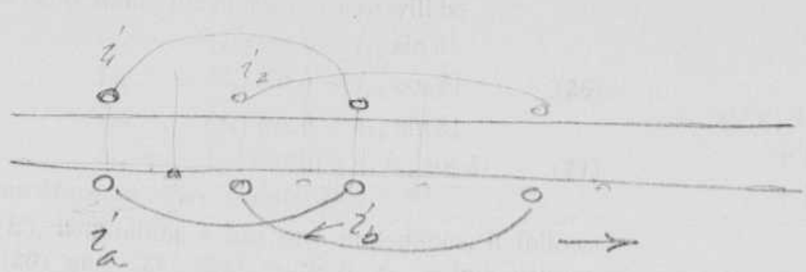


FIG. 4

phase for phase, and it therefore follows that the voltage vectors of each machine must coincide, as shown in Fig. 4.

Referring to the figure it will be seen that the direct and quadrature components of voltage of the two machines are subject to the mutual relations,

$$e_{hd} = e_{gd} \cos \delta - e_{gq} \sin \delta$$
$$e_{hq} = e_{gd} \sin \delta + e_{gq} \cos \delta \qquad (\mathbf{24})$$
$$e_{gd} = e_{hd} \cos \delta + e_{hq} \sin \delta$$
$$e_{gq} = - e_{hd} \sin \delta + e_{hq} \cos \delta \qquad (\mathbf{25})$$



$$\omega = \frac{d\theta}{dt}, \quad \theta = \int \omega\, dt.$$

$$e_1 = (r_1 + Lp)i_1 + Mp(i_a \cos\omega t) + Mp(i_b \sin\omega t).$$

$$e_2 = (r_2 + Lp)i_2 + Mp(i_b \cos\omega t) + Mp(i_a \sin\omega t).$$

$$e_a = (r_a + L_a p)i_a + Mp(i_1 \cos\omega t) + Mp(i_2 \sin\omega t).$$

$$e_b = (r_b + L_b p)i_b + Mp(i_2 \cos\omega t) + Mp(i_1 \sin\omega t).$$

[illegible]

[illegible] consideration of [illegible] [illegible] power factor it [illegible] be seen that the factor of proportionality must be [illegible]

[illegible]

Substituting from Equations (14) and (16) there results the useful relation, [illegible] (17)

[illegible] Torque [illegible]

[illegible] to determine the [illegible] on the rotor directly from the general relation,

(Total power output) = (mechanical power transferred across gap) − (rate of increase of total stored magnetic energy) − (total ohmic losses). (18)

However, since this torque depends uniquely only on the magnitudes of the currents in every circuit of the machine, it follows that a general formula for torque may be derived by considering any special case in which arbitrary conditions are imposed as to the way in which these currents are [illegible].

The simplest conditions to impose are that [illegible] and [illegible] remain constant as the rotor moves. In this case there will be no change in the stored magnetic energy of the machine as the rotor moves, and the power output of the rotor will be just equal in magnitude and opposite in sign to the rotor losses. It follows that under the special conditions assumed, Equation (18) becomes simply,

(armature power output) = (mechanical power across gap) − (armature losses)

or, $P = T\,p\,\theta - \frac{2r}{3}\,[\,\ldots\,]$

$= T\,p\,\theta - r\,[\,\ldots\,]$

Then,

T = per-unit instantaneous electrical torque

$= \dfrac{[\text{illegible}]}{p\,\theta}$

but subject to the conditions imposed

[illegible]

It therefore follows that,

[illegible] (19)

= vector product of [illegible] and [illegible]

[illegible] (19a)

[illegible] could have been established directly by [illegible] is obtained by [illegible] of self [illegible] coefficients in [illegible] machines.[illegible]

[illegible] speed.

[illegible] the constant [illegible] of the [illegible]

[illegible] ( )

[illegible]

From [illegible]

[illegible]

[illegible] (20)

[illegible] (21)

Solving gives,

[illegible]

[illegible] (22)

[illegible]

[illegible] (23)

where, [illegible]

[illegible] Equations [illegible]

Suppose that two machines which we will designate respectively by the subscripts [illegible] are connected together, but not to any other machines or circuits, and assume in addition that there are no [illegible]. In this case the voltages of each machine will be equal

Fig. [illegible]

phase for phase, and it therefore follows that the voltage vectors of each machine must coincide, as shown in Fig. [illegible]

Referring to the figure it will be seen that the direct and quadrature components of voltage of the two machines are subject to the mutual relations

[illegible]

[illegible] (24)

[illegible]

[illegible] (25)

On the other hand, for currents there will be

$$
\begin{aligned}
i_{hd} &= -\{i_{gd}\cos\delta - i_{gq}\sin\delta\} \\
i_{hq} &= -\{i_{gd}\sin\delta + i_{gq}\cos\delta\}
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
i_{gd} &= -\{i_{hd}\cos\delta + i_{hq}\sin\delta\} \\
i_{gq} &= -\{-i_{hd}\sin\delta + i_{hq}\cos\delta\}
\end{aligned} \tag{27}
$$

F. One Machine on an Infinite Bus

In (*E*), if machine h has zero impedance, it follows from (**20**) and (**21**) that $e_{hd} = 0$, e_{hq} = bus voltage say $= e$.

Then for machine g there is,

$$
\begin{aligned}
e_d &= e\sin\delta \\
e_q &= e\cos\delta
\end{aligned} \tag{28}
$$

G. Torque Angle Relations

From Equations (**11**), (**12**), and (**19**), there is,

$$T = \frac{I_q\,\psi_d}{x_q} - \frac{I_d\,\psi_q}{x_d} - \frac{x_d - x_q}{x_d\,x_q}\,\psi_d\,\psi_q$$

Then if the rotor leads the vector $\bar{\psi}$ by an angle $\bar{\delta}$ there is

$$
\begin{aligned}
\psi_q &= -\psi\sin\delta \\
\psi_d &= \psi\cos\delta
\end{aligned}
$$

$$T = \frac{I_q\,\psi}{x_q}\cos\delta + \frac{I_d\,\psi\sin\delta}{x_d} + \frac{x_d - x_q}{2\,x_d\,x_q}\,\psi^2\sin 2\,\delta \tag{29}$$

A derivation of this formula for steady load conditions has been previously given by Doherty and Nickle.[9]

H. Three-Phase Short Circuit with Constant Rotor Speed Maintained

Since a three-phase short circuit causes e_d and e_q to vanish suddenly, its effect with constant rotor speed maintained may be found by impressing $e_d = -e_{d0}$, $e_q = -e_{q0}$ in (**22**) and (**23**) where e_{d0} and e_{q0} are the values of e_d and e_q before the short circuit. The initial currents existing before the short circuit must be added to the currents found in this way in order to obtain the resultant current after the short circuit.

With $s = 0$ and E constant there is in detail,

$$
\begin{aligned}
i_d &= \frac{z_q(p)\,e_{d0} + x_q(p)\,e_{q0}}{D(p)}\,.\,1 + \frac{x_q\,E - r\,e_{d0} - x_q\,e_{q0}}{r^2 + x_d\,x_q} \\
i_q &= \frac{z_d(p)\,e_{q0} - x_d(p)\,e_{d0}}{D(p)}\,.\,1 + \frac{r\,E - r\,e_{q0} + x_d\,e_{d0}}{r^2 + x_d\,x_q}
\end{aligned} \tag{30}
$$

The working out of the formulas may be illustrated by consideration of the simple case of a machine with no rotor circuits in addition to the field. In this case there is

$$x_q(p) = x_q$$

$$x_d(p) = \frac{x_d'\,T_0\,p + x_d}{T_0\,p + 1}$$

$$D(p) = \left\langle \frac{x_d'\,T_0\,p + x_d}{T_0\,p + 1}\,p + r \right\rangle \langle x_q\,p + r\rangle + \frac{x_d'\,T_0\,p + x_d}{T_0\,p + 1}\,x_q$$

$$
\begin{aligned}
&x_d'\,x_q\,T_0\,p^3 \\
&\quad + [x_d'\,r\,T_0 + (x_d + r\,T_0)\,x_q]\,p^2 \\
&\quad + [r\,(x_d + x_q + r\,T_0) + x_d'\,x_q\,T_0]\,p \\
&= \frac{+\,r^2 + x_d\,x_q}{T_0\,p + 1} \\
&= \frac{d(p)}{T_0\,p + 1}
\end{aligned} \tag{31}
$$

By the expansion theorem there is, finally,

$$i_d = \frac{x_q\,E}{r^2 + x_d\,x_q} + \sum_{1}^{3} \frac{(T_0\,\alpha_n + 1)\,\langle (x_q\,\alpha_n + r)\,e_{d0} + x_q\,e_{q0}\rangle\,\epsilon^{-\alpha_n t}}{\alpha_n\,d'(\alpha_n)}$$

$$i_q = \frac{r\,E}{r^2 + x_d\,x_q} + \sum_{1}^{3} \frac{\langle x_d'\,T_0\,\alpha_n^2 + (x_d + r\,T_0)\,\alpha_n + r\rangle e_{q0} - (T_0\,\alpha_n\,x'_d + x_d)\,e_{d0}}{\alpha_n\,d'(\alpha_n)}\,\epsilon^{-\alpha_n t} \tag{32}$$

where the summation is extended over the roots of $d(\alpha) = 0$ and $d'(p) = \frac{d}{d\,p}\,d(p)$

The phase currents may, of course, be found from

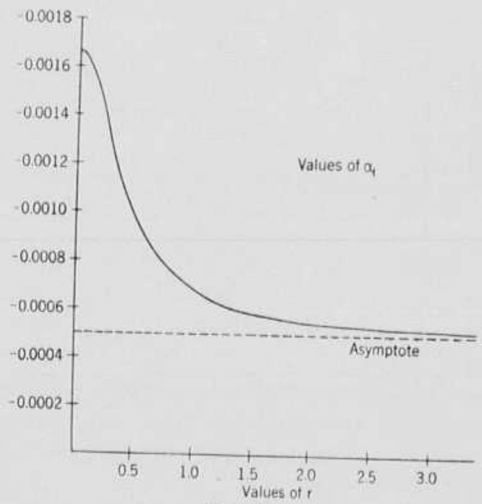


FIG. 5

Equations (**32**) by the application of Equations (**14**). For the particular case

$$T_0 = 2{,}000,\; x_d = 1.00,\; x_q = 0.60,\; x_d' = 0.30$$

the roots α_1, α_2, α_3 of the equation $d(p) = 0$, were found to be as shown in Figs. 5, 6, and 7, where

$$
\begin{aligned}
\alpha_2 &= \alpha_a + \alpha_b \\
\alpha_3 &= \alpha_a - \alpha_b
\end{aligned}
$$

It will be noted that, as would necessarily be the case, where $r = 0$, α_1 is equal to the reciprocal of the short circuit time constant of the machine, *i. e.*, for $r = 0$,

$$\alpha_1 = -\frac{x_d}{x_d'}\,\frac{1}{T_0} = -\,0.001667$$

while for $r = \infty$

$$\alpha_1 = -\frac{1}{T_0} = -\,0.000500$$

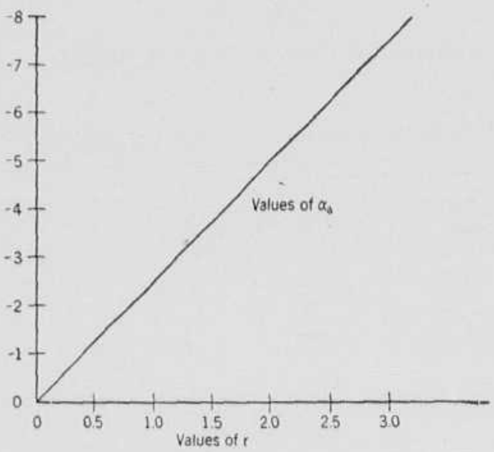


FIG. 6

The root α_a is found to be almost exactly equal to the value which it would have were $T_0 = \infty$, *i. e.*,

$$\alpha_a = \frac{r\,(x_d' + x_q)}{2\,x_d'\,x_q} \text{ approximately}$$

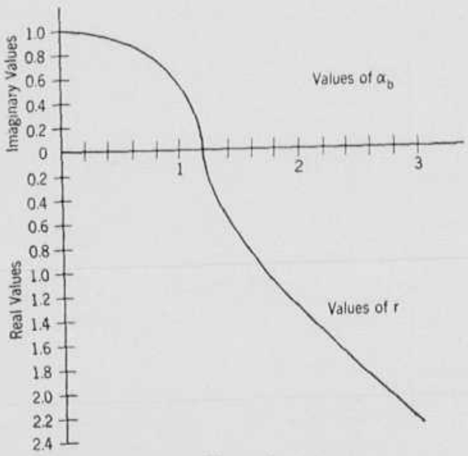


FIG. 7

Thus, in the special case considered this approximate formula gives

$$\alpha_a = \frac{(0.30 + 0.60)\,r}{2 \times 0.30 \times 0.60} = 2.50\,r$$

which checks the result found by the exact solution of the cubic.

I. Starting Torque

On infinite bus and with slip s, there will be, choosing $\delta = \frac{\pi}{2} - s\,t$, and referring to Equation (**28**),

$$e_d = \cos s\,t$$
$$e_q = \sin s\,t$$

If we now introduce a system of vectors rotating at s per-unit angular velocity there is

$$ed = 1.0$$
$$e_q = -j$$
$$p = j\,s \qquad (33)$$

Then from (**22**) and (**23**),

$$i_d = \{\, j\,s\,x_q'\,(j\,s) + r - j\,(1 - s)\,x_q'\,(j\,s) \,\} \div \{\, [j\,s\,x_d'\,(j\,s) + r]\,[j\,s\,x_q'\,(j\,s) + r] + (1 - s)^2\,x_d'\,(j\,s)\,x_q'\,(j\,s) \,\}$$

$$= \frac{j\,(1 - 2\,s)\,x_q'\,(j\,s) - r}{r^2 + (1 - 2\,s)\,x_d'(j\,s)\,x_q'(j\,s) + j\,s\,r[x_d'(j\,s) + x_q'(j\,s)]}$$

$$= \left\{ j\,x_q'\,(j\,s) - \frac{r}{1 - 2\,s} \right\} \div \left\{ x_d'\,(j\,s)\,x_q'\,(j\,s) + \frac{r}{1 - 2\,s}\,[r + j\,s\,(x_d'\,(j\,s) + x_q'\,(j\,s))] \right\} \qquad (34)$$

$$i_q = -\frac{[j\,s\,x_d'\,(j\,s) + r]\,(-\,j) - (1 - s)\,x_d'\,(j\,s)}{r^2 + (1 - 2\,s)x_d'(j\,s)x_q'(j\,s) + j\,s\,r[x_d'(j\,s) + x_q'(js)]}$$

$$= \left\{ x_d'\,(j\,s) + \frac{j\,r}{1 - 2\,s} \right\} \div \left\{ x_d'\,(j\,s)\,x_q'\,(j\,s) + \frac{r}{1 - 2\,s}\,[r + j\,s\,(x_d'\,(j\,s) + x_q'\,(j\,s))] \right\} \qquad (35)$$

The expressions for average power and torque then become,

$$P_{av} = 1/2\,[e_d \,.\, i_d + e_q \,.\, i_q]$$
$$T_{av} = 1/2\,[i_q \,.\, \psi_d - i_d \,.\, \psi_q]$$

where the dot indicates the scalar product, or

$$P_{av} = 1/2\,[1 \,.\, i_d - j \,.\, i_q]$$
$$= 1/2\,[\text{Real of } i_d - \text{Imaginary of } i_q] \qquad (36)$$

There is in general,

$$e_d + r\,i_d = p\,\psi_d - (1 - s)\,\psi_q$$
$$e_q + r\,i_q = (1 - s)\,\psi_d + p\,\psi_q$$

$$\psi_d = \frac{\begin{vmatrix} e_d + r\,i_d & -(1 - s) \\ e_q + r\,i_q & p \end{vmatrix}}{\begin{vmatrix} p & -(1 - s) \\ (1 - s) & p \end{vmatrix}}$$

$$= \frac{p\,(e_d + r\,i_d) + (1 - s)\,(e_q + r\,i_q)}{p^2 + (1 - s)^2} \qquad (37)$$

$$\psi_q = \frac{p\,(e_q + r\,i_q) - (1 - s)\,(e_d + r\,i_d)}{p^2 + (1 - s)^2} \qquad (38)$$

$$\psi_d = \frac{j\,s\,(e_d + r\,i_d) + (1-s)\,(e_q + r\,i_q)}{1 - 2\,s}$$

$$\psi_q = \frac{j\,s\,(e_q + r\,i_q) - (1-s)\,(e_d + r\,i_d)}{1 - 2\,s}$$

with $e_d = 1.0$, $e_q = -j$

$$\psi_d = \frac{j\,s + j\,s\,r\,i_d + (1-s)\,(-j) + (1-s)\,r\,i_q}{1 - 2\,s}$$

$$= \frac{-(1-2s)\,j + r\,[j\,s\,i_d + (1-s)\,i_q]}{1 - 2\,s}$$

$$= -j + \frac{r}{1-2\,s}\,[j\,s\,i_d + (1-s)\,i_q] \qquad (39)$$

$$\psi_q = \frac{j\,s\,(-j + r\,i_q) - (1-s) - r\,(1-s)\,i_d}{1 - 2\,s}$$

$$= \frac{-(1-2\,s) + r\,[j\,s\,i_q - (1-s)\,i_d]}{1 - 2\,s}$$

$$= -1 + \frac{r}{1-2\,s}\,[j\,s\,i_q - (1-s)\,i_d] \qquad (40)$$

Thus,

$$T_{av} = 1/2 \left[\begin{array}{l} i_q\,.\,(-j) + i_q\,.\,\dfrac{r}{1-2\,s}\,\langle j\,s\,i_d + (1-s)\,i_q\rangle \\ -\,i_d\,.\,(-1) - i_d\,.\,\dfrac{r}{1-2\,s}\,\langle j\,s\,i_q - (1-s)\,i_d\rangle \end{array} \right]$$

$$= P_{av} + \frac{r}{2(1-2\,s)} \left[\begin{array}{l} (1-s)\,\langle i_q^2 + i_d^2\rangle \\ +\,2\,s\,i_q\,.\,j\,i_d \end{array} \right]$$

$$= P_{av} + \frac{r}{2}\,(i_q^2 + i_d^2) + \frac{r\,s}{2(1-2\,s)} \left[\begin{array}{l} i_q^2 + i_d^2 \\ +\,2\,i_q\,.\,j\,i_d \end{array} \right]$$

$$= P_{av} + r\,\frac{i_q^2 + i_d^2}{2} + \frac{r\,s}{2(1-2\,s)}\,(i_q + j\,i_d)^2 \qquad (41)$$

Mr. Ralph Hammar, who has been engaged in the application of the general method of calculation outlined above, to the predetermination of the starting torque of practical synchronous motors, has suggested an interesting modification of formulas (**36**) and (**41**), based upon the fact that, since the total m. m. f. consists of direct and quadrature components pulsating at slip frequency, it may be resolved into two components, one moving forward at a per-unit speed $1 - s + s = 1.0$, and the other moving backward at a per-unit speed $1 - s - s = 1 - 2\,s$. Thus from this standpoint half of both the direct and quadrature components will move forward, and half backward. Since the quadrature axis is ahead of the direct it follows that as far as concerns the forward component the quadrature current i_q is equivalent to a d-c. $j\,i_q$, while as regards backward component it is equivalent to a direct component $-\,j\,i_q$. It follows that the vector amounts of forward and backward m. m. f. or current are

$$\text{forward current} = i_f = \frac{1}{2}\,(i_d + j\,i_q)$$

$$\text{backward current} = i_b = \frac{1}{2}\,(i_d - j\,i_q) \qquad (42)$$

If we define by analogy,

$$\text{forward voltage} = \frac{1}{2}\,(e_d + j\,e_q)$$

$$\text{backward voltage} = \frac{1}{2}\,(e_d - j\,e_q) \qquad (43)$$

There is,

$$i_f = \frac{1}{2}\left\{ \frac{-2\,r}{1-2\,s} + j\,[x_d'\,(j\,s) + x_q'\,(j\,s)] \right\} \div \left\{ x_d'\,(j\,s)\,x_q'\,(j\,s) + \frac{r}{1-2\,s}\,\langle r + j\,s\,[x_d'\,(j\,s) + x_q'\,(j\,s)]\rangle \right\}$$

$$i_b = \frac{1}{2}\left\{ j\,[x_q'\,(j\,s) - x_d'\,(j\,s)] \right\} \div \left\{ x_d'\,(j\,s)\,x_q'\,(j\,s) + \frac{r}{1-2\,s}\,\langle r + j\,s\,[x_d'\,(j\,s) + x_q'\,(j\,s)]\rangle \right\}$$

$$e_f = 1.0 \qquad (44)$$

$$e_b = 0 \qquad (45)$$

$$P_{av} = e_f\,.\,i_f = \text{real of } i_f \qquad (46)$$

$$T_{av} = P_{av} + r\,i_f^2 + \frac{r}{1-2\,s}\,i_b^2 \qquad (47)$$

J. Zero Armature Resistance, One Machine Connected to an Infinite Bus

Assume that a machine of negligible armature resistance is operating from an infinite bus of per-unit voltage e, at synchronous speed, with a steady excitation voltage E_0, and displacement angle δ_0. At the instant $t = 0$, let δ and E change.

There is,

$$i_d = \frac{E_0 - \psi_{d0}}{x_d} - \frac{1}{x_d\,(p)}\,\Delta\,\psi_d + \frac{G\,(p)}{x_d\,(p)}\,\Delta\,E$$

$$i_q = -\,\frac{\psi_{q0}}{x_q} - \frac{1}{x_q\,(p)}\,\Delta\,\psi_q$$

$$\psi_d = e\cos\delta$$

$$\psi_q = -\,e\sin\delta$$

From which there is, by obvious re-arrangement,

$$i_d = \frac{E - e\cos\delta}{x_d} + e\,\frac{x_d - x_d(p)}{x_d\,x_d(p)}(\cos\delta_0 - \cos\delta) - \frac{x_d(p) - G(p)\,x_d}{x_d\,x_d(p)}\,\Delta E$$

$$i_q = \frac{e\sin\delta}{x_q} + e\,\frac{x_q - x_q(p)}{x_q\,x_q(p)}(\sin\delta - \sin\delta_0) \qquad (\mathbf{48})$$

Then,

$$T = \frac{E\,e\sin\delta}{x_d} + \frac{x_d - x_q}{2\,x_d\,x_q}\,e^2\sin 2\,\delta + e^2\cos\delta\,\frac{x_q - x_q(p)}{x_q\,x_q(p)}(\sin\delta - \sin\delta_0) + e^2\sin\delta\,\frac{x_d - x_d(p)}{x_d\,x_d(p)}(\cos\delta_0 - \cos\delta) - e\sin\delta\,\frac{x_d(p) - x_d\,G(p)}{x_d\,x_d(p)}\,.\,\Delta E \qquad (\mathbf{49})$$

But quantities a_{dn}, a_{qn}, α_{dn}, α_{qn}, b_n, β_n may be found such that

$$\frac{x_q - x_q(p)}{x_q(p)}\,.\,1 = \frac{x_q - x_q''}{x_q''}\sum a_{dn}\,\epsilon^{-\alpha_{dn}t}$$

$$\frac{x_d - x_d(p)}{x_d(p)}\,.\,1 = \frac{x_d - x_d''}{x_d''}\sum a_{qn}\,\epsilon^{-\alpha_{qn}t} \qquad (\mathbf{50})$$

$$\frac{x_d(p) - x_d\,G(p)}{x_d(p)}\,.\,1 = \sum b_n\,\epsilon^{-\beta_n t}$$

$$x_q'' = x_q(\infty)$$
$$x_d'' = x_d(\infty)$$
$$\Sigma\,a_{dn} = 1.0$$
$$\Sigma\,a_{qn} = 1.0$$
$$\Sigma\,b_n = 1.0$$

It therefore follows from the operational rule that,

$$f(p)\,F(t) = F(o)\,\phi(t) + \int_0^t \phi(t-u)\,F'(u)\,d\,u \qquad (\mathbf{51})$$

where,

$$\phi(t) = f(p)\,.\,1$$

that if

$$\delta = \delta(t)$$
$$p\,\delta = \delta'(t)$$
$$\Delta E = \Delta E(t)$$
$$p\,\Delta E = \Delta E'(t)$$

Equations (**48**) and (**49**) may be rewritten in the form,

$$i_d = \frac{E - e\cos\delta}{x_d} + e\,\frac{x_d - x_d''}{x_d\,x_d''}\sum a_{dn}\,\epsilon^{-\alpha_{dn}t}\int_0^t \epsilon^{\alpha_{dn}u}\sin\delta(u)\,\delta'(u)\,d\,u - \frac{1}{x_d}\sum b_n\,\epsilon^{-\beta_n t}\int_0^t \epsilon^{\beta_n u}\,\Delta E'(u)\,d\,u \qquad (\mathbf{48a})$$

$$i_q = \frac{e\sin\delta}{x_q} + e\,\frac{x_q - x_q''}{x_q\,x_q''}\sum a_{qn}\,\epsilon^{-\alpha_{qn}t}\int_0^t \epsilon^{\alpha_{qn}u}\cos\delta(u)\,\delta'(u)\,d\,u$$

$$T = \frac{E\,e\sin\delta}{x_d} + \frac{e^2(x_d - x_q)}{2\,x_d\,x_q}\sin 2\,\delta + e^2\frac{x_d - x_d''}{x_d\,x_d''}\sin\delta\sum a_{dn}\,\epsilon^{-\alpha_{dn}t}\int_0^t \epsilon^{\alpha_{dn}u}\sin\delta(u)\,\delta'(u)\,du + e^2\frac{x_q - x_q''}{x_q\,x_q''}\cos\delta\sum a_{qn}\,\epsilon^{-\alpha_{qn}t}\int_0^t \epsilon^{\alpha_{qn}u}\cos\delta(u)\,\delta'(u)\,du - \frac{e\sin\delta}{x_d}\sum b_n\,\epsilon^{-\beta_n t}\int_0^t \epsilon^{\beta_n u}\,\Delta E'(u)\,d\,u \qquad (\mathbf{49a})$$

Formula (**49a**) may be used to determine starting torque and current with zero armature resistance, by introducing $\delta(t) = s\,t$, $\delta'(t) = s$. Thus the average component of torque is found to be,

$$T_{av} = \frac{1}{2}\,\frac{x_d - x_d''}{x_d\,x_d''}\sum a_{dn}\,\frac{\alpha_{dn}\,s}{\alpha_{dn}^2 + s^2} + \frac{1}{2}\,\frac{x_q - x_q''}{x_q\,x_q''}\sum a_{qn}\,\frac{\alpha_{qn}\,s}{\alpha_{qn}^2 + s^2} \qquad (\mathbf{52})$$

Since

$\dfrac{\alpha\,s}{\alpha^2 + s^2}$ is never greater than ½, and

$$\Sigma\,a_{dn} = \Sigma\,a_{qn} = 1.0$$

it follows that T_{av} is never greater than

$$\frac{1}{4}\left\{\frac{x_d - x_d''}{x_d\,x_d''} + \frac{x_q - x_q''}{x_q\,x_q''}\right\} \qquad (\mathbf{53})$$

Equation (**53**) thus provides a very simple criterion of the maximum possible starting torque of a synchronous motor of given dimensions, when armature resistance is neglected.

The same formula may also be used to obtain a simple expression for the damping and synchronizing components of pulsating torque due to a given small angular pulsation of the rotor.

Thus if the angular pulsation is

$$\Delta\,\delta = [\Delta\,\delta]\sin(s\,t)$$

and if the pulsation of torque is expressed in the form

$$\Delta T = T_s \, \Delta \delta + T_d \frac{d}{dt} \Delta \delta$$

there results,

$$T_s = T_{s0} + e^2 \sin^2 \delta_0 \frac{x_d - x_d''}{x_d \, x_d''} \sum \frac{a_{dn} \, s^2}{(\alpha_{dn})^2 + s^2}$$

$$+ e^2 \cos^2 \delta_0 \frac{x_q - x_q''}{x_q \, x_q''} \sum \frac{a_{qn} \, s^2}{(\alpha_{qn})^2 + s^2} \qquad \textbf{(54)}$$

$$s \, T_d = e^2 \sin^2 \delta_0 \frac{x_d - x_d''}{x_d \, x_d''} \sum \frac{a_{dn} \, s \, \alpha_{dn}}{(\alpha_{dn})^2 + s^2}$$

$$+ e^2 \cos^2 \delta_0 \frac{x_q - x_q''}{x_q \, x_q''} \sum \frac{a_{qn} \, s \, \alpha_{qn}}{(\alpha_{qn})^2 + s^2}$$

where,

$$T_{s0} = \frac{e \, I_{d0} \cos \delta_0}{x_d} + \frac{e^2 \, (x_d - x_q)}{x_d \, x_q} \cos 2 \, \delta_0$$

δ_0 = average angular displacement, *i. e.*, total angle $= \delta = \delta_0 + \Delta \delta$.

It can be shown that for the case of no additional rotor circuits, Equations (**54**) are exactly equivalent to Equations (**24**) and (**25**) in Doherty and Nickle's paper, *Synchronous Machines III*. The new formulas herein developed are, however, very much simpler in form, especially since in the case which Doherty and Nickle have treated, there is only one term in the summation; that is, $n = 1$, and α is merely the reciprocal of the short circuit time constant of the machine, expressed in radians.

K. The Equivalent Circuit of Synchronous Machines Operating in Parallel at No Load, Neglecting the Effect of Armature Resistance

Let, δ_a = angle of rotor a and bus
θ_0 = angle of rotor a in space

In general, the shaft torque of a machine depends on its acceleration and speed in space, and the magnitude and rate of change of the bus voltage as a vector. If all of the machines are operating at no load and if there is no armature resistance, a small displacement of any one machine will change the magnitude of the bus voltage only by a second order quantity; consequently for small displacements the magnitude of the bus voltage may be regarded as fixed, and only the angle of the bus and rotor need be considered. Furthermore, the electrical torque may be found in terms of (δ) by employing an infinite bus formula. But Equation (**49a**) implies the alternative general operational form,

$$T = \frac{e \, I_{d0} \sin \delta}{x_d} + \frac{e^2 \, (x_d - x_q) \sin 2 \, \delta}{2 \, x_d \, x_q}$$

$$- \frac{x_d - x_d''}{x_d \, x_d'} e^2 \sin \delta \sum \frac{a_{nd} \, p}{p + \alpha_{nd}} . \cos \delta \qquad \textbf{(49b)}$$

$$+ \frac{x_q - x_q''}{x_q \, x_q''} e^2 \cos \delta \sum \frac{a_{nq} \, p}{p + \alpha_{nq}} . \sin \delta$$

Therefore in the case under consideration there is for machine a,

$$T_a = \left[\frac{e \, I_a}{x_{da}} + e^2 \frac{(x_{da} - x_{qa})}{x_{da} \, x_{qa}} \right] \delta_a$$

$$+ \frac{x_{qa} - x_{qa}''}{x_{qa} \, x_{qa}''} e^2 \sum a_{nqa} \frac{p}{p + \alpha_{nqa}} . \delta_a \qquad \textbf{(55)}$$

where: e = per-unit bus voltage
I_a = per-unit excitation of machine a, etc.

This equation can be represented by Fig. 8, in which the charge through the circuit represents (δ_a) and the

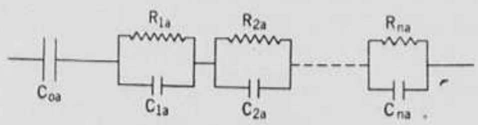


Fig. 8

voltage across the circuit represents the electrical torque of the machine (T_a).

The capacitances and resistances must be chosen so that

$$C_{0a} = \frac{x_{da} \, x_{qa}}{e \, I_a \, x_{qa} + e^2 \, (x_{da} - x_{qa})} \qquad \textbf{(56)}$$

$$C_{na} = \frac{x_{qa} \, x_{qa}''}{e^2 \, a_{nqa} \, (x_{qa} - x_{qa}'')}$$

$$R_{na} = \frac{1}{C_{na} \, \alpha_{nqa}}$$

The equation for the mechanical torque is

$$T_{sa} = T_a + M_a \, p \, s_a \qquad \textbf{(57)}$$

where:

M_a = inertia factor of machine a in radians

$$= \frac{2 \times \text{stored mech. energy at normal speed}}{\text{base power}}$$

$$= 2 \, \pi \, \mathrm{f} \, \frac{0.462 \;\; W R^2 \left(\dfrac{\text{rev. per min.}}{1000} \right)^2}{\text{base kw.}}$$

s_a = per-unit speed of machine a

t = time in seconds $\left(p = \dfrac{d}{dt} \right)$

But, $s_a = p \, \theta_a$

Thus there is

$$T_{sa} = T_a + M_a \, p^2 \, \theta_a \qquad \textbf{(57a)}$$

which corresponds to the equivalent circuit of Fig. 9, in which change $= \theta_a$

$$L_a = M_a$$

The machine operating on an infinite bus can be represented by the equivalent circuit of Fig. 10, since the condition

$$\theta_a = \delta_a = 0$$

is fulfilled.

Several machines in parallel on the same bus may be

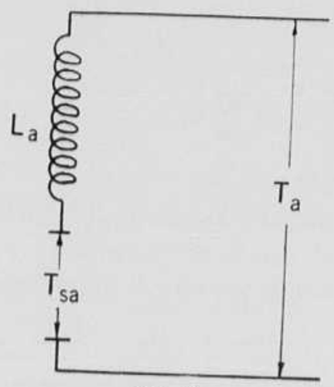


FIG. 9

represented by the diagram of Fig. 11, since the conditions

$$\theta_a - \delta_a = \theta_b - \delta_b = \ldots (= \text{bus angle in space})$$

$$T_a + T_b + T_c, \text{ etc.} = \text{bus power output} = 0$$

A transmission line may be represented by a condenser.

Thus two machines connected by a line of reactance (x) would be represented by the circuit of Fig. 12, where

$$C = \frac{x}{e^2} \tag{58}$$

Shaft torques are, of course, represented by voltages.

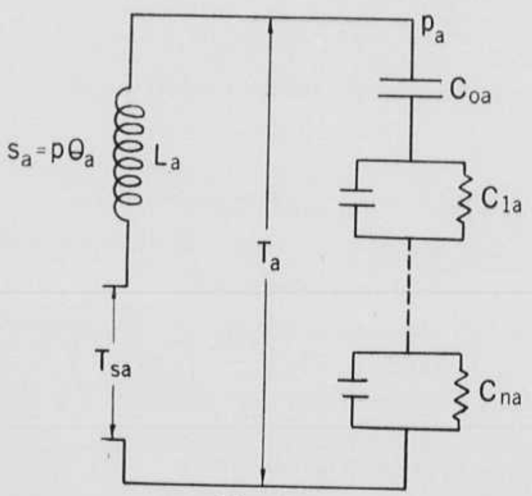


FIG. 10

Mechanical damping, such as that due to a fan on a motor shaft or that due to the prime mover, is represented by resistance in series with the inductance (L) as in Fig. 13. (R) must be chosen equal to the rate of decrease in available driving torque with increase in speed.

Governors and other prime mover characteristics may also be represented by connecting their circuits in the inductive branch of the circuit. Thus a governor which acts through a single time constant may be represented by the circuit of Fig. 14, where

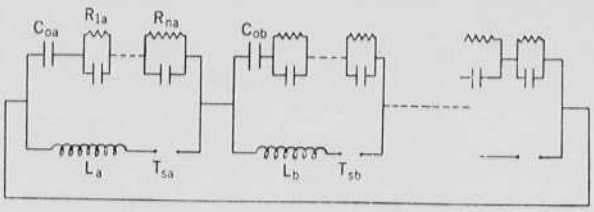


FIG. 11

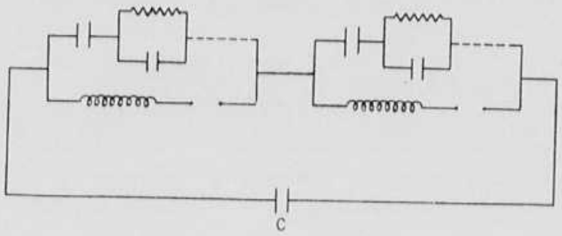


FIG. 12

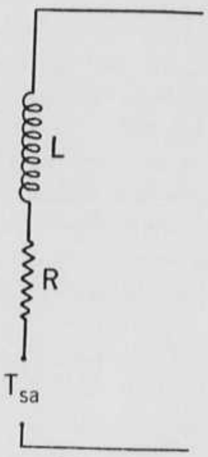


FIG. 13

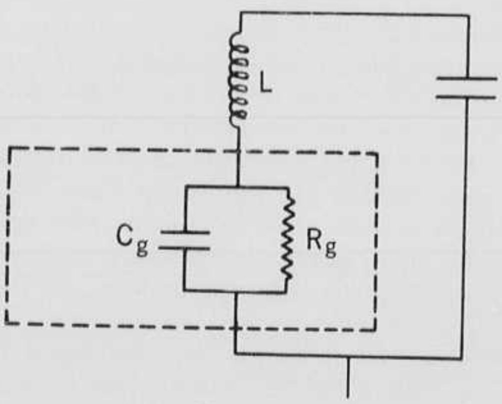


FIG. 14

$$R_g = \frac{1}{\text{regulation}}$$

$$C_g = \frac{\text{time constant of governor in elec. radians}}{R_g} \tag{59}$$

An induction motor is represented by the simple circuit of Fig. 15 and is precisely the circuit of a synchronous machine with only one time constant and $C_0 = \infty$ on account of $I = 0$.

Results similar to these have been previously shown by Arnold, Nickle,[10] and others, but simpler and more approximate circuits were used, the branches of the several circuits were not directly evaluated in terms of machine constants, and the derivation was incomplete in that the limitation to no load and zero resistance was not appreciated.

L. Torque Angle Relations of a Synchronous Machine Connected to an Infinite Bus, for Small Angular Deviations from an Average Operating Angle

There is, in general,

$$T = T_0 + \Delta T = (\psi_{d0} + \Delta \psi_d)(i_{q0} + \Delta i_q) - (i_{d0} + \Delta i_d)(\psi_{q0} + \Delta \psi_q)$$

For small angular deviations,

$$\begin{aligned} \Delta T &= i_{q0}\,\Delta \psi_d + \psi_{d0}\,\Delta i_q - i_{d0}\,\Delta \psi_q - \psi_{q0}\,\Delta i_d \\ &= \{\psi_{d0} + i_{d0}\,x_q(p)\}\,\Delta i_q - \{\psi_{q0} + i_{q0}\,x_d(p)\}\,\Delta i_d \end{aligned} \quad \textbf{(60)}$$

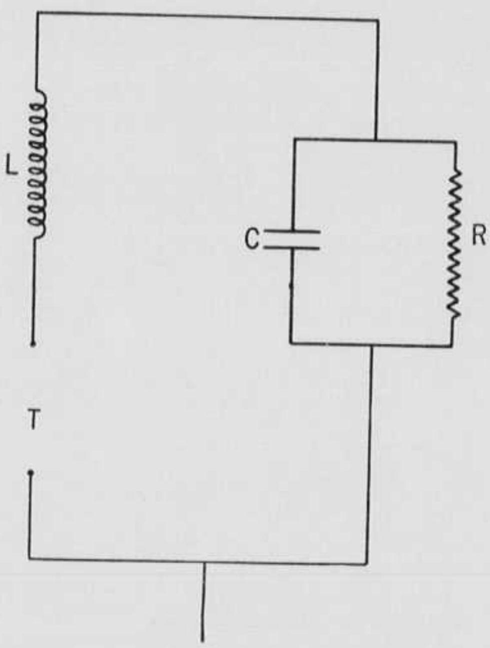


FIG. 15

$$e_{d0} + \Delta e_d = p\,\Delta \psi_d - r(i_{d0} + \Delta i_d) - (\psi_{q0} + \Delta \psi_q)(1 + p\,\Delta \delta)$$

$$e_{q0} + \Delta e_q = p\,\Delta \psi_q - r(i_{q0} + \Delta i_q) + (\psi_{d0} + \Delta \psi_d)(1 + p\,\Delta \delta)$$

$$\Delta e_d = p\,\Delta \psi_d - r\,\Delta i_d - \psi_{q0}\,p\,\Delta \delta - \Delta \psi_q$$

$$\Delta e_q = p\,\Delta \psi_q - r\,\Delta i_q + \psi_{d0}\,p\,\Delta \delta + \Delta \psi_d$$

from which there is

$$z_d(p)\,\Delta i_d - x_q(p)\,\Delta i_q = -\Delta e_d - \psi_{q0}\,p\,\Delta \delta$$

$$z_q(p)\,\Delta i_q + x_d(p)\,\Delta i_d = -\Delta e_q + \psi_{d0}\,p\,\Delta \delta$$

$$\Delta i_d = \frac{z_q(p)\langle -\Delta e_d - \psi_{q0}\,p\,\Delta \delta\rangle + x_q(p)\langle -\Delta e_q + \psi_{d0}\,p\,\Delta \delta\rangle}{D(p)}$$

$$\Delta i_q = \frac{z_d(p)\langle -\Delta e_q + \psi_{d0}\,p\,\Delta \delta\rangle - x_d(p)\langle -\Delta e_d - \psi_{q0}\,p\,\Delta \delta\rangle}{D(p)} \quad \textbf{(61)}$$

where,

$$D(p) = z_d(p)\,z_q(p) + x_d(p)\,x_q(p)$$

but from Equations (**28**),

$$\begin{aligned} e_{d0} + \Delta e_d &= e \sin(\delta_0 + \Delta \delta) \\ e_{q0} + \Delta e_q &= e \cos(\delta_0 + \Delta \delta) \\ \Delta e_d &= e \cos \delta_0\,\Delta \delta \\ \Delta e_q &= -e \sin \delta_0\,\Delta \delta \end{aligned} \quad \textbf{(62)}$$

$$\Delta i_d = \frac{-(e \cos \delta_0 + \psi_{q0}\,p)\,z_q(p) + (e \sin \delta_0 + \psi_{d0}\,p)\,x_q(p)}{D(p)} \,.\, \Delta \delta$$

$$\Delta i_q = \frac{(e \sin \delta_0 + \psi_{d0}\,p)\,z_d(p) + (e \cos \delta_0 + \psi_{q0}\,p)\,x_d(p)}{D(p)} \,.\, \Delta \delta \quad \textbf{(63)}$$

$$\Delta T = \frac{[\psi_{d0} + i_{d0}\,x_q(p)]\left\{\begin{matrix}(e \sin \delta_0 + \psi_{d0}\,p)\,z_d(p) \\ +(e \cos \delta_0 + \psi_{q0}\,p)\,x_d(p)\end{matrix}\right\} + [\psi_{q0} + i_{q0}\,x_d(p)]\left\{\begin{matrix}(e \cos \delta_0 + \psi_{q0}\,p)\,z_q(p) \\ -(e \sin \delta_0 + \psi_{d0}\,p)\,x_q(p)\end{matrix}\right\}}{D(p)} \,.\, \Delta \delta \quad \textbf{(64)}$$

say,

$$\Delta T = f(p)\,.\,\Delta \delta$$

From (**57a**) the equation for shaft torque becomes

$$\Delta T_s = (M\,p^2 + f(p))\,.\,\Delta \delta$$

Thus,

$$\Delta \delta = \frac{1}{M\,p^2 + f(p)}\,.\,\Delta T_s \quad \textbf{(65)}$$

Appendix

Formula for Linkages and Voltage in Field Circuit with no Additional Rotor Circuits

In this case the per-unit field linkages will depend linearly on the armature and field currents. That is, in general,

$$\Psi = a\,I - b\,i_d$$

Then if normal linkages are defined as those existing at no load there must be $a = 1.0$.

The quantity b may be found by suddenly impressing terminal linkages ψ_d with no initial currents in the machines and $E = 0$.

By definition there is, initially

$$i_d = -\frac{\psi_d}{x_d'}$$

but also there must be from the definition of $x_d{}^2$

$$i_d = \frac{I - \psi_d}{x_d}$$

hence there must be an initial induced field current of amount

$$I = \psi_d \left(1 - \frac{x_d}{x_d'} \right)$$

But, initially the field linkages are zero, thus

$$\Psi = \psi_d 1 - \frac{x_d}{x_d'} + \frac{b}{x_d'} = 0$$

hence $$b = x_d - x_d'$$

Similarly, there will be

$$E = \text{per-unit field voltage}$$
$$= c\,p\,\Psi + d\,I$$

Normal field voltage will be here defined as those existing at no load and normal voltage. This requires that $d = 1$. The quantity c may then be recognized as the time constant of the field in radians when the armature is open circuited, since with the field shorted under these conditions there is

$$(T_0\,p + 1)\,I = 0$$
$$c\,p\,\Psi + I = 0$$
$$\psi = I$$

$c = T_0$ = time constant of field with armature open circuited.

Bibliography

1. Doherty, R. E. and Nickle, C. A., *Synchronous Machines V*, A. I. E. E. Quarterly Trans., Vol. 48, No. 2, April, 1929.

2. Park, R. H. and Robertson, B. L., *The Reactance of Synchronous Machines*, A. I. E. E. Quarterly Trans., Vol. 47, No. 2, April, 1928, p. 514.

3. Park, R. H., "Definition of an Ideal Synchronous Machine and Formula for the Armature Flux Linkages," *General Elec. Rev.*, June, 1928, Vol. 31, pp. 332-334.

4. Alger, P. L., *The Calculation of the Reactance of Synchronous Machines*, A. I. E. E. Quarterly Trans., Vol. 47, No. 2, April, 1928, p. 493.

5. Doherty, R. E. and Nickle, C. A., *Synchronous Machines IV*, A. I. E. E. Quarterly Trans., Vol. 47, No. 2, April, 1928, p. 457, Discussion p. 487.

6. Wieseman, R. W., *Graphical Determination of Magnetic Fields; Practical Application to Salient-Pole Synchronous Machine Design*, A. I. E. E. Trans., Vol. XLVI, 1927, p. 141.

7. Doherty, R. E. and Nickle, C. A., *Synchronous Machines III, Torque-Angle Characteristics Under Transient Conditions*, A. I. E. E. Trans., Vol. XLVI, 1927, p. 1.

8. Bekku, S., "Sudden Short Circuit of Alternator," Researches of the Electrotechnical Laboratory No. 203, June, 1927.

9. Doherty, R. E. and Nickle, C. A., *Synchronous Machines I and II, An Extension of Blondel's Two-Reaction Theory*, A. I. E. E. Trans., Vol. XLV, pp. 912-47.

10. Nickle, C. A., *Oscillographic Solution of Electromechanical Systems*, A. I. E. E. Trans., Vol. XLIV, 1925, pp. 844-856.

11. Dreyfus, L., "Ausgleichvorgange Beim Plotzlichen Kurzschluss von Synchron Generatoren," *Archiv f. Electrotech.*, 5 S 103, 1916.

12. Dreyfus, L., "Ausgleichvorgange in der Symmetrischen Mehrphasenmaschine," *Elektrotech. u. Maschinenbau*, 30 S 25, 121, 139, 1912.

13. Dreyfus, L., "Freie Magnetische Energie Zwischen Verketteten Mehrphasensystemen," *Elektrotech u. Maschinenbau*, 29 S. 891, 1911.

14. Dreyfus, L., "Einfuhrung in die Theorie der Selbsterregten Schwingungen Synchroner Maschinen," *Elektrotech. u. Maschinenbau*, 29 S. 323, 345, 1911.

Mar 1, 1930
H. E. Edgerton

The exponent $\varepsilon^{j\theta}$ can be thought of as an operator which takes into account the speed of the rotor. For instance, in the first ~~of~~ the four equations, the voltages are in terms of drop with respect to the stator current. The current in the rotor, of slip frequency for an induction motor in the steady state, is speeded up by a frequency of $(1-S)$ so that its apparent frequency from the stator side is that of the fundamental. Likewise in the expressions for the rotor voltage, the stator currents ~~have~~ their frequency reduced to that of the rotor.

STEADY STATE

BALANCED STATOR AND ROTOR. BAL. VOLTAGE.

With balanced voltage on the stator we know that $E_1 = e_{+1} + e_{-1}$ and $E_2 = -j\,e_{+1} + j\,e_{-1}$ or solving for the components.

$e_{+1} = \dfrac{E_1 + jE_2}{2}$ and $e_{-1} = \dfrac{E_1 - jE_2}{2}$ Also the rotor will run at a constant speed, i.e. θ has a const. rate of increase.

$\theta = (\omega - n)t$ n = electrical angular velocity of the rotor.

In the steady-state it is known that the current in both the rotor and the stator are sinusoids of frequencies determined by the applied potential E and the slip $\left[s = \dfrac{(\omega - n)}{\omega}\right]$.

Thus the components of current are of the form:

$$i'_{+2} = I_{+2}\,\varepsilon^{j(\omega-n)t} \qquad i'_{-2} = I_{+2}\,\varepsilon^{-j(\omega-n)t}.$$

$$i'_{+1} = I_{+1}\,\varepsilon^{j\omega t} \qquad i'_{-1} = I_{-1}\,\varepsilon^{-j\omega t}$$

When the rotor is shorted $E_{+2} = E_{+1} = 0$.

this in the differential equations for the + components;

$$e_{+1} = z_1 I_{+1}\varepsilon^{j\omega t} + Mp\left(I_{+2}\,\varepsilon^{j\omega t}\right) \qquad z_1 = (r_1 + j\omega L_1)$$

$$e_{-1} = 0 = z_2 I_{+2}\,\varepsilon^{-j(\omega-n)t} + Mp\left(I_{+1}\,\varepsilon^{j(\omega-n)t}\right). \qquad z_2 = (r_2 + j\omega s L_2).$$

cancel exponentials

$$E_1 = z_1 I_{+1} + j\omega M I_{+2}$$

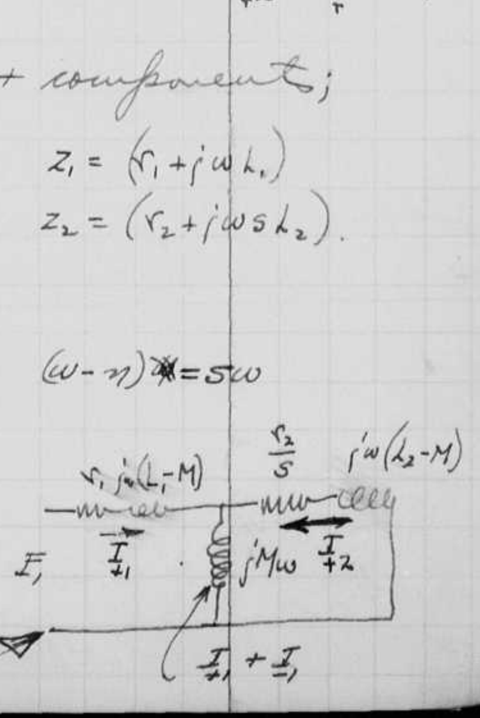
$$0 = z_2 I_{+2} + j\omega M s I_{+1} \qquad s = \frac{(\omega - n)}{\omega} \qquad (\omega - n) = s\omega$$

add and subtract

$$E_1 = \left[r_1 + j\omega(L_1 - M)\right] I_{+1} + j\omega M\left(I_{+2} + I_{+1}\right).$$

$$0 = \left[r_2 + js\omega(L_2 - M)\right] I_{+2} + j\omega M\left(I_{+1} + I_{+2}\right)$$

Which gives the ordinary steady state equations for the ind. motor and the equivalent circuit.



Mar. 1, 1930
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Ind. Motor with 1φ Rotor.

A three phase induction motor with a ~~single~~ single phase rotor circuit has a differential equation that is given by the following component equations. (this is given in Ku's paper A.I.E.E. 1929 Jan-Feb).

$$(r_1 + L_1 p) i_+' + \frac{M}{2} p\, i_2' \,\varepsilon^{jnt} = v_+ \qquad (1)$$

$$(r_1 + L_1 p) i_-' + \frac{M}{2} p\, i_2' \,\varepsilon^{-jnt} = 0 \leftarrow \text{Bal. volts.} \qquad (2)$$

$$(r_2 + L_2 p) i_2' + \frac{3}{2} M p \left(i_+' \varepsilon^{-jnt} + i_-' \varepsilon^{jnt}\right) = 0. \qquad (3).$$

Proceeding as before to pick out the steady state solutions, we select the form of i_2' since we know that it will be sinusoidal. The magnitude I_2 is undetermined but the frequency is that of the slip.

$$i_2' = I_2 \left(\frac{\varepsilon^{j\omega ts} + \varepsilon^{-j\omega ts}}{2}\right). \qquad s = \text{slip}.$$

When this is used in (1) it is found that i_+' must have two frequencies of components, one of rated frequency (that of v_+) and the other of $(1-2s)$ or $(2n-\omega)$.

From (1)

$$\cancel{(r_1 + L_1 p) i_+' + \frac{M}{2} I_2 \left[j\omega \frac{\varepsilon^{j\omega ts}}{2} - j\frac{\omega}{2} \varepsilon^{-j\omega ts} \right] \varepsilon^{jnt}}$$

$$(r_1 + L_1 p) i_+' = v_+ - \frac{M}{2} p\, I_2 \left(\frac{\varepsilon^{j(\omega s+n)t} + \varepsilon^{-j(\omega s - n)t}}{2}\right) \qquad \omega - n - n = \omega - 2n$$

$$(\omega - n + n) = \omega$$

$$(r_1 + L_1 p)\, i_+' = E\varepsilon^{j\omega t} - \frac{M}{4} I_2 \left(j\omega\, \varepsilon^{j\omega t} + j(2n-\omega)\varepsilon^{j(2n-\omega)t} \right)$$

i_+' is of the form $I_A \varepsilon^{j\omega t} + I_B \varepsilon^{j(2n-\omega)t}$.

Equating each frequency.

$$(r_1 + j\omega L_1) I_A \varepsilon^{j\omega t} = E\varepsilon^{j\omega t} - \frac{M I_2}{4} j\omega\, \varepsilon^{j\omega t}$$

$$(r_1 + j(2n-\omega)) I_B\, \varepsilon^{j(2n-\omega)t} = -j(2n-\omega)\frac{M I_2}{4}\, \varepsilon^{j(2n-\omega)t}.$$

Cancelling exponentials

$$Z_1 I_A = E - j\frac{\omega M}{4} I_2$$

$$Z_{11} I_B = -j(2n-\omega)\frac{M I_2}{4}.$$

In a similiar manner the expressions for the steady state values of i_2 and i_+ can be put into equation (3), thus determining the form of equation that i' must have.
From equation (3):

$$(r_2 + js\omega L_2) I_2 + \frac{3}{2} M p \left[I_A \varepsilon^{j\omega t - nt} + I_B \varepsilon^{j[(2n-\omega)t - nt]} \right] + \frac{3}{2} M p \left[I \quad ? \quad \right] = 0$$

$$I_2 [r_2 + j\omega L_2] \varepsilon^{j\omega s t} + I_2 (r_2 - j\omega L_2) \varepsilon^{-j\omega s t} + \frac{3}{2} M j\omega s I_A \varepsilon^{j(\omega-n)t} + \frac{3}{2} M (-j\omega s) I_B \varepsilon^{-j(\omega-n)t} + \quad ? \quad = 0.$$

$$i' = I_C \varepsilon^{-j\omega t} + I_D \varepsilon^{-j(2n-\omega)t}$$

Separateking the different freq. and cancelling ~~freq.~~ exponentials

$$I_2 [r_2 + j\omega L_2] \cancel{\varepsilon^{js\omega t}} + \frac{3}{2} M j\omega s I_A \cancel{\varepsilon^{js\omega t}} + \frac{3}{2} j s\omega M I_D \cancel{\varepsilon^{js\omega t}} = 0.$$

and
$$I_2 [r_2 - j\omega L_2] \cancel{\varepsilon^{-js\omega t}} - \frac{3}{2} M j\omega s I_B \cancel{\varepsilon^{-js\omega t}} - \frac{3}{2} j s\omega M I_C \cancel{\varepsilon^{-js\omega t}} = 0$$

I_A see below

?

I_B

$$I_2 Z_2 + jX I_A + jX I_D = 0.$$

$$I_2 (Z_2)_c - jX I_B - jX I_C = 0.$$

Equation (2) with known value of i_2 from assumption of the steady state.

$$(r_1 + L_1 p) i' + \frac{M}{2} p \frac{I_2}{2} \left(\varepsilon^{j\omega s t - n} + \varepsilon^{-j\omega s t - nt} \right) = 0$$

$$\varepsilon^{j(\omega - 2n)t} \qquad \varepsilon^{-j\omega t}$$

$$(r_1 + L_1 p) i' + \frac{M I_2}{4} \left(-j\omega \varepsilon^{-j\omega t} - j(2n-\omega) \varepsilon^{-j(2n-\omega)t} \right) = 0.$$

i' must have two frequencies and be of the form.

$$i' = I_A \varepsilon^{-j\omega t} + I_B \varepsilon^{-j(2n-\omega)t}$$ since also $i_+ + i' = \underline{\text{real}} = i_1$

Equating equal freq. and cancelling exponents.

$$(r_1 - j\omega L_1) I_A - \frac{M I_2}{4} j\omega = 0$$

$$(r_1 - j(2n-\omega) L_1) I_B - \frac{M I_2}{4} j(2n-\omega) = 0.$$

Collecting various vector expressions.

$$\mathrm{I}\ \begin{cases} Z_1 I_A = E - j\omega \dfrac{M I_2}{4} \\ Z_{11} I_B = -j \dfrac{M I_2 (2n-\omega)}{4} \end{cases}$$

From + sequence

$$\mathrm{II}\ \begin{cases} \cancel{Z_{1c} I_A = j\omega \dfrac{M I_2}{4}} \\ \cancel{Z_{11c} I_B = j(2n-\omega) \dfrac{M I_2}{4}} \end{cases}$$

Dont need these.

$$\mathrm{III} \quad Z_2 I_2 + j\tfrac{3}{2} M\omega s\left[I_A + I_B\right] = 0$$

From I and III.

$$I_2 = \frac{-E\, j\frac{3}{2}M\omega s/Z_1}{\left[Z_2 + \frac{3}{8}\frac{M^2\omega^2 s}{Z_1} + \frac{3}{8}\frac{M^2\omega(2n-\omega)s}{Z_{11}}\right]}$$

$$= \frac{-E\, j\frac{3}{2}M\omega s/Z_1}{Z_2 + \frac{3}{8}M^2\omega^2 s\left[\frac{1}{Z_1} + \frac{2n-\omega}{\omega}\frac{1}{Z_{11}}\right]}$$

$$\frac{2n-\omega}{\omega} = 1-2s$$

since

$$\frac{2n-2\omega+\omega}{\omega} = \frac{\omega}{\omega} - 2\frac{(\omega-n)}{\omega} = 1-2s.$$

$$= \frac{-E\, j\frac{3}{2}M\omega s/Z_1}{Z_2 + \frac{3}{8}M^2\omega^2 s\left[\frac{Z_{11}+(1-2s)Z_1}{Z_1 Z_{11}}\right]} = \frac{+E}{\mathcal{Z}_2}$$

$$\frac{1}{\mathcal{Z}} = \frac{1}{j\left[\frac{2Z_1Z_2}{3\, s\omega M} + \omega M\left[\frac{1}{4} + \frac{1}{4}(1-2s)\frac{Z_1}{Z_{11}}\right]\right]}$$

$$Z_1 = r_1 + j\omega L_1$$

$$Z_{11} = r_1 + j(2n-\omega)L_1$$

$$= r_1 + j\omega L_1(1-2s)$$

$$I_A = \frac{E - j\frac{\omega M}{4}\left(\frac{+E}{\mathcal{Z}_2}\right)}{Z_1} = \frac{E\left(1 - j\frac{\omega M}{4\mathcal{Z}_2}\right)}{Z_1} = \frac{E}{\mathcal{Z}_A}$$

$$I_B = -j\frac{M}{4}\frac{(2n-\omega)}{Z_{11}}\left(\frac{+E}{\mathcal{Z}_2}\right) = -j\frac{M}{4}\frac{(1-2s)\omega}{Z_{11}}\frac{E}{\mathcal{Z}_2} = -j\frac{\omega M}{4}\frac{(1-2s)E}{Z_{11}\mathcal{Z}_2} = \frac{E}{\mathcal{Z}_B}$$

$$\mathcal{Z}_A = \frac{Z_1}{1 - j\frac{\omega M}{4\mathcal{Z}_2}}$$

$$\mathcal{Z}_B = \frac{Z_{11}}{-(1-2s)j\frac{\omega M}{4\mathcal{Z}_2}} =$$

Mar 2 1930
H. E. Edgerton

Currents in an Ind. Motor with a 1 phase rotor.

In the preceding three pages, I have obtained expressions for the magnitudes (vector) of the current in the rotor and the two components of current in the stator. This ~~is~~ was done by assuming the current in the rotor to be a sinusoidally varying one, the magnitude being unknown. By inspection of the differential equations it was found that two current components must exist in the stator, one of fundamental frequency and one of (1-2S) fund. freq. This was also known experimentally but it could be deduced from the differential equations. Now the various frequencies ~~were~~ of currents were given unknown magnitudes and placed in the differential equations for the steady state. In this case the p (or $\frac{d}{dt}$) is replaced by the $j\omega$ or $j(2n-\omega)$ for the (1-2S) frequency. In this manner three equations are obtained with three unknowns after the different frequency components are equated and the exponentials cancelled. The three unknowns (I_2, I_A, and I_B) are then solved in terms of

r_1 L_1 r_2 L_2 M S. E.

stator resis
" inductance
rotor resis
" inductance
mutual
slip
applied stator potential

L_1 = synchronous self inductance of stator per phase. It is the self inductance of one phase (stator), plus the mutual effect of each of the other stator phases.

L_2 = self inductance of the ~~field or~~ rotor.

The current in the rotor is then:

$$i_2' = \frac{I_2}{\cdot}\overset{\cos}{\sin}\,\omega s t = \frac{E}{Z_2}\overset{\cos}{\sin}\,s\omega t \quad \text{where } Z_2 = j\left[\frac{2}{3}\frac{z_1 z_2}{s\omega M} + \omega M\left(\frac{1}{4}+\frac{1}{4}(1-2s)\frac{z_1}{z_{11}}\right)\right]$$

$$z_1 = r_1 + j\omega L_1$$

$$z_2 = r_2 + j\omega L_2 S. \qquad z_{11} = r_1 + j\omega L_1(1-2s).$$

Mar. 10, 1930

The ~~stator~~ current in phase one is equal to

$$i_1' = i_+' + i_-'$$

$$= 2(I_A \cos\omega t + I_B \cos(2n-\omega)t)$$

$$= 2[I_A \cos\omega t + I_B \cos\omega(1-2s)t]$$

$\cancel{\left(\frac{2n-\omega}{\omega}\right)\omega}$

$\cancel{\omega\left(-1+\frac{2n}{\omega}\right)}$

$\cancel{s = \frac{\omega - n}{\omega}}$

$\left(\frac{2n - 2\omega + \omega}{\omega}\right)\omega$

$= (1-2s)\omega$. ✓

And the rotor current as demonstrated on the previous page is

$$i_2' = I_2 \cos s\omega t.$$

In these expressions I_2, I_A, and I_B are vectors which are determined by the constants of the motor that is under question. (and the slip).

$$I_2 = \frac{E}{Z_2} \qquad Z_2 = j\left[\frac{2}{3}\frac{z z_2}{s\omega M} + \omega M\left[\frac{1}{4} + \frac{1}{4}(1-2s)\frac{Z_1}{Z_{11}}\right]\right]$$

$$I_A = \frac{E}{Z_A} \qquad Z_A = \frac{z_1}{1 - j\frac{\omega M}{4 Z_2}}$$

$$I_B = \frac{E}{Z_B} \qquad Z_B = \frac{z_{11}}{-(1-2s)\,j\frac{M\omega}{4 Z_2}}$$

Needless to say the above calculations are quite difficult to make since they involve quite a few vector expressions.

Major oral exam in Elect Eng. Apr. 2, 1930 10-200

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

____ negative strip(s)

3 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 18 and 19.

Item(s) now housed in accompanying folder.

Mar. 10, 1930.

The ~~stator~~ current in phase one is equal to

$$i_1' = i_f' + i_b'$$

$$= 2(I_A \cos \omega t + I_B \cos (2n - \omega)t)$$

$$= 2\left[I_A \cos \omega t + I_B \cos \omega(1-2s)t\right]$$

And the rotor current as demonstrated on the previous page is

$$i_2' = I_2 \cos s\omega t.$$

$\cancel{\frac{(2n-\omega)}{\omega}\omega}$

$\cancel{\omega\left(-1+\frac{2n}{\omega}\right)}$

$\cancel{s = \frac{\omega - n}{\omega}}$

$\left(\frac{2n - 2\omega + \omega}{\omega}\right)\omega$

$= (1-2s)\omega.$ ✓

In these expressions I_2, I_A, and I_B are vectors which are determined by the constants of the motor that is under question. (and the slip).

$$I_2 = \frac{E}{Z_2} \qquad Z_2 = j\left[\frac{2}{3}\frac{z z_2}{s\omega M} + \omega M\left[\frac{1}{4} + \frac{1}{4}(1-2s)\frac{z_1}{z_{11}}\right]\right]$$

$$I_A = \frac{E}{Z_A} \qquad Z_A = \frac{z_1}{1 - j\frac{\omega M}{4 Z_2}}$$

$$I_B = \frac{E}{Z_B} \qquad Z_B = \frac{z_{11}}{-(1-2s)\,j\,\frac{M\omega}{4 Z_2}}$$

Needless to say the above calculations are quite difficult to make since they involve quite a few vector expressions.

Major oral exam in Elect Eng. Apr. 2, 1930 10 - 200

Notebook # 3

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1

Equation for torque

$$T = P\, j' M \left(i'_{a2}\, i'_{b1}\, \varepsilon^{j\theta} - i'_{a1}\, i'_{b2}\, \varepsilon^{j\theta} \right)$$

with the single phase rotor.

$$i'_{b1} + i'_{b2} = I_b \cos s\omega t = I_b \frac{\varepsilon^{j\omega t s} + \varepsilon^{-j s\omega t}}{2.}$$

$$-j i'_{b1} + j i'_{b2} = 0$$

$$i'_{b1} = \frac{I_b}{2} \cos s\omega t$$

$$i'_{b2} = \frac{I_b}{2} \cos s\omega t.$$

$(2n - \omega - \omega + \omega$
$2(n-\omega)$
$\omega - \cdot - s2 + \omega$
$\omega - 2s.$

$-j2n + j\omega + j\omega - js$
$-j2(\omega - n) - js.$
$-j2s - js$
$-j3s.$

$$T = j PM \left(i'_{a2}\, \varepsilon^{j\theta} - i'_{a1}\, \varepsilon^{-j\theta} \right) \frac{I_b}{2} \cos s\omega t.$$

$$i'_{a1} = I_A\, \varepsilon^{j\omega t} + I_B\, \varepsilon^{+j(2n-\omega)t.}$$

$$i'_{a2} = I_A\, \varepsilon^{-j\omega t} + I_B\, \varepsilon^{-j(2n-\omega)t}$$

$$\theta = (\omega - s)t$$

$$T = j PM \Big(I_A\, \varepsilon^{-j\omega t}\, \varepsilon^{j(\omega - s)t} + I_B\, \varepsilon^{-j(2n-\omega)t}\, \varepsilon^{j(\omega - s)t}$$

$$- \text{conj} \Big) \frac{I_b}{2} \cos s\omega t.$$

$$= j PM \left(I_A\, \varepsilon^{-jst} + I_B\, \varepsilon^{-j3st} \right) \frac{I_b}{2} \cos s\omega t.$$

$T =$ real $\times 2$

1950

Glued in Apr 21 1930. H.E.S.

1930

[illegible]

A salient-pole motor when brought up to speed as an induction motor usually pulls into synchronism and operates as a synchronous motor, if the shaft load is small and [illegible]. The salient poles follow the rotating field of the armature by an angle sufficient to supply the load on the shaft. This reluctance torque is a function of the angle is a sin 2θ term. The reason that it depends upon the double angle is because the polarity of the salient poles is not definite but depends upon the position. In other words the salient pole is a mass of iron that endeavors to place itself in a place where the magnetic flux will be a maximum and it does not depend on whether the field is north or south.

The synchronous torque expressions for the salient-pole machine as given by [illegible] and [illegible] at the A.I.E.E. convention in June 1931, are

$$P = \frac{EV}{x_d} \sin\theta + \frac{V^2 (x_d - x_q)}{x^2} \sin 2\theta$$

$$+ \frac{V^2}{x^2} (1 - \cos 2\theta)$$

The first term (sin θ) is the synchronizing power due to the field current. E is the induced e.m.f. in the stator, V is the applied potential, x_d is the reactance in the direct axis. [illegible] where x_q is the reactance in the quadrature axis and r is the armature resistance.

The second term (sin 2θ) is the reluctance torque. The last term represents the I^2R losses that occur in the armature. The sum of these components equals the input to the motor.

Because the field current is [illegible] the input to the motor is equal to

$$P = \frac{V^2 (x_d - x_q)}{x^2} \sin 2\theta + \frac{V^2 r}{x^2}$$

[illegible] has been neglected here. [illegible] can be

[illegible] Apr 21 1930.

PULLING INTO STEP OF A SALIENT-POLE SYNCHRONOUS
MOTOR UNDER LIGHT LOAD CONDITIONS.

A salient-pole motor when brought up to speed as an induction motor usually pulls into synchronism and operates as a reluctance motor, if the shaft load is small and other conditions favorable. The salient-poles follow the rotating m.m.f. of the armature by an angle sufficient to supply the load on the shaft. This reluctance torque as a function of the angle is a sin 2θ term. The reason that it depends upon the double angle is because the polarity of the salient poles is not definite but depends upon the position. In other words the salient pole is a mass of iron that endeavors to place itself in a place where the maximum flux will exist and it does not depend on whether the field is north or south.

The synchronous torque expressions for the salient-pole machine as given by Doherty and Nickle at the A.I.E.E. convention in June 1926, are

$$P = \frac{EV\,x_d}{z^2}\sin\theta + \frac{V^2(x_d - x_q)}{z^2}\sin 2\theta + \frac{rV}{z^2}(V - E\cos\theta).$$

The first term (sin θ) is the synchronizing power due to the field current. E is the induced e.m.f. in the stator. V is the applied potential. x_d is the reactance in the direct axis. $Z^2 = \cancel{x_d x_q}$ $r^2 + x_d x_q$ where x_q = the reactance in the quadrature axis and r = the armature resistance.

The second term (sin 2 θ) is the reluctance power. The last term corrects for the losses that exist in the armature. The sum of these components equals the input to the motor.

Before the field current is connected the input to the motor is equal to

$$P = \frac{V^2(x_d - x_q)}{z^2}\sin 2\theta + \frac{r\,V^2}{z^2}$$

Core loss has been neglected here. In case measurements can be made then the core loss should be subtracted from P before it is equated to the other quantities.

$$P - (\text{core loss}) = \frac{V^2 (x_d - x_q)}{z^2} \sin 2\theta + \frac{r V^2}{z^2}$$

The shaft load being small (only windage and friction), sin 2 θ will be small and satisfies the equation in four positions for each 360 electrical degrees of displacement. Two of these angles are unstable (90 and 270 degrees) since the slope of the torque curve is negative. The two possible operating angles are 0 and 180 degrees. When the motor pulls into step as a reluctance motor it may be at either of these angles (0 pr 180 degrees.)

Thus when the field is connected to a supply of d-c. there are three possibilities; (1) the motor will operate at zero angular displacement but with a much smaller armature current.

(2) The motor will operate at 180 degrees of angular displacement but with <u>increased</u> armature current.

(3) The motor will not operate continuously as case (2) if the field current is large enough. In this case the motor must slip a pole, resulting in voilent pulsations of current and power.

The three attached oscillograms were taken for the three cases that are listed above.

December 18th, 1929.

H. E. Edgerton.

Massachusetts Institute of Technolo gy.

Notebook # 3

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Usc. No. 1.

made then the core loss should be subtracted from P before it is equated to the other quantities.

$$P - (\text{core loss}) = \frac{V^2 (x_d - x_q)}{z^2} \sin 2\theta + \frac{r V^2}{z^2}$$

The shaft load being small (only windage and friction), sin 2 θ will be small and satisfies the equation in four positions for each 360 electrical degrees of displacement. Two of these angles are unstable (90 and 270 degrees) since the slope of the torque curve is negative. The two possible operating angles are 0 and 180 degrees. When the motor pulls into step as a reluctance motor it may be at either of these angles (0 pr 180 degrees.)

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(1) the motor will operate at zero angular displacement but with a much smaller armature current.

(2) The motor will operate at 180 degrees of angular displacement but with increased armature current.

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Osc. No. 1.

Nov 15

Machine 804 A + B

With only Wind & Friction on set & small machine driving

Watts	Vt	Ia	RPM
1580	220	4.18	1204
1520	218	4.1	1204

With large machine excited for core loss.

Watts input	V_t 804 B	Ia	I_f 804 A	RPM	E_a 804 A
2020	220	5.34	7.3	1210	211
2160	219	5.64	9.2	1208	244
2320	218.5	6.05	10.65	1202	266
2528	217.5	6.70	12.5	1201	287
2980	215.0	7.90	16.0	1196	326

Short circuit on 804 A.

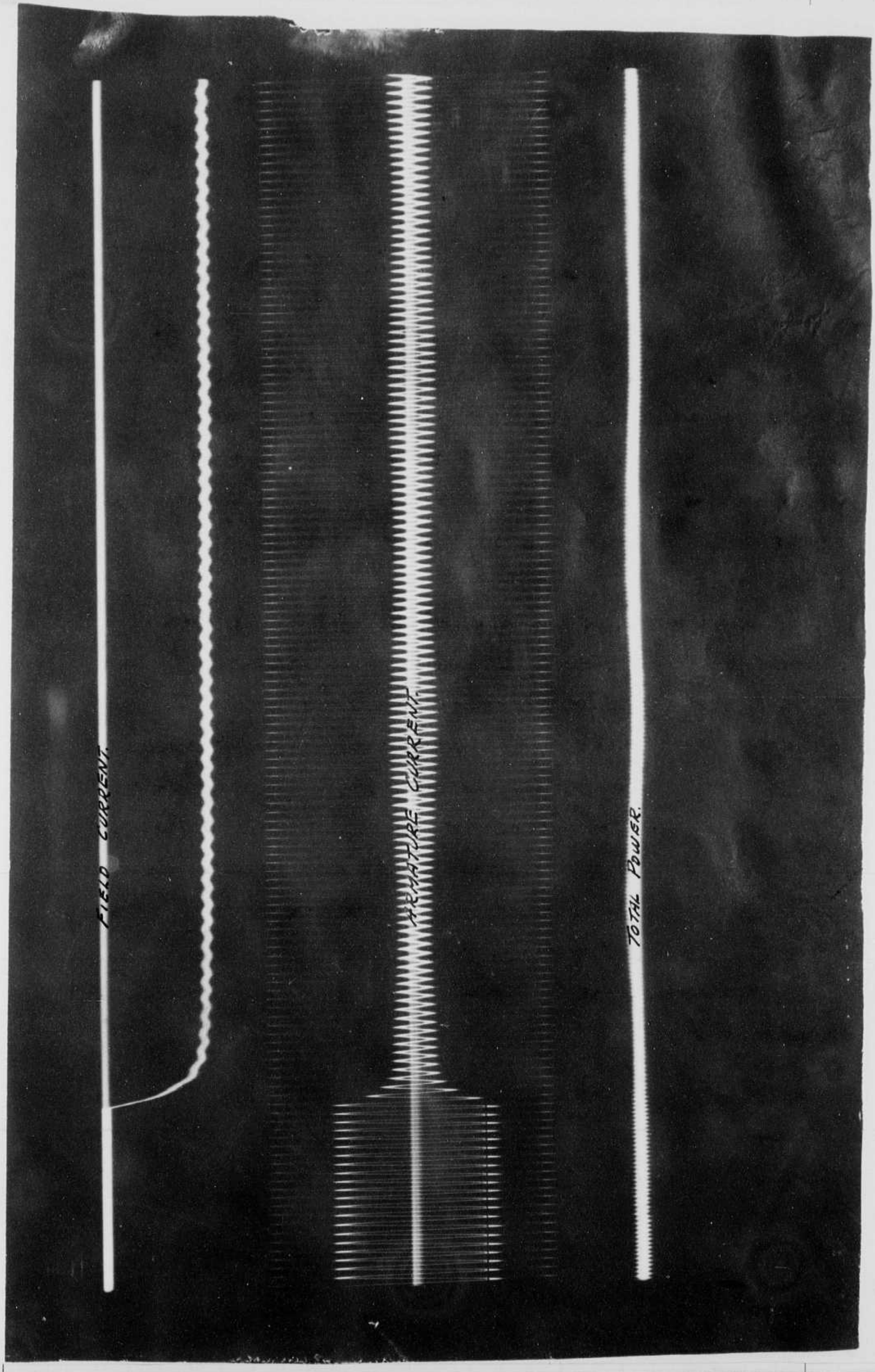
Watts input	V_t 804 B	I_a 804 B	I_f 804 A	Speed	I_a 804 A
1880	218	5	5.05	1204	76.8
2840	216.8	7.5	9.25	1205	141
2140	217.5	5.65	6.50	1205	99.2
3600	2170	9.6	11.45	1206	174
1440	217.5	3.85	0	1202	—

With added reactance on 804 A & shortcircuited

I_f	I_{a_1}	I_{a_2}	I_{a_3}	RPM	Ea to N	X_s	
4.6	22	21.8	21.5	1200	79	3.62	Iron core Reactors
7.7	37	36.5	36.2		129.5	3.54	
10.1	47.6	46.5	46.1		152	3.26	
12.3	55.9	54.3	54.1		167.5	3.06	
4.15	51.2	50.8	49.9		72	1.42	air core reactors
5.20	64	63.7	62.8		89.5	1.41	
6.40	78.4	77.8	77.3		108.5	1.395	

GB

USC. No. 1.



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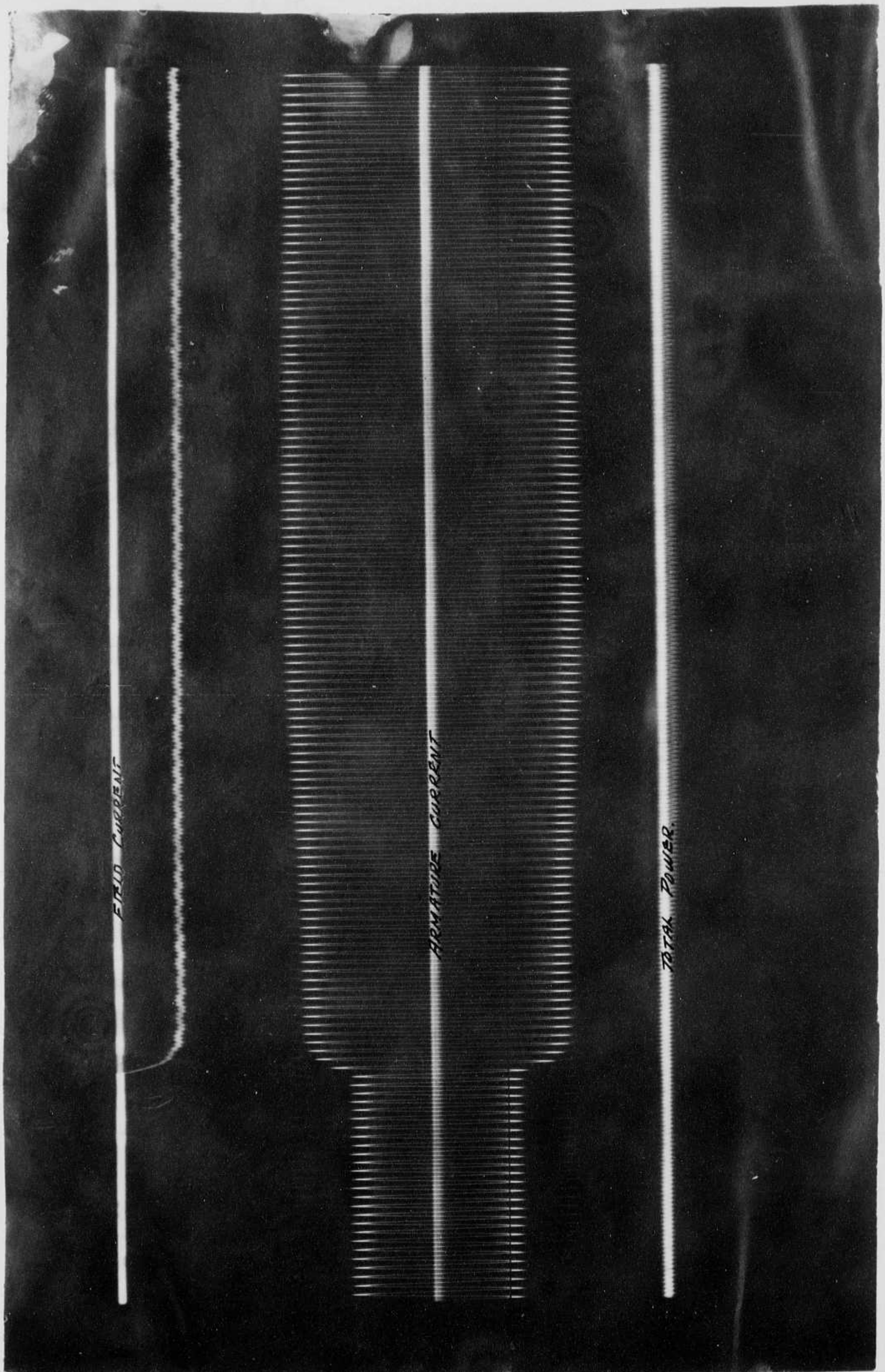
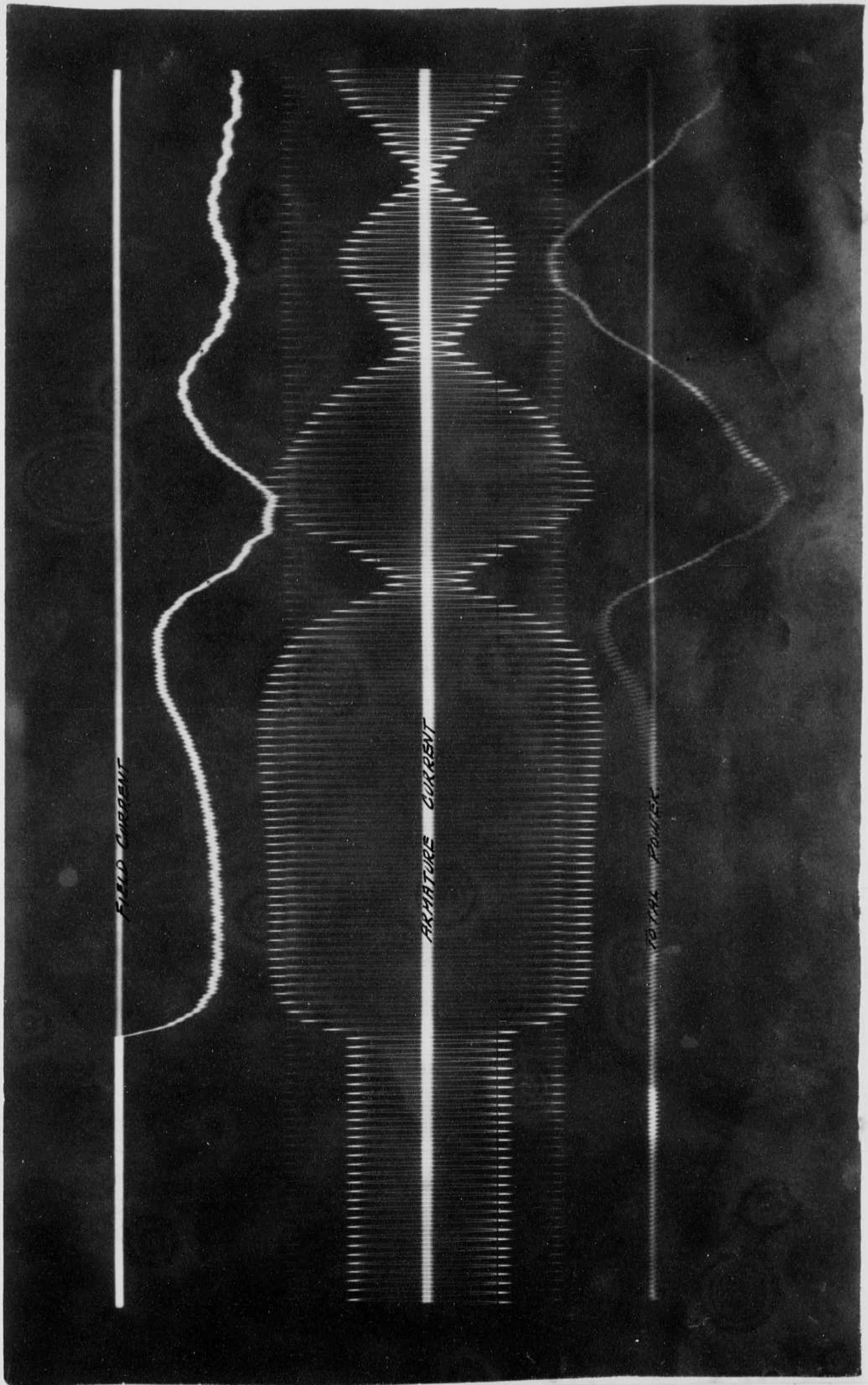


Osc. No. 2

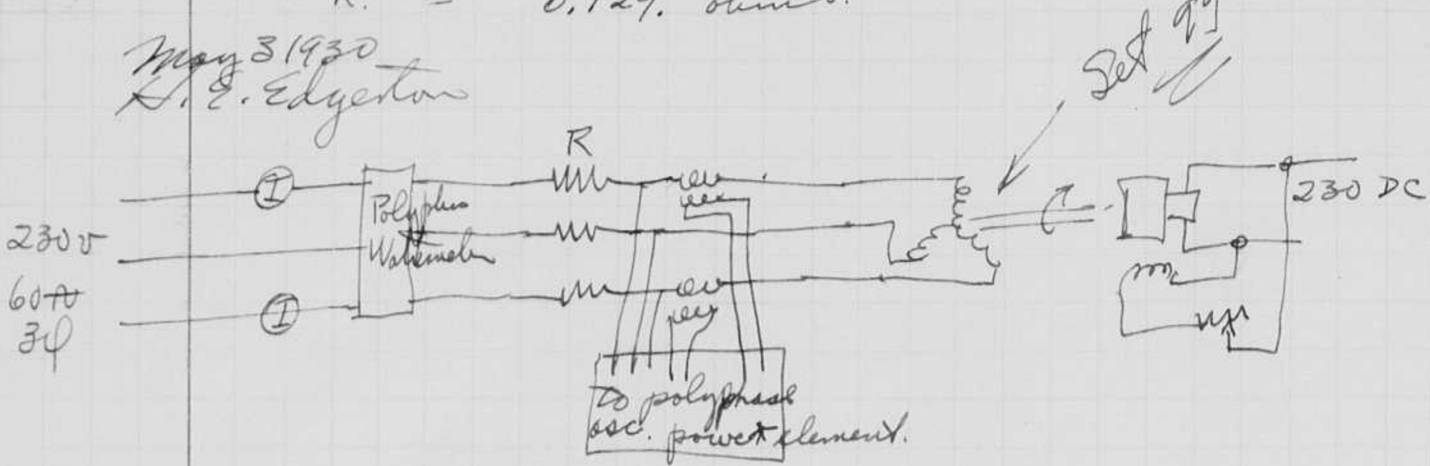


Osc. No. 3.

May 2 1930.
H. E. Edgerton

Resistance of leads for measuring resistance
= ~~0.153~~ ohms. loose contact.
R. = 0.129. ohms.

May 3 1930
H. E. Edgerton



When R was about 0.7 of an ohm the damping was small but still positive.

$R_1 = 1.46 - .13 = 1.33$ ohms.
$R_2 = 1.82 - .13 = 1.69$
$R_3 = 1.62 - .13 = 1.49$

With this resistance the motor has very little damping with E = V. Excitation on fld 1 with 110 volts as source.

Could not show neg power running as ind motor with 1φ rotor.

$R_1 = 3.23 - .13$
$R_2 = 2.88 - .13$
$R_3 = 2.49$

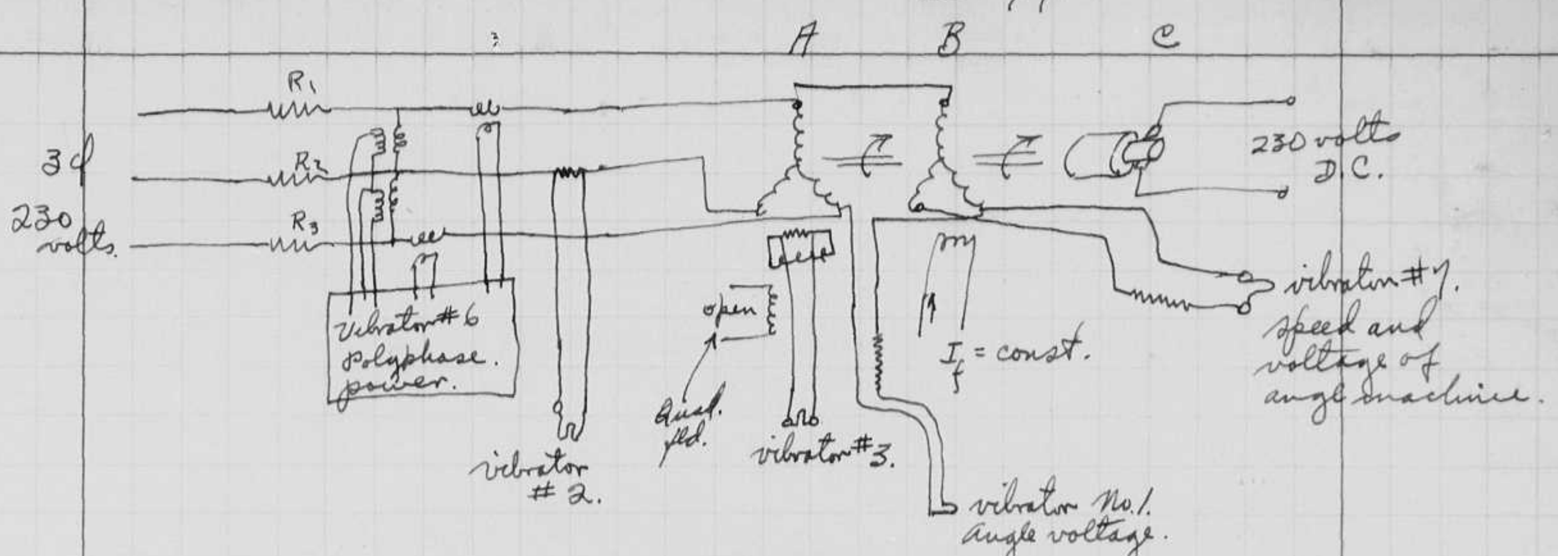
Osc No 7. No excitation 1φ rotor. (Regular field winding short-circuited)
D.C. Machine delivering power into lines.
The rotor current vibrator was sticking

Osc No. 8. Repeat. No zeros for V_{angle} and V_{speed}.

Osc No 9, calib reduced on Vibrator #3 (rotor amps).

Hot. $R_1 = 2.29 - .13 = 2.16$
$R_2 = 2.40 - \quad = 2.27$
$R_3 = 2.32 - \quad = 2.19$
3 | 6.62
2.21 ohms avg.

Set 99



Osc.	H.W	I_1	I_2	V	I_f	E.	
10	547400 × 4	656691	656760	24873		24873	$R_1 R_2$ and R_3 shorted for calib. run.
Calibration	19.2	19.0	20.7	228		252	Fld No 1 (West)
	7.68 kW.						

Failed to record the zero line of the power. Same as film No. 9.

Windage Friction (coreloss of 99 A and 99 B).

	× 2						
	5.6	3.±	3±	228		226	Fld No 1 (West).

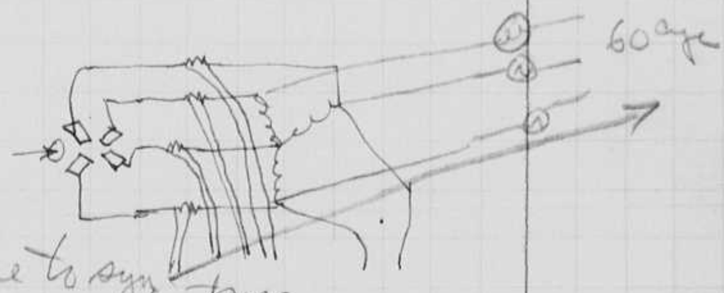
Osc — Self oscillation. Quad fld open.

the Power element put ahead of the resistances.

11.						230.	zero of P-Power shifted. other calib same.
12.	—	—	—	234		220	

1

13 S.S. test on new switch



V	I
98.	~~9.96~~ 9.1

Slow osc. osc due to syn arrangement, too osc.

14. Same except time delay relay set on fastest point.

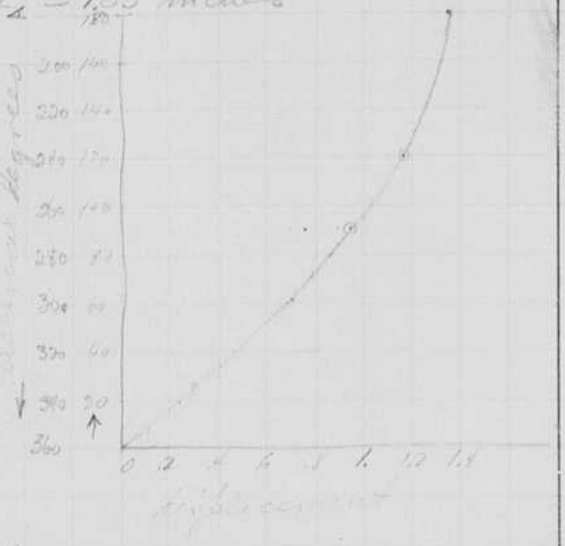
Calculation of Oscillograms.

May 4 - 1930
H. E. Edgerton

From film no 9. when θ = 180 electrical degrees
E_L = 1.35 inches

Osc. 11.

P inch	Osc.	E_L inch	P inch	I inch	θ	P kw.	KVA.
-.39	1	-.23	-.60	.20	-20	-4.61	2.97
.09	2	+.07½	.39	.09	6	-3.0	1.33
.71	3	.40	+.06	.36	35	+.46	5.35
1.25	4	.65	+.71	.60	58	5.46	8.92
1.49	5	.78	1.25	.70	72	9.6	10.4
1.52	6	.86	1.49	.72	79	11.45	10
1.41	7	.79	1.52	.65	71	11.7	9.65
1.15	8	.68	1.41	.53	61	10.8	7.87
.68	9	.40	1.15	.32	35	8.83	4.75
.09	10	+.08	.68	.07	7	5.225	1.04
-.39	11	-.27	+.09	.20	-19	-.692	2.97
-.60	0	-.45	-.39	.37	-36	-3.0	5.5



From osc. 10
1 inch of Poly Power
= 7.68 KW.

0.5 inch = 188 amp for 7.42
1 inch = 14.84 KVA.

Calc of Evans and Sels chart.

$V = 228$
$R = 2.31 + 0.23 = 2.54$ ohms per phase
$X = 4.53$ ohms
$x_3/x_0 = 0.14 \times 3$
$Z = \sqrt{2.54^2 + 4.53^2} = 5.2$ ohms.

$P = \frac{V^2 R}{Z^2} = \frac{228^2 \; 2.54}{(1000) \; 27} = \underline{4.79 \text{ Kw}}$

$Q = \frac{V^2 X}{Z^2} = \frac{228^2 \; 4.53}{27} = 8.73$ KW.

$P_{cd} = \frac{228 \cdot 228}{5.2 \cdot 1000} = 10.1$ kw

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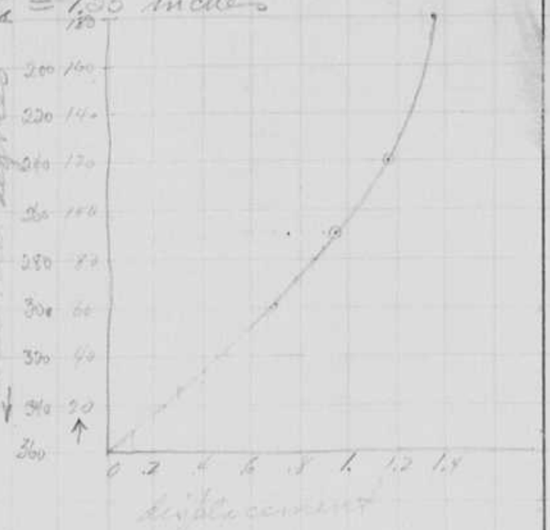
Calculation of Oscillograms.

May 4. 1930
H. E. Edgerton

From film no 9. when θ = 180 electrical degrees
E_L = 1.35 inches

Osc. 11.

P inch	Pos.	E_L inch	P inch	I inch	θ	P kW.	KVA.	1.1
-.39	1	-.23	-.60	.20	-20	-4.61	2.97	3.26
.09	2	+.07	.39	.09	6	-3.0	1.33	1.46
.71	3	.40	+.06	.36	35	+.46	5.35	5.88
1.25	4	.65	+.71	.60	58	5.46	8.92	9.82
1.49	5	.78	1.25	.70	72	9.6	10.4	11.45
1.52	6	.86	1.49	.70	79	11.45	10.40	11.45
1.41	7	.79	1.52	.65	71	11.7	9.65	10.6
1.15	8	.68	1.41	.53	61	10.8	7.87	9.66
.68	9	.40	1.15	.32	35	8.83	4.75	5.22
.09	10	+.08	.68	.07	7	5.225	1.04	1.15
-.39	11	-.27	+.09	.20	-19	-.692	2.97	3.26
-.60	0	-.45	-.39	.37	-36	-3.0	5.5	6.06



From osc. 10

1 inch of Poly Power = 7.68 KW.

0.5 inch = 18.8 amp or 7.42

1 inch = 14.84 KVA.

Calc of Evans and Sels chart.

$V = 228$

$R = 2.21 + 0.2^2 = 2.54$ ohms per phase

$X = 4.53$ ohms

$X_e / X_g = 0.14 \times 3$

$Z = \sqrt{2.54^2 + 4.53^2} = 5.2$ ohms.

$P = \frac{V^2 R}{Z^2} = \frac{228^2 \, 2.54}{(1000)\, 27} = 4.79 \text{ Kw}$

$Q = \frac{V^2 X}{Z^2} = \frac{228^2 \, 4.53}{27} = 8.73 \text{ KW.}$

$R_d = \frac{228 \times 230}{5.2 \; 1000} = 10.1 \text{ kw}$

0.45
73.45
37.00

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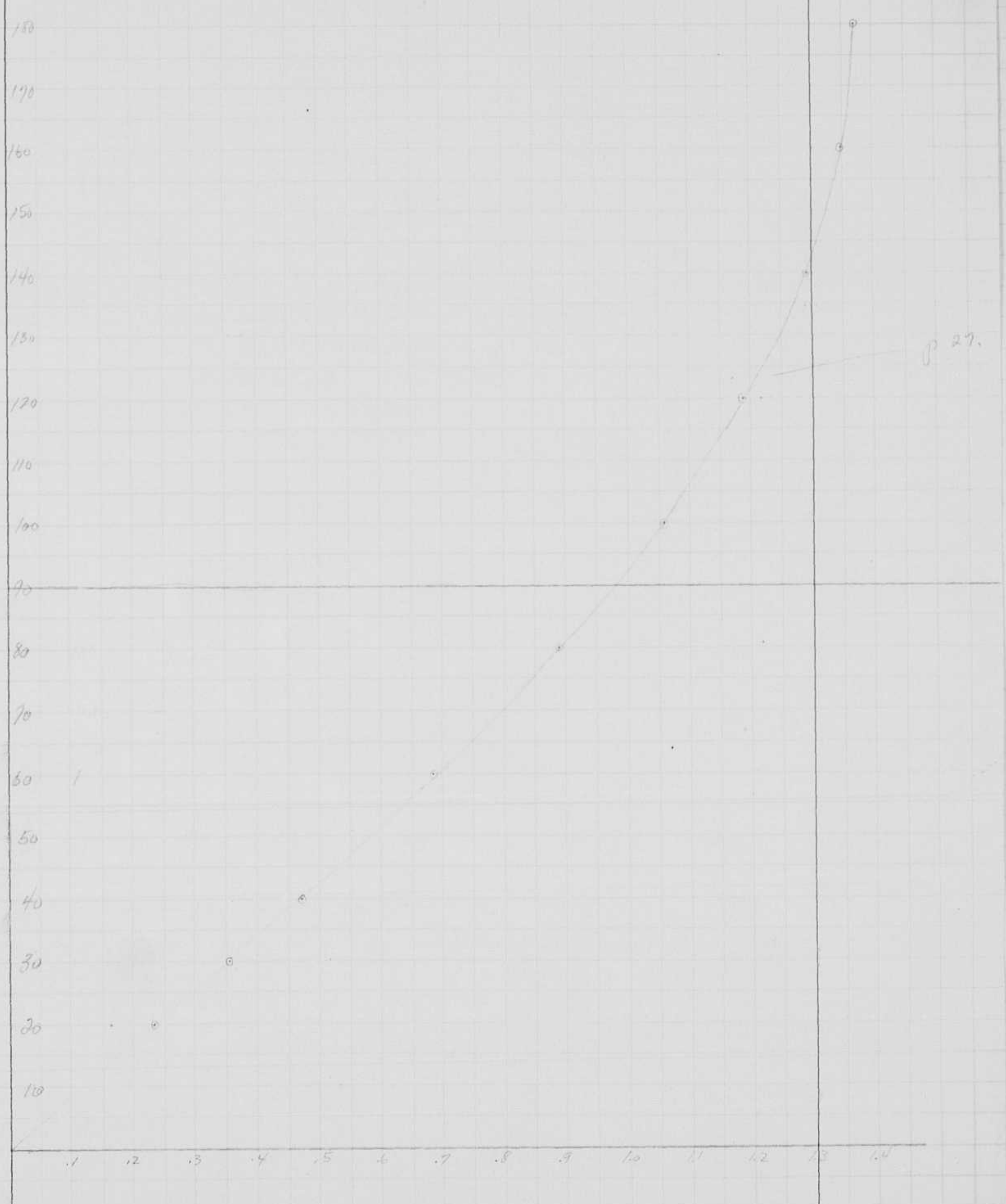
180
170
160
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.1
1.2
1.3
1.4
ρ 27.

Calculation of Oscillogram No. 9.

May 4 1930
H. E. Edgerton

No.	angle volts inches	θ_{deg}	P inch	P kw.
1	1.32	+147	.20	1.53
2	1.22	124	.35	2.69
3	1.1	107	.55	4.22
4	.97	88	.66	5.07
5	.82	72	.53	4.07
6	.61	52	.21	1.61
7	.41	35	-.03	-.23
8	.22	18	-.16	-1.23
9	.00	00	-.12	-0.922
10	.24	21	+.03	+.23
11	.42	36	.18	1.38
12	.60	51	.33	2.54
13	.79	69	.54	4.15
14	.97	88	.69	5.30
15	1.10	107	.57	4.38
16	1.20	122	.32	2.46
17	1.29	140	.05	.384
18	1.35	161	-.13	-1.0
19	1.37	180	-.15	-1.15
20	1.35	161	-.02	-.154
21	1.32	147	+.16	+1.23
22	1.25	130	.30	2.3
23	1.15	114	.53	3.84
24	1.02	94	.66	5.07
25	.85	75	.58	4.45

Calc of slip.

180 degrees = 15 cycles of time

$\frac{d\theta}{dt} = \frac{180}{.25} = 720$ elect deg/sec.

$\omega = 360 \times 60 =$ syn speed in elect deg/sec $= 21{,}600.$ elect deg/sec.

$s = \frac{\frac{d\theta}{dt}}{\omega} = \frac{720}{21600} = 3.33$ percent slip.

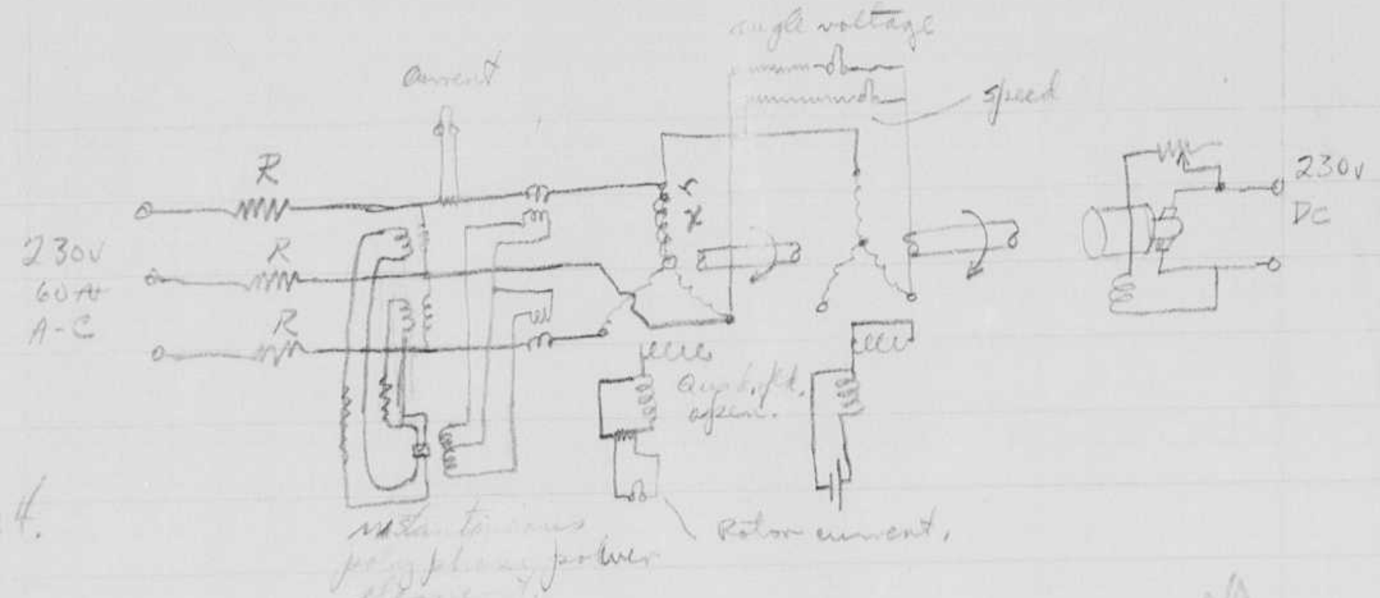

Fig 4.

wiring diagram for oscillogram no ~~4~~ 7 8 9 ?

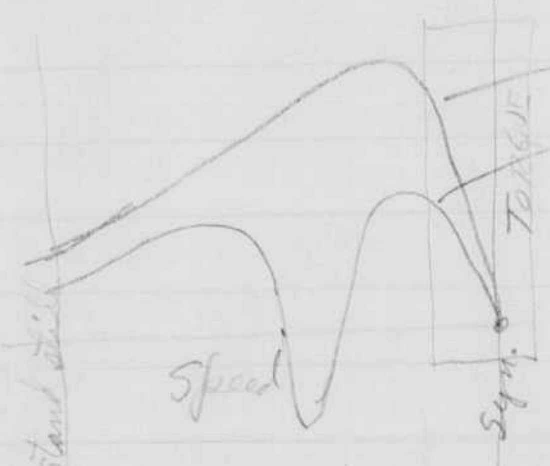

Fig 1.

polyphase rotor winding

single phase rotor winding (average torque curve. The instantaneous torque varies at double slip frequency).

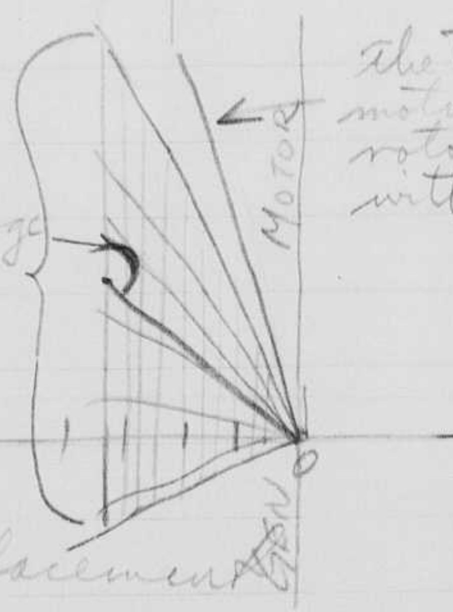

Fig 2.

The torque from a motor with a single phase rotor varies as a function of angular displacement (see fig 3)

average

The torque from a motor with a polyphase rotor does not vary with angular displacement.

Fig 3. θ - Power.

Fig 5 osc.

Notebook # 3

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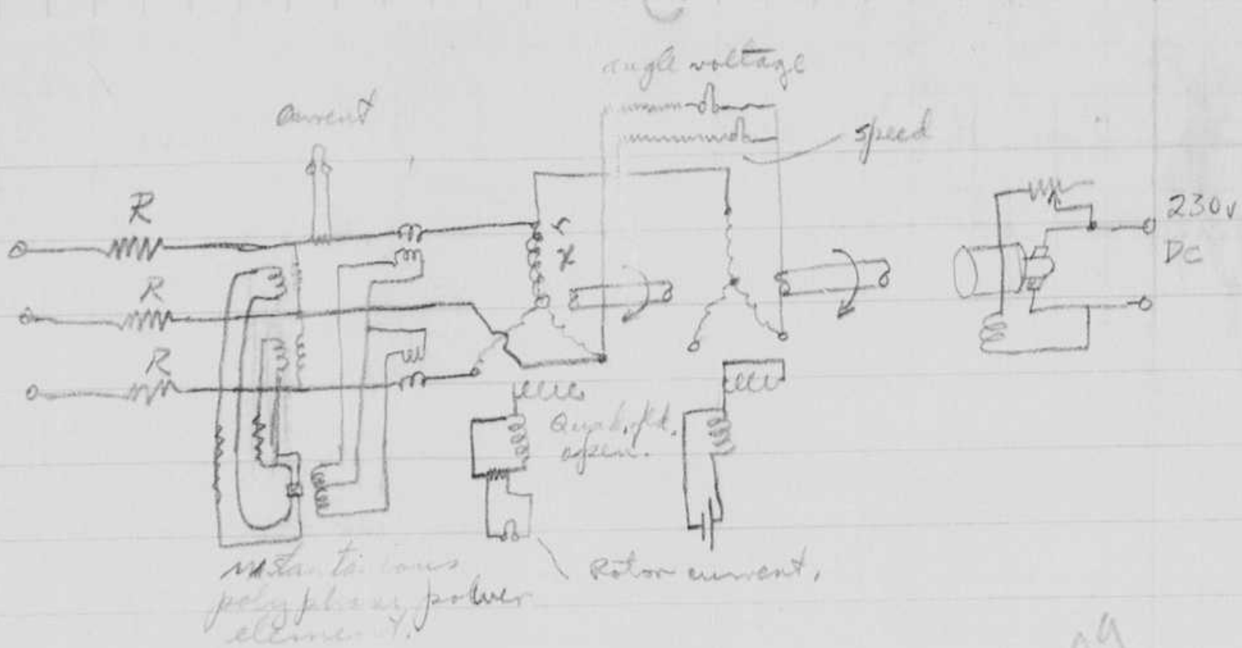

Fig 4.

wiring diagram for oscillogram no ?

Fig 1.

polyphase rotor winding

single phase rotor winding (average torque curve. the instantaneous torque varies at double slip frequency).

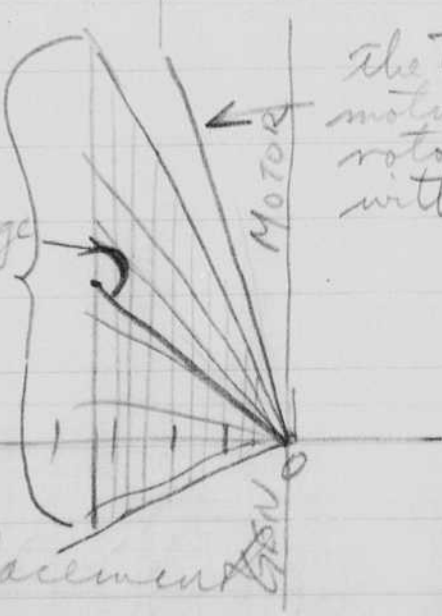

Fig 2.

the torque from a motor with a polyphase rotor does not vary with angular displacement.

The average torque from a motor with a single phase rotor varies as a function of angular displacement (see fig 3)

Fig 3. θ-Power.

Fig 5 osc.

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

____ negative strip(s)

3 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 28 and 29.

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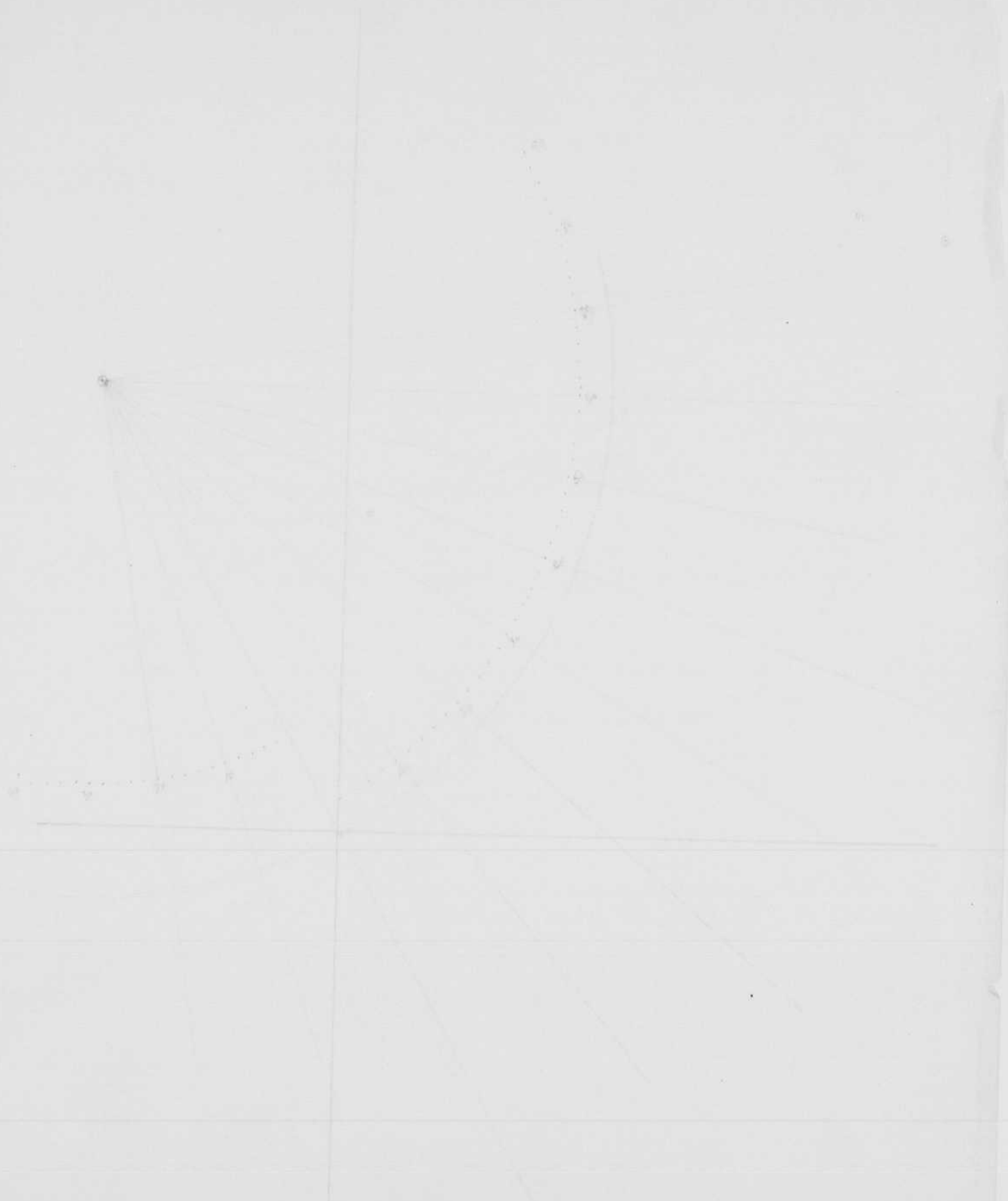
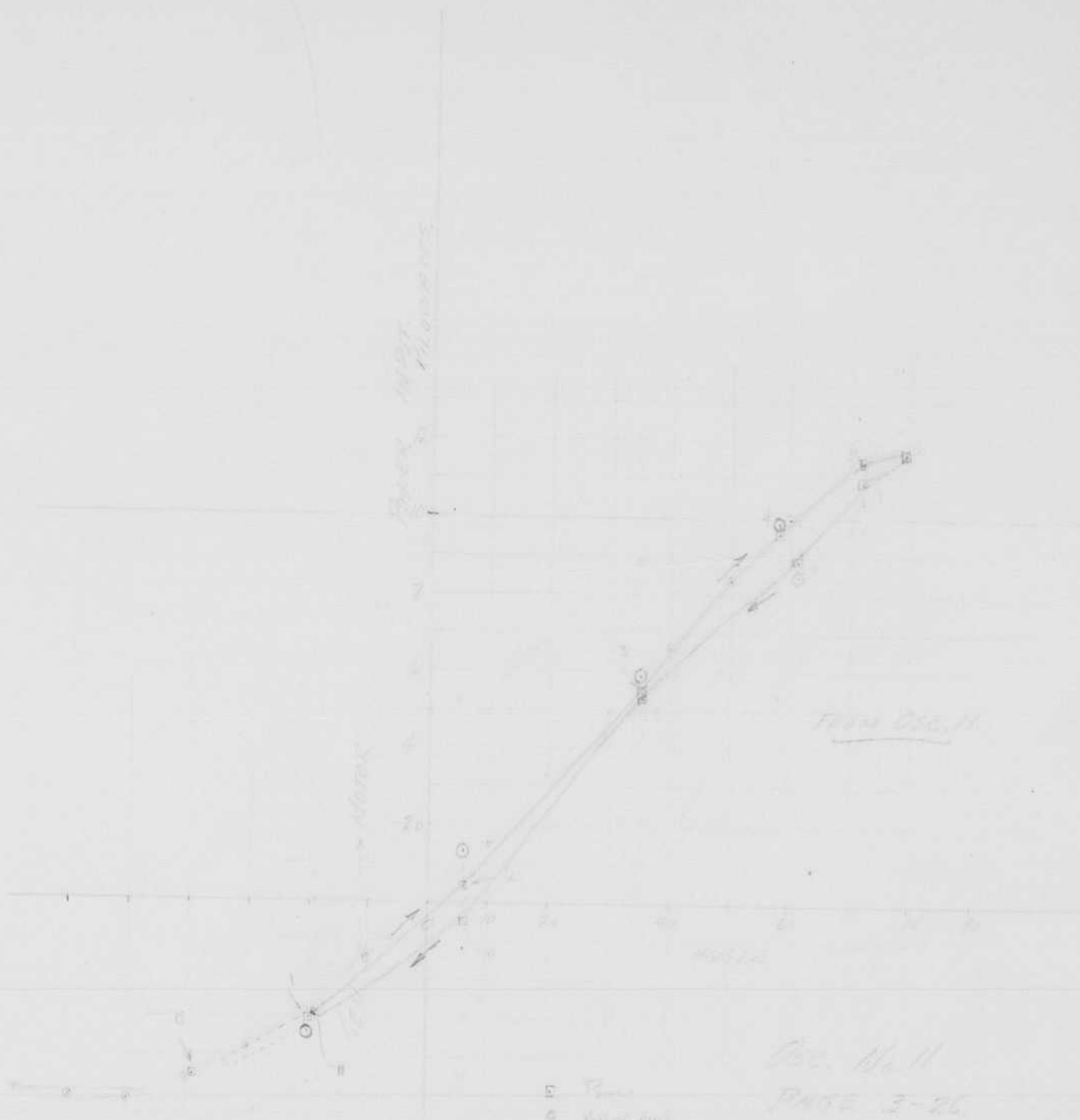
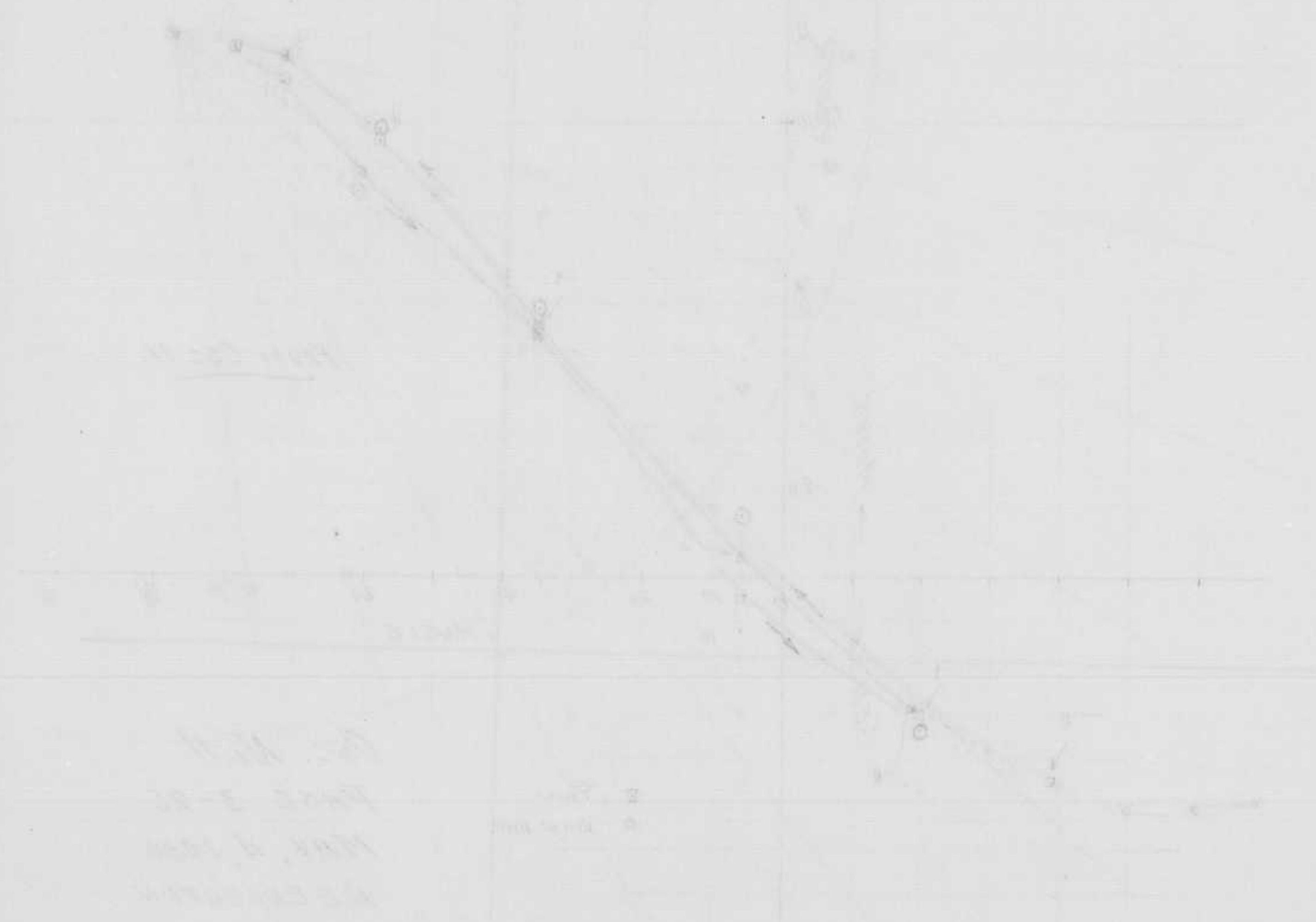




Osc. No. 11
Page 3-26.
May. 11, 1930.
H.C. Robertson



OSC. No. 11
PAGE 3-26
MAY 4, 1930
H.E. EDGERTON

May 6 1930
H. E. Edgerton
John Ross.

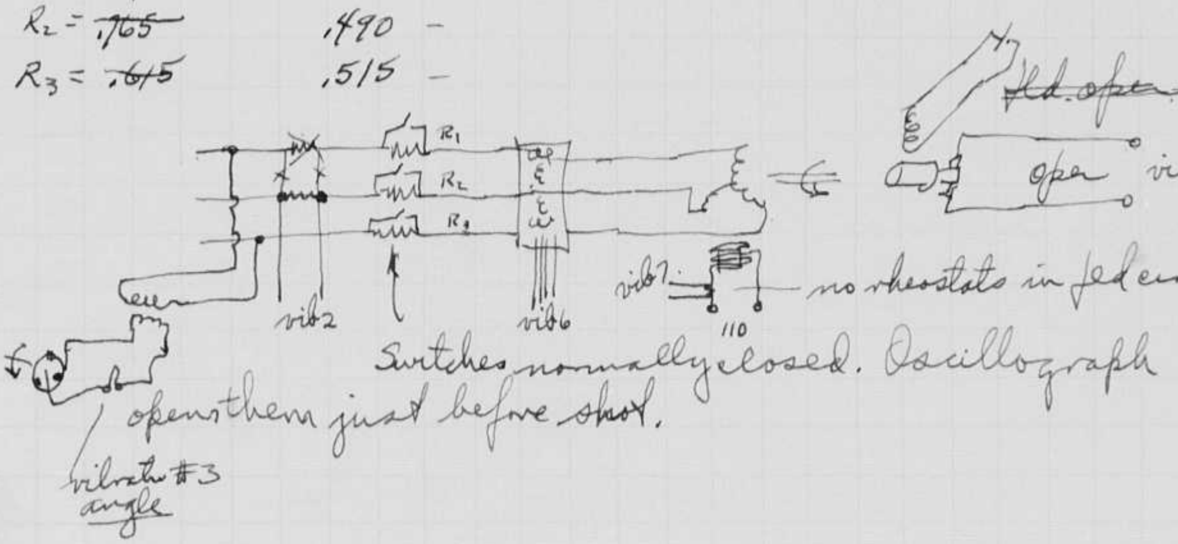
Self-Oscillation of 96.

Salient pole syn. motor # 305997.
G.E Co ATB-6-37.5-1200 Form PB
Pf-80 3φ 90-180 amps 30 KW. 60~. ~~37.5~~
Direct Connected to -
1016477 Shunt type RC B3 A Form
230 volts amps 160 1200 R.P.M.

lead resistance.

R_1 = ~~.507 - .128~~ .466 - .128
R_2 = ~~.765~~ .490 -
R_3 = ~~.615~~ .515 -

Osc. 15



vib 1. to give speed

no rheostats in fed circuit.

Switches normally closed. Oscillograph opens them just before shot.

16

17

18

19. Calibration

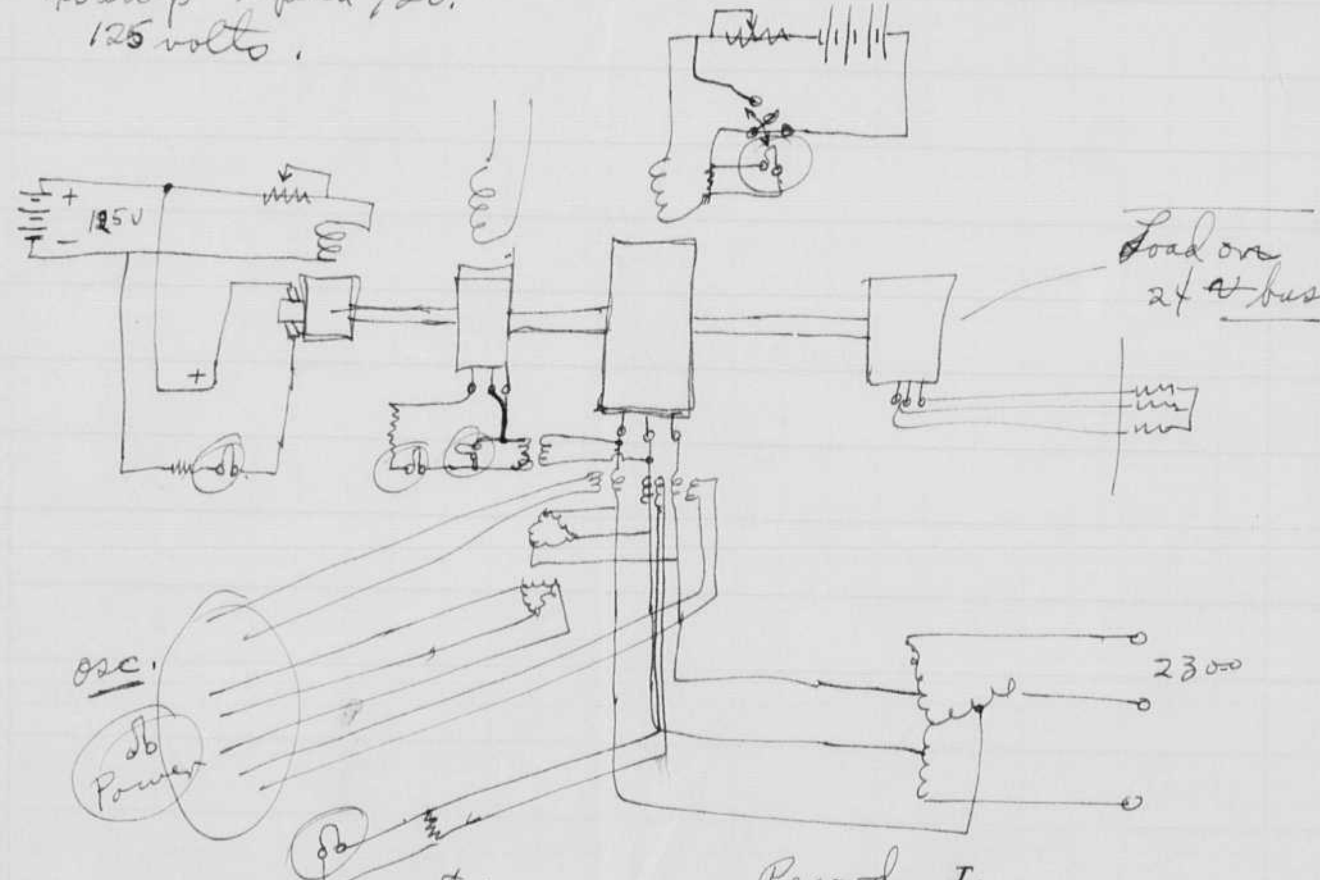
I	V	W	I_f on 230 v.		Dc motor I_f	E	I
50.3	221	×20 ~~.246~~ .981	2.82		2.22	228	69.0

Salient-Pole Motor
Outline of Pullin tests

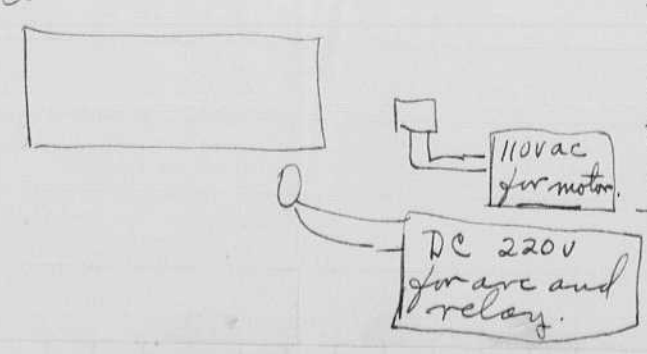
~~May~~ 12, 1930
H. E. Edgerton

G.E.	G.E.	G.E.
A.C. Generator.	Synchronous Motor.	A.C. Generator
#842053	#302707.	#842052
ATI-10-62.5M 720 C	ATI-10-160M 720	ATI-4-62.5-M-720 C
50KW 720 P.f. .8	40 amp 2300v 60~	50KW speed 720 P.f. = .8.
157 amp volts 230	H.P. output 160 speed 720	amp 157. Volts 230.
	P.f. = .8	
	125 KVA.	

G.E.
D-C Exciter
502320
type FF-4-5-720 form E
40 amp speed 720.
125 volts.



Record
I_f
I_a
V_a
V_{angle}.
P_{input}.
Speed.



40
10,000
240
100 × 1. = 100 volt amp.

May. 14, 1930
H. Edgerton

WR^2 from ~~Ira~~ A. Terry's letter of May 5, 1930 and reactances.

	WR^2	x_d	x_q
# 842053	960	.362	0.173 .173.
# 302707	1175	.651	.32
# 842052	627	0.409	0.209
total WR^2	2762 pound feet squared.		

$$P_j = \frac{18.6\ WR^2 f}{P^2} \times 10^{-6} = \frac{18.6\ \ 2762\ \ 60}{10 \times 10} \times 10^{-6} = 0.0308 \text{ kw/elect. deg/sec}^2$$

Reluctance power at $\frac{1}{2}$ voltage.

$V = .5$

$x_d = .651 \quad x_q = .32$

$$P = \frac{V^2(x_d - x_q)}{2\,x_d x_q} = \frac{.5^2(.651 - .32)}{2\ .651\ .32} = 0.198$$

$$P_{KW} = \overset{125}{\cancel{160}}\ .198 = \overset{32}{\cancel{8}}\ KW.\ \ 25.\ KW$$

$160\ .198 = \quad 31.8\ KW.$

Synchronous power.

$$P_m = \left(\frac{VE\ \cancel{x_q}}{\cancel{x_d x_q}}\right)\overset{126}{\cancel{160}} = \left(\frac{.5\ \ .5}{.651}\right)\overset{125}{\cancel{160}} = \cancel{125}\ \cancel{61.3}\ 48.0\ KW \quad \cancel{31.3}\ KW. \quad 61.5\ KW$$

160 160

1100 volts - 8.8 amps field current. from sat. curve.

$$\frac{31.3\ KW \times 1.000}{\sqrt{3}\ \ 1100} = 16.4$$ amperes required from a.c. 1100 volt source.

The motor requires 160 amps. exciting current.

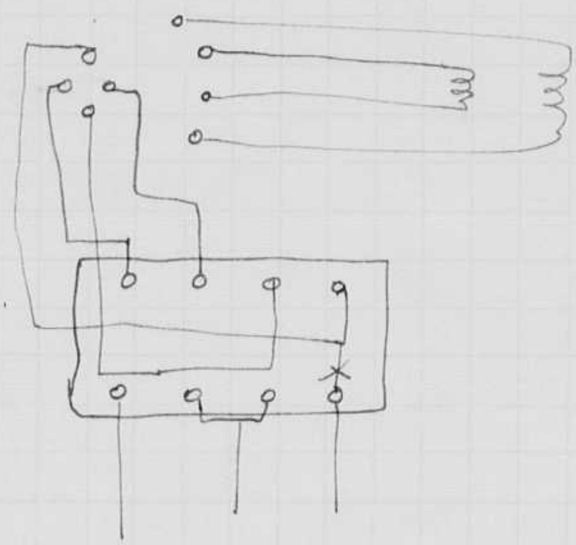
May 15 1930
A. E. Elgentin
Fritz Brodevick

Test on 302707.

C.T. 40:1
C.T. ~~200~~:1 ratio
P.T. 20:1.

V. 55 on start 111 – 115+ start.
58 on run.
I less than 1 amp run.
3 amps on ~~start~~ . $\frac{40}{}$ $\frac{120}{}$
32. = 1.6 amps × ~~100~~ 20 running light.
.32. amps.

Connections of polyphase ~~watt~~ element of oscillographs



Field Resistance. 9.5 ohms fld + Rheo on board.
5.2 ohms fld only with Rheo out.

$$S \leqq \frac{600 \pm}{n}\sqrt{\frac{P_m}{f\,(WR^2)}}$$ n syn speed.

$P_m = 50$
$f = 60$
$WR^2 = 2762$
$n = 720$

$$S < \frac{6}{720}\sqrt{\frac{50}{60 \quad 2762}} < 1.45\%$$

61.5

see page 37 for corrected.

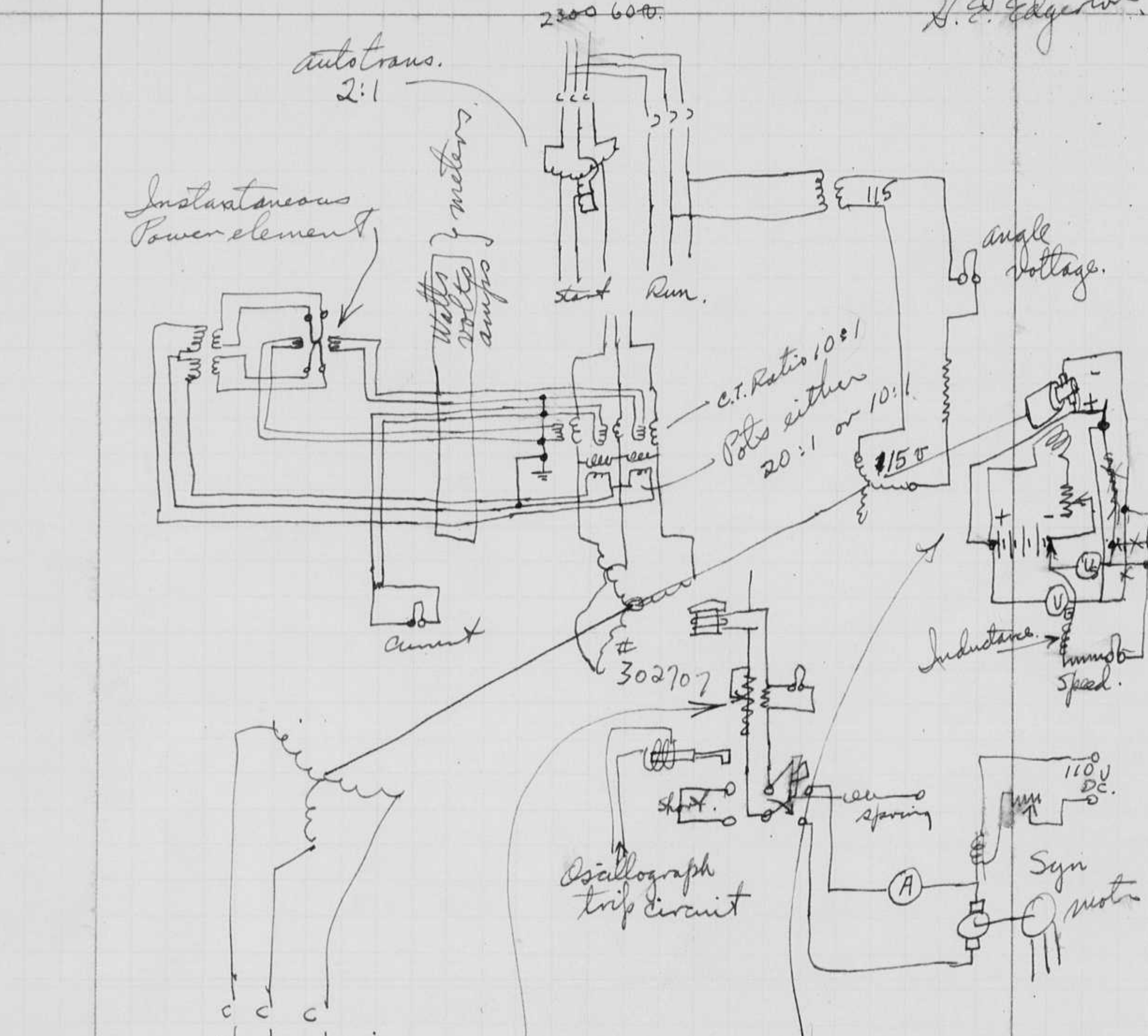
Steady-State Power-Angle Curves on # 302707.

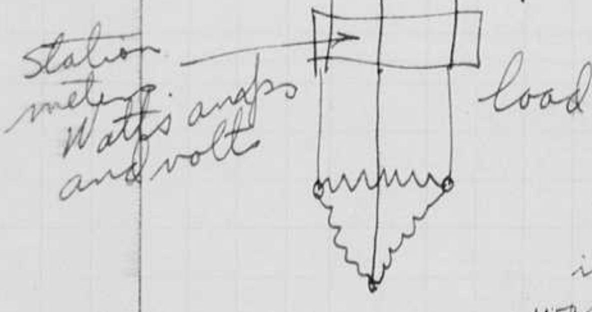
May 16 1930.

H. E. Edgerton.
Paul Fourmarier.
Sam Levine
Chas Kingsley.

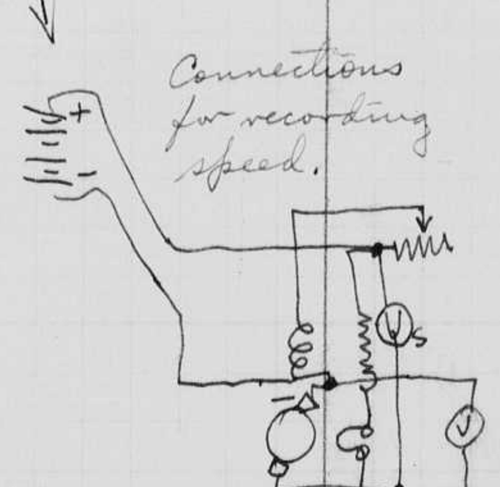
θ	V_1 A.Cline	V_2 Gen	Angle. 32270.	V_1 X20 24873.	KW X10 X20	I_1 X10	I_2 X10	I_f D.	P_{out} KW meter on board.	V	
	116.5	116.5	59.0	115.3	1.7	1.08	1.05	18.9	24	130.	
			.253 14.7 29.4		34.						
			56.5	115.0	2.28	1.34	1.33	18.8	34	155	
			.242 14.1 28.2		45.6						
			56.0	115	2.44	1.43	1.42	18.95	38	164	
			.240 13.9 27.8		48.8						
			53.9	115.3	2.85	1.61	1.6	18.8	46.	180	
			.2315 13.4 26.8		57.0						
			61.0	115.2	1.37	1.—	.95-	18.8	18.	112.	
			.262 15.2 30.4		27.4						
			64.3	115.2	.90	.75	.75±	18.8	8.0	70	
			.276 16.0 32.0		18.0						
			66.0	115.2	~~.40~~	.65	.65	18.9	0	0	
$2\sin^{-1}\frac{.285}{\frac{66.5}{233.0}} = 16.54° \times 2 = 33.07°$			.283 16.45 32.9		8.0						
			66.5	115.2	~~.5~~ .47	—	—	18.9	0	220	core loss of 24 KW
			.285 33.08		$\boxed{9.4 KW}$						
			52.	115.0	3.46	1.86	1.88	18.9	57	230.	
			.223 12.9 25.8		69.2						
			50.	115.0	3.92	2.1	2.1	18.9	66	250.	
			.2145 12.4 24.8		78.4						
			66.5	115.2.	.5	3.46	3.46	9.2	0	0	
			64.0	115.0	1.18	3.5	3.51	9.25	9.0	97	
	shut down		60.0	115.3	1.61	3.55	3.56	9.25.	20.4	138	
			56.	115.0	2.06	3.71	3.70	9.20	28.5	166.	
			54.5	115.0	2.30	3.77	3.74	9.20	33.2	176	
			52.5	115.0	2.64	3.84	3.82	9.20	39.5	193.	
			49.5	115.0	3.17	3.97	3.95	9.20	49.0	213	
	meters interchanged		47.5 ↔	115.0	3.47	4.07	4.07	9.2	55.0	227	
			45.5	116.0	3.82	4.18	4.20.	9.1	60.	239	
			43.0	115.8	4.17	4.33	4.33.	9.1	66.5	252	
			≶	1/2 volts on autotrans	3.2? guess from yesterdays tests			shorted thru 5 ohms	32. ↓ 16.	Reluctance pullout / Pullin.	

May 17 1930
H. E. Edgerton





The entire resistance
in the field circuit
was used for all of the
tests. This resistance
is the one on the
control panel.



Pulling-into Step Oscillograms on # 302707.
and Sliptorque Curve of 302707.

H. E. Edgerton
May 17, 1930

Kingsley Jr.
Paul Hourmarie
S. Levine

V angle machine		V x10	I x10	HW x10x10	I_f	V_B	V_s max min.	Pout	V_{out}	
		115	3.03	.80	0		.22 .33	0		
			5.+	3.– 4.7			.1 – .3	25	175.	
		114	5.+	1.3 – 4.0				18	148.	
slip avg.	Osc 20.		1.71	3.4	9.3			25.5	175	
	21.	115	1.69	3.3	9.4			25	173	
	22	115	1.82	3.61	9.4			27.5	182	
1.5 1.49	23. 68" 66 67	115	1.84	3.61	9.2			27.5	157	↑ 0 ohms in series with V_B
	24	Off scale – failed to pull in						27.5	155	2 Ω in series with V_s.
1.47 off.	25	(trip out too soon)						27.3	152	
		114.5	1.84	3.7	9.2		19.8	↓		
1.54 140" 1.585%	26 66 1.515	115	1.86	3.69	9.2		19.8	27.8	154	
1.49 140" 1.497%	27 66 1.515	115	1.84	3.67	9.25		19.8	27.7	153	
1.73 161" 1.717%	28 65" 1.54	115	1.86	3.72	9.55		19.8	28.2	155	
1.49 142" 1.51%	29	115	1.86	3.66	9.4		19.8	27.8	153	
	30	115	2.11	3.98	8.55	.2ᵛ 19.5ᵛ R	24.5ᵛ angle speed	31.0	161	calibration

slip from angle curve

These readings were made immediately after synchronization.

Slip of film 30 $\frac{... 0.2}{19.5} = 1.025\%. = \left(\frac{1.07 \text{ to } .80}{2}\right)$ inches variation .94 ± in.

Calculation of slip from number of cycles per 360 elect degrees
x cycles / elect rev.

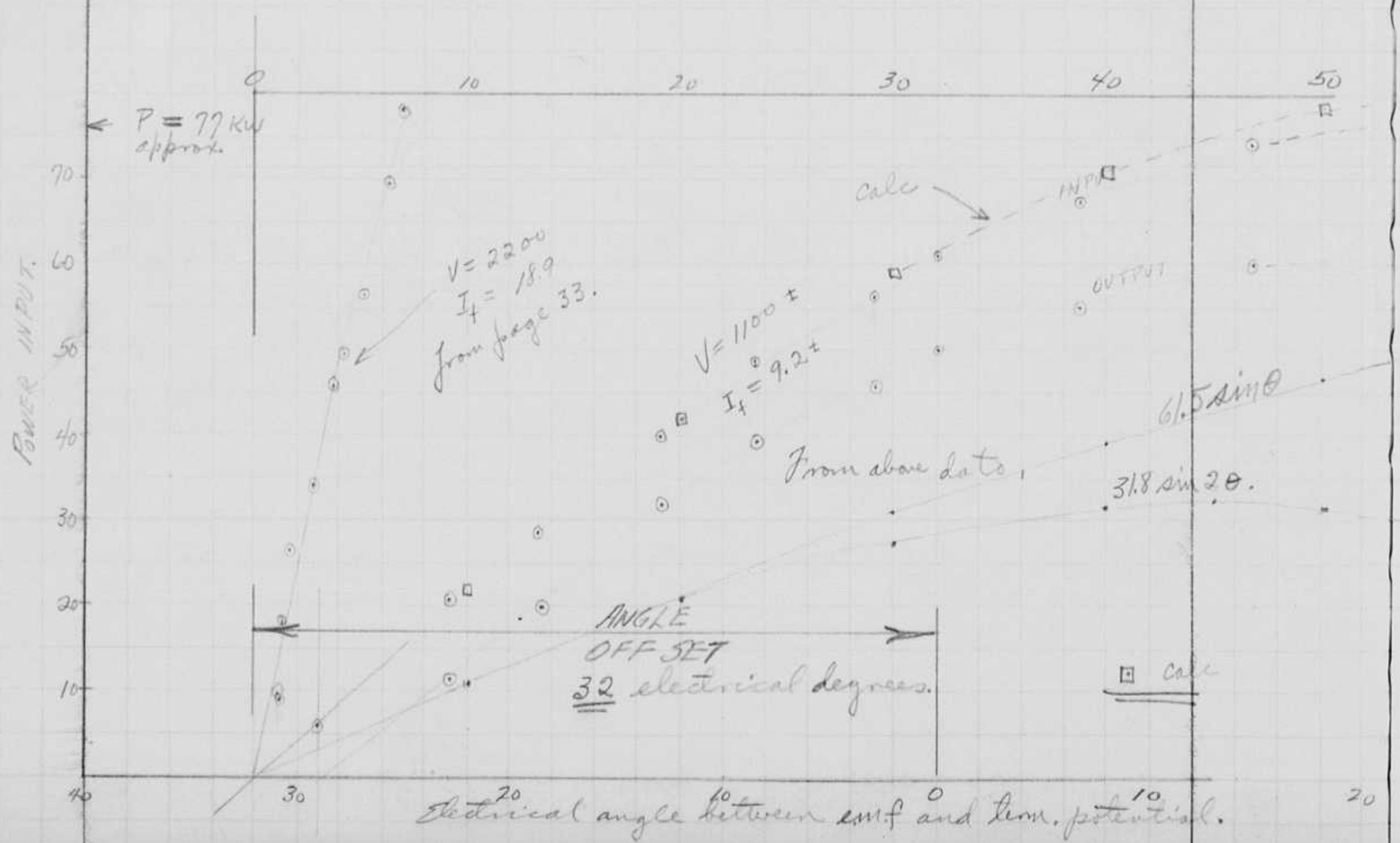
$$\text{slip} = \frac{60}{x} \text{ cycles/sec} \qquad \% \text{ slip} = \frac{60 \times 100}{60x} = \frac{1}{x} \times 100$$

Steady-state,

May 17, 1930.

Power-Angle Curve 302707

V_{ac} ×10	i_{ac} ×10	i_{ac} ×10	kW ×10×10	$V_{ang.}$	P_{out}	V_{out}	i_f	$\frac{V_{ang}}{2\times115.5}$	θ	θ −32
115.2	–	–	0.61	59.	0	0	9.2	.255	14.55	29.1
115.5	1.05	1.02	2.05	46	12	105	9.1	.197	11.35	22.7
115.1	1.46	1.44	2.82	38	20	132	9.1	.163	9.39	18.78
114.9	2.03	2.05	4.00	26	32	161	9.2	.111	6.4	12.8
114.6	2.53	2.56	4.87	17	39	181	9.2	.0729	4.18	8.36
115.0	3.04	3.02	5.65	6±	46	196	9.2	.0256	1.45	2.90
114.2	3.33	3.32	6.15	0±	50	205	9.15	0	0	0
114.3	3.74	3.75	6.72	13	55	215	9.2	.0557	3.2	6.4
114.2	4.40	4.42	7.40	30	60	225	9.±	.129	7.35	14.7



Probability of Synchronization

May 17 1930.

V	I_f	P output at syn speed.	V. output.	x = synchronized o - failed.
115	9.2	24.5	143	X X

Discontinued due to lateness of hour.

From page 31,

Calculated $P_{reluctance} = 31.8$

" $P_{syn.} = 61.5.$

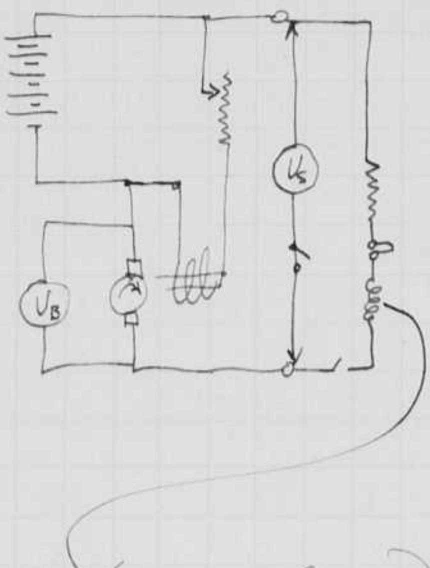
Condition for limiting slip

$$s \leq \frac{600}{720}\sqrt{\frac{61.5}{60\ 2762}}$$

$\leq$ .0160 or 1.6 percent slip.

Calibration of speed vibrator

May 19, 1930.
H. E. Edgerton
Paul Bourmarier



V_B	V_s	Def in inches.	Slip	slip/inch
19.4	.20	.96		
19.4	.30	1.31		
19.4	.30	1.22		
19.4	.20	.80	after the commutator was cleaned.	
	.32	2.03		

(220 volts
.3 amps ±
60 cycles.
Less than .5 of an amp.

these show that the speed cannot be calibrated by this means.

With 1/3 of inductance 220 v 3.0 amps 60 ~.

Average slip may be computed from the angle voltage, then this should give the average of the speed record. It also may be computed from the record of field current since the fundamental of this is at slip frequency.

There was too much inductance in series with the speed vibrator for the tests taken on the 17 th. However the records show the tendency of the speed which is important and sufficient for us.

Synchronization Tests

#302707

May 19, 1930
H. E. Edgerton
Paul Fourmaier.

x = Syn.   O = did not syn.

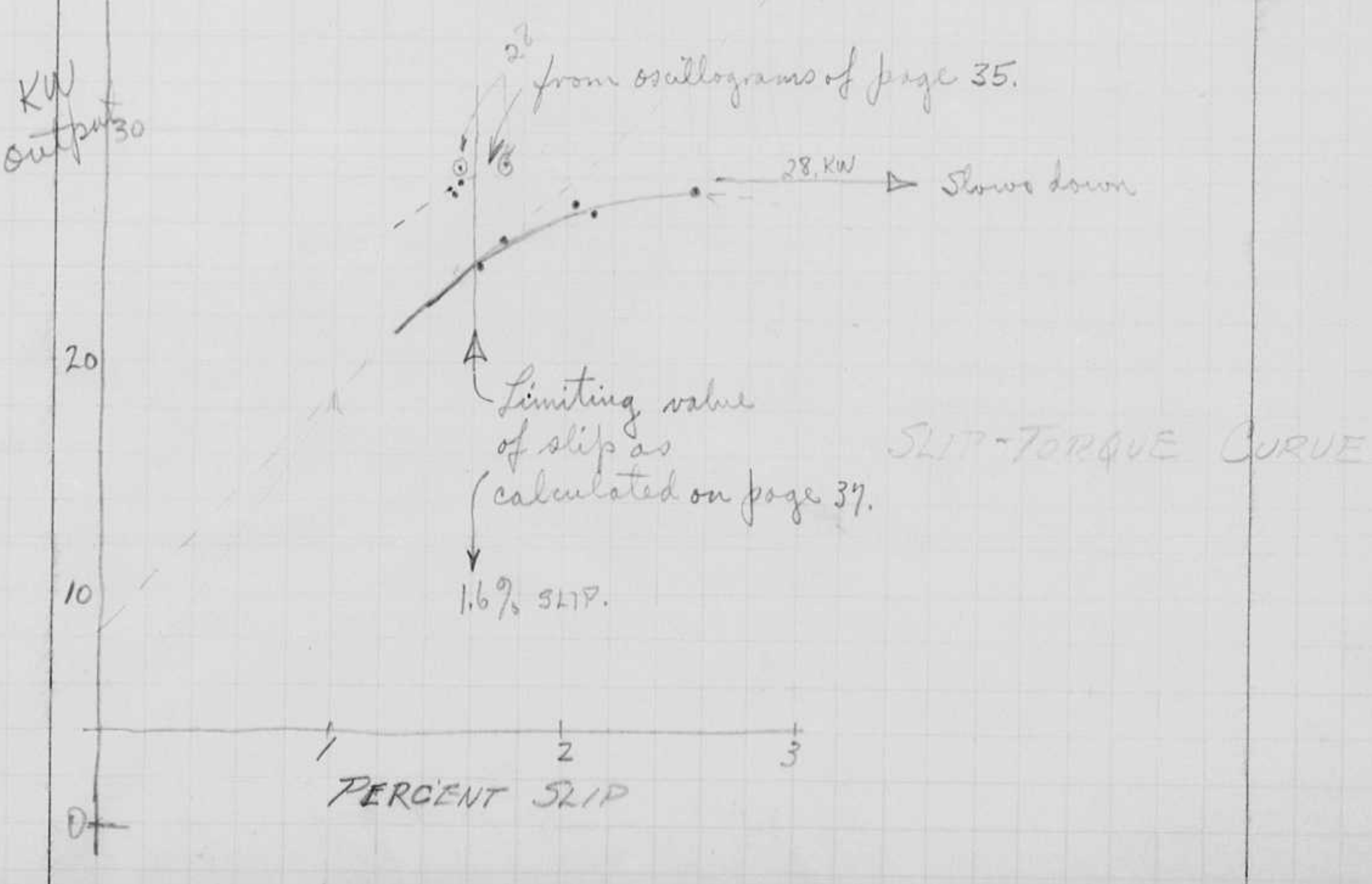
V ×10 ✓	I_f ✓	HW output ×10 ×10	I_a ×10	P output	V. out.
	9.2 -			28. -	0

A load of 28 KW causes the slip to be much larger than it was ~~yesterday~~ Sat.

		slip %		
	29 beats in 1/2 min.	1.61	24.1	158
113.6	31 "	1.724	25.3	164.5
113.6	38 ..	2.11	26.5	171.0
113.5	46	2.56	27.6	176.0
	37 .32 - volts / 194	1.65 / 2.05	26.5	170.0 ±

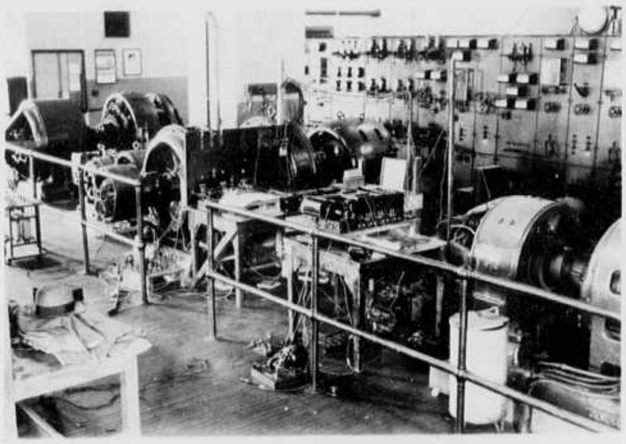
$\% \text{ Slip} = \frac{\text{beats}}{36.00} \%$

with fld shorted the motor pulls out at 26.5 kw.

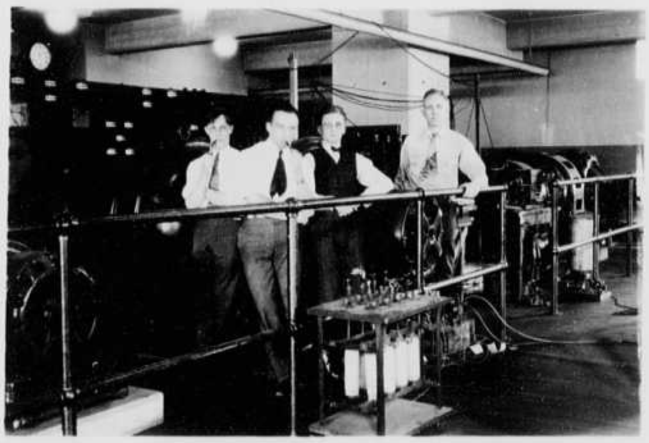


May 22 1930
H. E. Edgerton

Photographs taken May 17, 1930
During tests of pages 34-35-



Chas Kingsley Sam Levine Paul Fourmarier Harold Edgerton

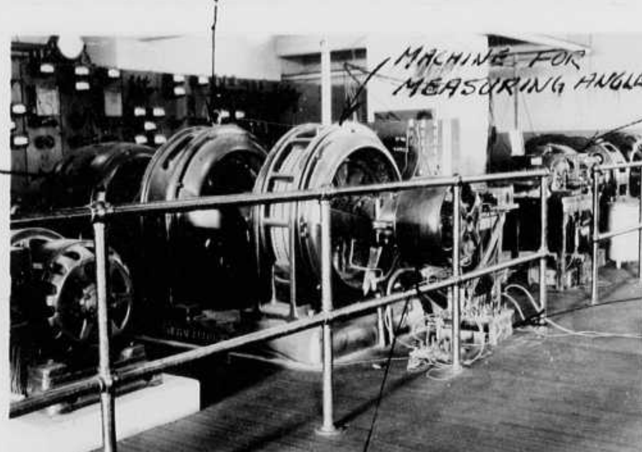


302707
160 KW MOTOR BEING TESTED.

62.5 KW
LOAD
MACHINE

MACHINE FOR
MEASURING ANGLE

oscillograph



change-over
switch.

Exciter used to
measure slip.

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

____ negative strip(s)

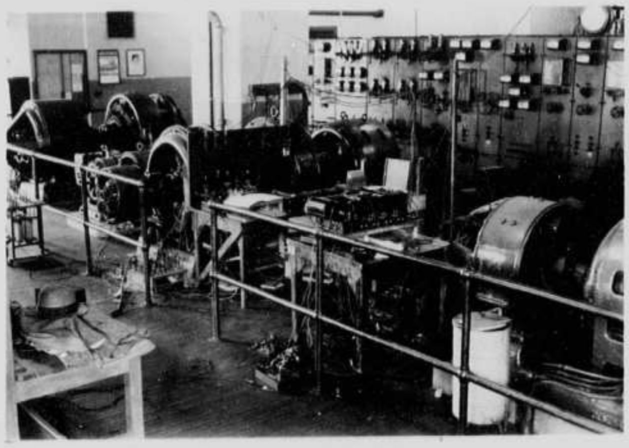
1 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 40 and 41.

Item(s) now housed in accompanying folder.

May 22 1930
H. E. Edgerton

Photographs taken May 17, 1930
During tests of pages 34-35-



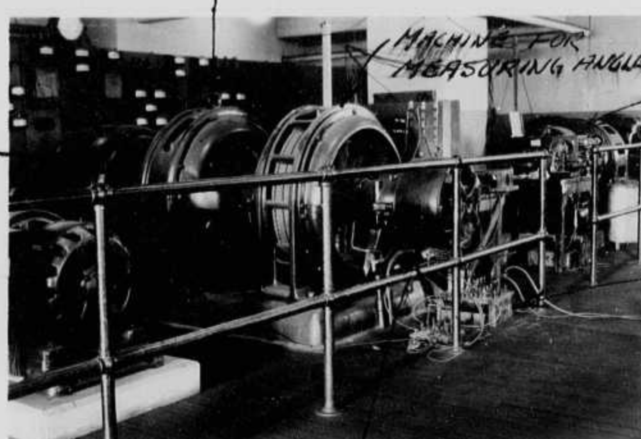
Chas Kingsley Sam Levine Paul Fourmarier Harold Edgerton

302707
160 KW MOTOR BEING TESTED.

MACHINE FOR MEASURING ANGLE

~~50 KW~~
62.5 KW
LOAD MACHINE

Oscillograph



change-over switch.

Exciter used to measure slip.

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

____ negative strip(s)

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(notes, drawings, letters, etc.)

was/were filmed where originally located between page 40 and 41.

Item(s) now housed in accompanying folder.


.25
.20
.15
.10
.05
0
K = .0177
R
∠R.

From page 68 Fournarier's ~~thesis~~.

Osc No 1. 27 excitation -40°

2 26 130°

3 23 110°

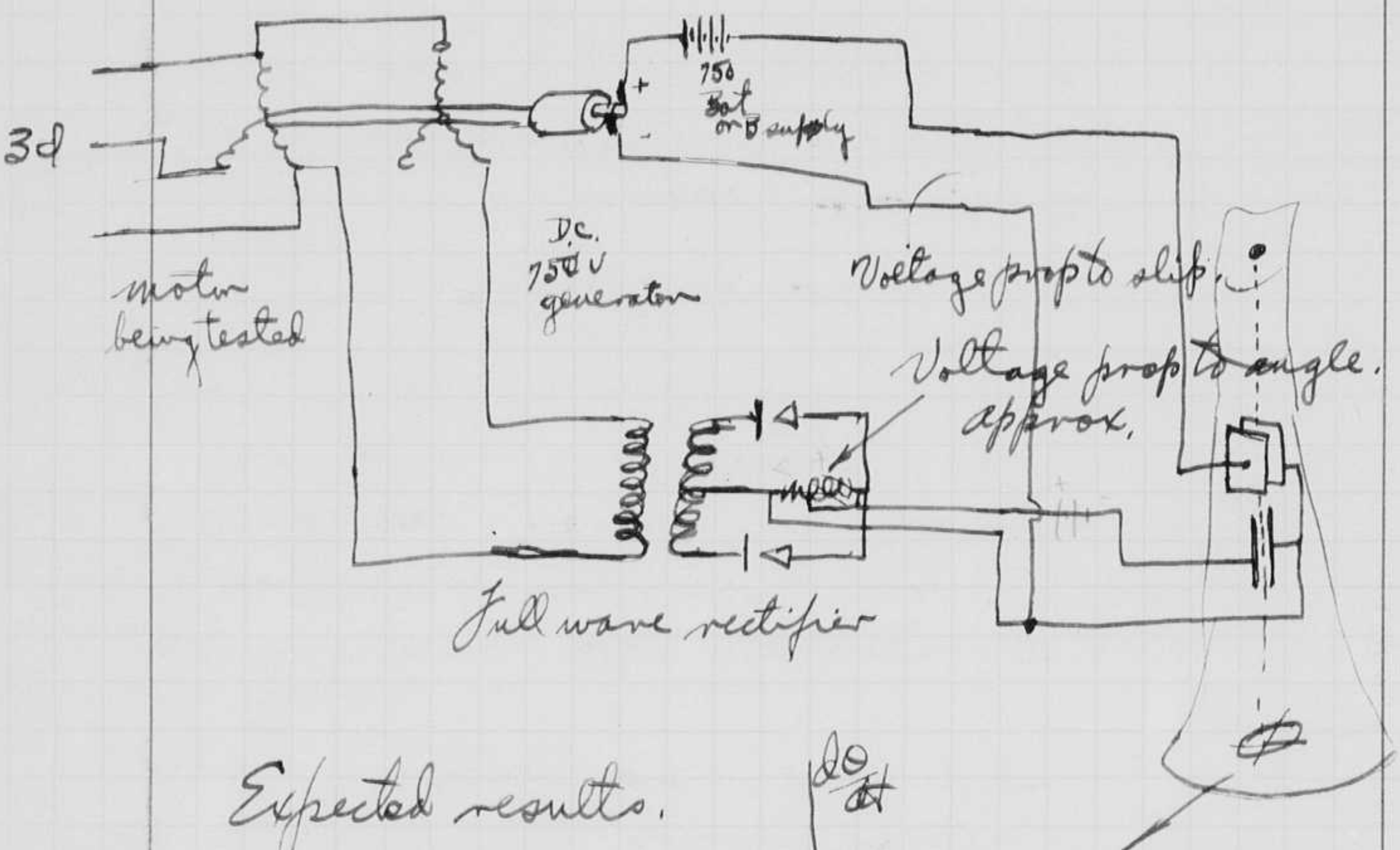
4 28 150.

$g = 0.7$

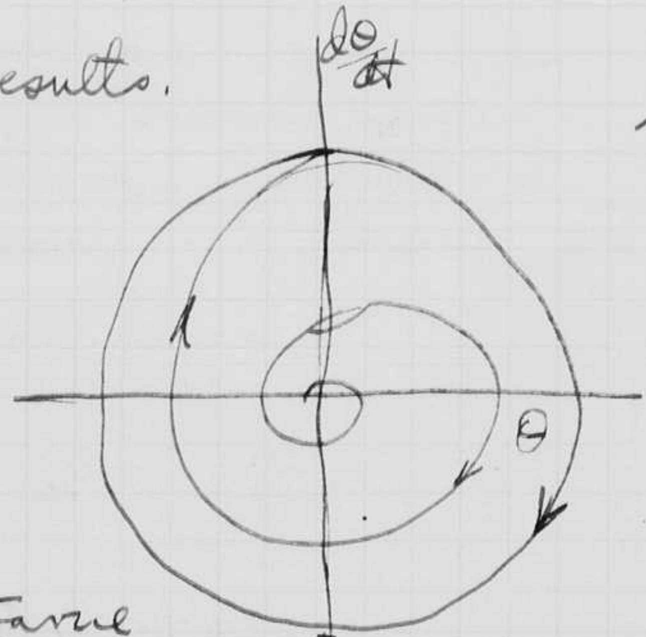
Oct 22, 1930
H. E. Edgerton

Method of Measuring Slip-angle curves for a hunting motor.

A W.E. Braun tube can be used to give a plot of slip against angle by the method shown.



Expected results.



The spot will give an increasing spiral if the damping is negative. If the damping is positive ~~the~~ any disturbance will give a spiral that decreases to a point at the center.

Determination of the Moment of Inertia of a Synchronous Machine by Measuring its Hunting Period.

The motional differential equation of synchronous torques that determines the angular transients of a synchronous machine that is connected to an infinite bus is the following:

$$P_j \frac{d^2\theta}{dt^2} + P_m \sin\theta = \text{load (any function of time)}$$

The electrical damping term has been neglected since it is usually small. The first term represents the torque that is due to the acceleration or deceleration of the combined rotating mass of the motor and load. The second represents the synchronizing torque between the two components of magnetic field which are due to the applied armature voltage and the field current.

The coefficients P_j and P_m will now be determined in units of kilowatts, electrical degrees, and seconds.

Inertia torque = $J\alpha$ in pound feet

where $J = \frac{WR^2}{g}$ = the moment of inertia in poundals

α = the angular acceleration in mechanical radians per second^2.

Inertia power = $\frac{2\pi n}{60}(J\alpha) = \frac{2\pi n}{60}\left(\frac{WR^2}{g}\right)\alpha$ in pound feet per second.

If we express the acceleration in electrical degrees per second^2 then

$$\alpha = \frac{2\pi}{180p}\frac{d^2\theta}{dt^2}$$

since 2π mechanical radians = 180 p electrical degrees.

The expression for the inertia power can be now written in terms of kilowatts, electrical degrees, and seconds as

$$\text{Inertia power in kilowatts} = \frac{2\pi n}{60}\left(\frac{WR^2}{g}\right)\left(\frac{2\pi}{180p}\right)\frac{d^2\theta}{dt^2}\,\frac{.746}{550}$$

$$= P_j\frac{d^2\theta}{dt^2} \qquad (2)$$

154 .5

where $P_j = 0.154 \times 10^{-6}\,\dfrac{n(WR^2)}{p}$ or $18.6 \times 10^{-6}\,\dfrac{f}{p^2}\,(WR^2)$

since $n = f\,\dfrac{120}{p}$ r.p.m.

The maximum synchronizing power for a three-phase round-rotor synchronous machine which has negligible armature resistance is

$$P_m = \frac{3\ VE}{1000\ x}\ \text{kilowatts} \qquad (3)$$

where V = terminal phase voltage

E = induced phase voltage due to the field current.

x = synchronous reactance per phase.

If the magnitude of the angular oscillation is not large then the slope of the power angle curve may be considered as a straight line. This assumption makes equation 1 a linear differential equation. The slope of the power-angle curve is

$$\text{Slope} = P_m\frac{\pi}{2}\,\frac{\cos\theta}{90}\ \text{kilowatts per electrical degree}$$

and the linear differential equation is

$$P_j\frac{d^2\theta}{dt^2} + P_m\frac{\pi}{2}\,\frac{\cos\theta}{90}\,\theta = \text{load in kilowatts.}$$

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

____ negative strip(s)

1 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 44 and 45.

Item(s) now housed in accompanying folder.

The expression for the inertia power can be now written in terms of kilowatts, electrical degrees, and seconds as

$$\text{Inertia power in kilowatts} = \frac{2\pi n}{60}\left(\frac{WR^2}{g}\right)\left(\frac{2\pi}{180p}\right)\frac{d^2\theta}{dt^2}\,\frac{.746}{550}$$

$$= P_j \frac{d^2\theta}{dt^2} \qquad (2)$$

154 .5

where $P_j = 0.154 \times 10^{-6} \dfrac{n(WR^2)}{p}$ or $18.6 \times 10^{-6} \dfrac{f}{p^2}(WR^2)$

since $n = f\,\dfrac{120}{p}$ r.p.m.

The maximum synchronizing power for a three-phase round-rotor synchronous machine which has negligible armature resistance is

$$P_m = \frac{3\ VE}{1000\ x} \text{ kilowatts} \qquad (3)$$

where V = terminal phase voltage

E = induced phase voltage due to the field current.

x = synchronous reactance per phase.

If the magnitude of the angular oscillation is not large then the slope of the power angle curve may be considered as a straight line. This assumption makes equation 1 a linear differential equation. The slope of the power-angle curve is

$$\text{Slope} = P_m \frac{\pi}{2}\,\frac{\cos\theta}{90} \text{ kilowatts per electrical degree}$$

and the linear differential equation is

$$P_j \frac{d^2\theta}{dt^2} + P_m \frac{\pi}{2}\,\frac{\cos\theta}{90}\,\theta = \text{load in kilowatts.}$$

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

____ negative strip(s)

1 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 44 and 45.

Item(s) now housed in accompanying folder.

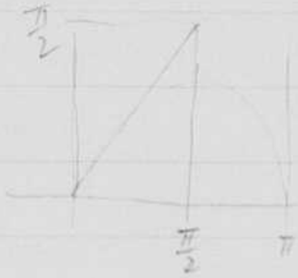
$$P_R = \frac{P}{\theta}$$

$$f = \cancel{120}\ \frac{pn}{120}$$

$$n = \frac{120 f}{p}$$

$$F = 0.268 \sqrt{\left(\frac{VE}{x}\cos\theta\right) \frac{p^2}{f}\left(\frac{1}{WR^2}\right) \frac{(120)^2 f}{(120)^2 f}}$$

$$= 0.268 \sqrt{\frac{VE}{x}\cos\theta \frac{(120)^2 f}{\cancel{120}\, n^2 \cancel{f}}}$$



$$= \frac{0.268 \times 120}{n}$$

$$= \frac{1}{2\pi} \sqrt{\frac{1}{18.5 \times 10^{-6} \frac{f}{p^2} WR^2} \cdot \frac{P_m \frac{\pi}{2} \cos\theta}{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{90}}$$

$$= \frac{1}{2\pi} \sqrt{\frac{1}{18.5 \times 10^{-6}}} \sqrt{\frac{p^2}{f\,(WR^2)} \frac{f\,(120)^2}{f\,(120)^2}}\ P_R$$

$$n^2$$

$$= \frac{120 \times 10^3}{2\pi \cancel{10^3}} \sqrt{\frac{1}{18.5} \frac{\pi}{180}}\ \frac{1}{n}\sqrt{\frac{f}{WR^2}}\ P_R.$$

$$.054$$

$$= 0.233$$

2400

$$\sqrt{\frac{\pi}{180}} \quad \frac{120 \times 10^3}{6.28} \times .233 = \quad 4430.$$

cyc second

588. cyc per second

3540 cyc per min

.01774 3.00 .132

The differential equation is analagous to a series circuit of inductance L and capacity C which has the differential equation

$$L \frac{d^2 q}{dt} + \frac{q}{c} = E.$$

The natural frequency of oscillation of an electrical circuit is known to be

$$F = \frac{1}{2\pi} \sqrt{\frac{1}{Lc}} \qquad \text{cycles per second.}$$

Similarly the frequency of mechanical oscillation is

$$F = \frac{1}{2\pi} \sqrt{\frac{1}{P_j} \quad \frac{P_m \frac{\pi}{2} \cos\theta}{90}}$$

$$= 0.268 \sqrt{\frac{VE\ p^2 \cos\theta}{x\ f\ (WR^2)}}$$

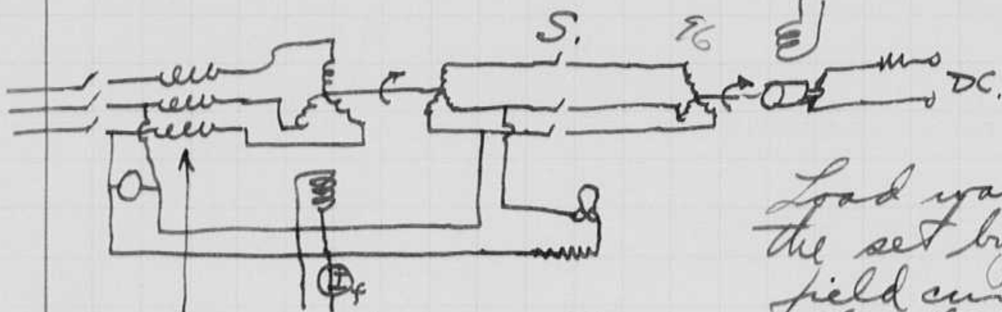
Solving for the moment of inertia

$$WR^2 = 0.072 \ \frac{VE\ p^2 \cos\theta}{x\ f\ F^2}$$

H. E. Edgerton
M.I.T. Nov. 15, 1930

H. S. Edgerton.
Nov. 16, 1930.

WR^2 from Hunting tests.



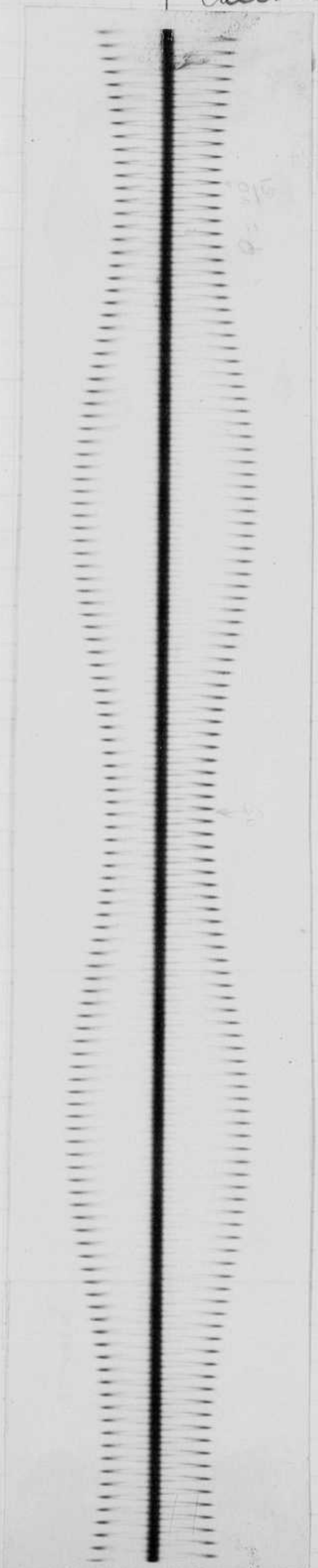
Reactors series connection good for 30 amperes.

Load was put on the set by changing field current on the d-c machine. then switch S was opened and this sudden removal of load allowed the set to swing about its no load angle. the quadrature field was open circuited and so the damping was negligible.

An oscillogram was taken when the circuit conditions were as follows.

$V = 232$ volts line to line.
$I_f = 8.4$ amperes.
$I_{ac} = 5 \pm$ amps.

Calculation of WR^2 from data of preceding page.



$V = \frac{232}{\sqrt{3}}$

$E =$

$p = 6$ (number of poles)

$\cos\theta = 1$

$x =$

$f = 60$ cycles/sec

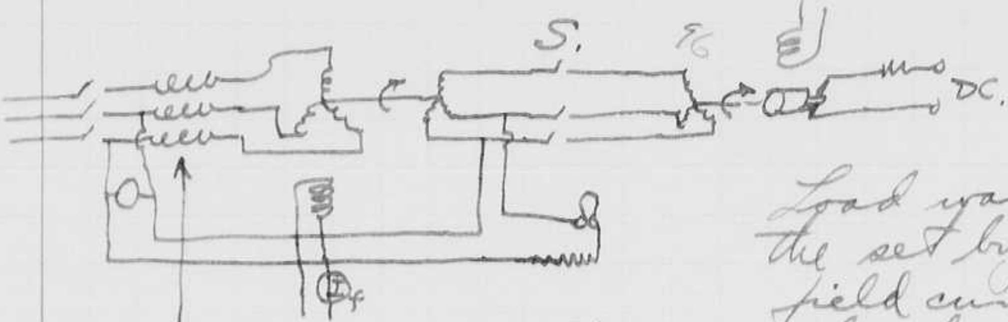
$F =$ cycles/sec.

$$0.012 \frac{V E p^2 \cos\theta}{x f F^2} =$$

See next page

H. S. Edgerton
Nov. 16, 1930.

WR² from Hunting tests.



Reactors series connection good for 30 amperes.

Load was put on the set by changing field current on the d-c machine, then switch S was opened and this sudden removal of load allowed the set to swing about its no load angle. the quadrature field was open circuited and so the damping was negligible.

An oscillogram was taken when the circuit conditions were as follows.

$V = 232$ volts line to line.
$I_f = 8.4$ amperes.
$I_{ac} = 5 \pm$ amps.

Calculation of WR^2 from data of preceding page.

$V = \frac{232}{\sqrt{3}}$

$E =$

$p = 6$ (number of poles)

$\cos\theta = 1$

$x =$

$f = 60$ cycles/sec

$F =$ cycles/sec.

$$0.012\,\frac{V E\, p^2 \cos\theta}{x f\, F^2} =$$

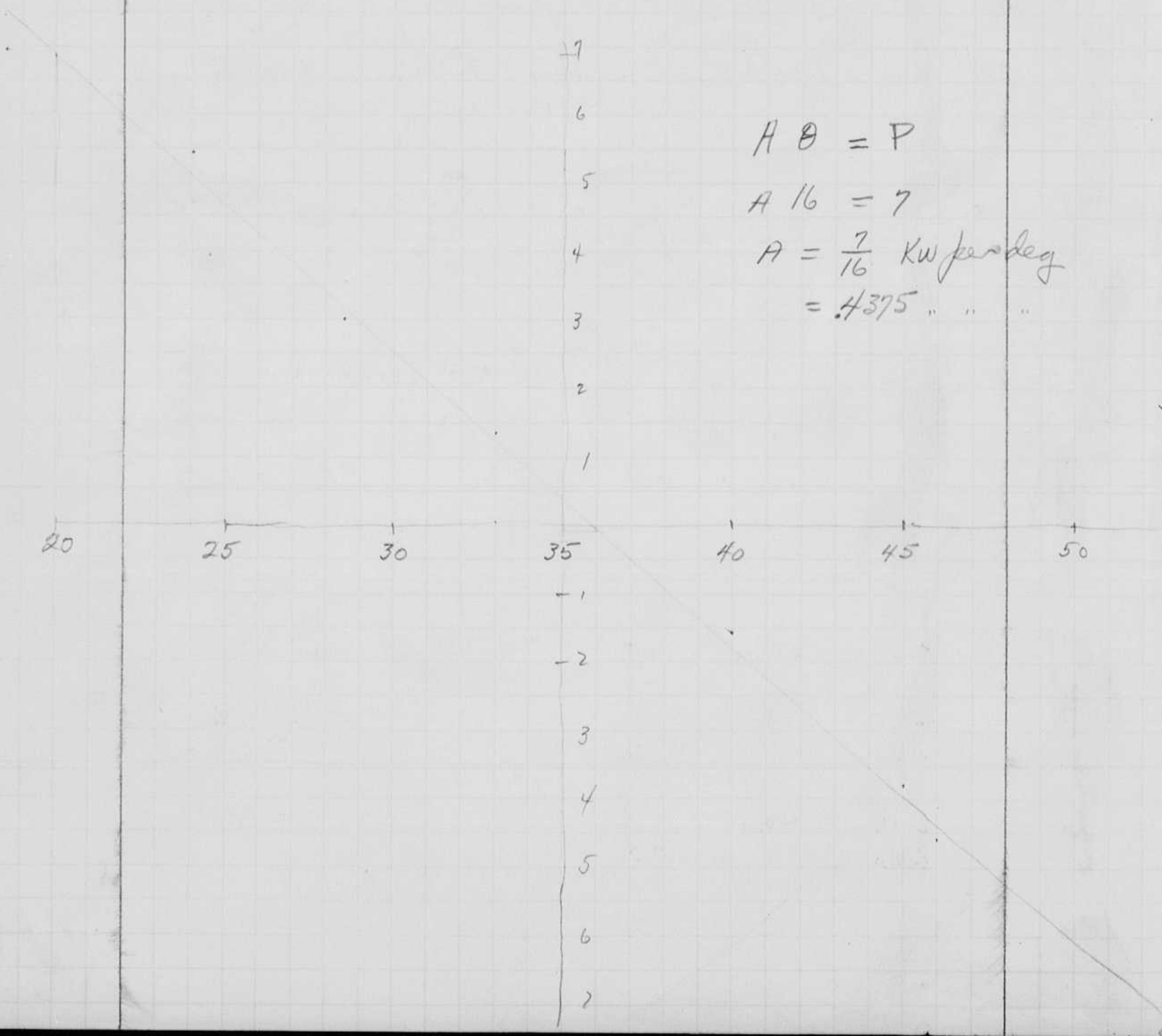
See next page.

2KVA   804 B. Syn Generator.

Starting Condition

60 cycles	V_{stator}	18	17	16	23.5	28.0	27.0
	V_{field}	186 volts			285		

When cold this motor will not start with 102 volts



$A\,\theta = P$

$A\,16 = 7$

$A = \frac{7}{16}$ KW per deg

$= .4375$ " "

Power-Angle Curve of 804 A.

Nov 17 1930

Reactors in series 804A.

θ	I_1	I_2	I_3	KW_1	KW_2	V	I_f	E_{KW}
+40.0	4.2	–	3.75	x2 .37 M	x2 .43 M	227	8.15	-1.60
+45.0	9.8	9.5	9.0	x2 .94 M	x2 .95 M.	227	8.15+	-3.78
+33.0	3.25	—	3.65	x2 .29 G	x2 .39 G	226.5	8.10	+1.36
28.5	7.50	7.5	7.8	x2 .68 G	x2 .82 G			+3.0
24.0	–	12.8	13.0	x4 .565 G	x4 .71 G	228	8.05	+5.4
19.0 18.0	–	17.6	17.9	x4 .74 G	x4 .985 G	226	8.0	+6.9
46.0	–	11.8	11.0	x4 .59 M	x4 .56 M	225.5	8.05	-4.6
53.0	–	19.2	18.3	x4 .98 M	x4 .875 M	226.	8.05	-7.42

$\frac{232.8.4}{226\,8.1}$

$1\ \frac{50}{60}$ ~~cycles~~

$F = \frac{60}{50}$

Short circuit of 804 A + reactors as used in tests.

I_f	I_a	E
2.05	16.2	
3.55	28.2	
3.60	28.2	109
4.95	38.9	
4.99		150
6.16	48.2	
6.16		182

$$F = \frac{1}{2\pi}\sqrt{\frac{1}{P_j}\,0.4375}$$

$$P_j = \left(\frac{1}{2\pi}\right)^2 \frac{1}{F^2}\,0.4375$$

$$WR^2 = \left(\frac{1}{2\pi}\right)^2 \frac{1}{F^2}\,0.4375\,\frac{P^2 \times 10^6}{18.6 f}$$

$$WR^2 = \left(\frac{50}{2\pi\,\cancel{60}}\right)^2 0.4375\,\frac{\cancel{36} \times 10^6}{18.6\ 60}$$

$= 247$ pound feet^2

$= 267.$

Dec 12 1930
H. O. E. Elmer Jr.
Chas. Kingsley Jr.
movies by hand.
J. Out A.

Synchronizing tests
movies of pulling into step phenomena.

No.	P_{kW}		I_f	I_m	Remarks	
1	0		0		regular start,	
2	33 KW	185 V	9.6	15.5	pull in ok. after start.	10 ft
3	36	195	10.0	15.5		10 ft
4		Ditto.		15.0		5 ft
5		Ditto		15		5 ft
6		Ditto		14. amps.	failed to pull in.	8 ft.
7				14.	pull in OK.	
8	35.5	195	10.0	13.9	Did not syn.	
9.	35.5	195	10.0	13.8	" " "	
10.	28.0	170	8.5	12.2	two shots.	
11.	0		0	10±2	Brought up to step pulling in a select motor with wrong polarity.	

Two 1000 watt lamps were hung above the set and were used in taking movies of the starting. Mr. Kingsley posed for the picture at the starting panel.

The film speed was 16 frames per sec. lens opening F 1.9. 16 mm film Panchromatic. Eastman.

Dec 15 1930. These films came back last Thursday and they were fine. I put on a show Friday evening in the lab.

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

1 negative strip(s) inside mounted envelope Pg. 51

____ unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 50 and 51.

Item(s) now housed in accompanying folder.

Dec 10 1930 — H. O. E. Elzey Jr.; Chas. Kingsley Jr.; Movies by Paul J. Ont?

Synchronizing tests & movies of pulling into step phenomena.

No.	P_{kW}		I_f	I_m	Remarks	
1	0		0		regular start,	
2	33 KW	185 V	9.6	15.5	pulls in ok. after start.	10 ft
3	36	195	10.0	15.5		10 ft
4		Ditto.		15.0		5 ft
5		Ditto		15		5 ft
6		Ditto		14. amps	failed to pull in.	8 ft.
7				14.	pull in OK.	
8	33.	[illegible]	10	13.9	Did not syn.	
9.	33.5	198	10.0	13.8	" " "	
10.	28.0	170	8.5	12.2	two shots.	
11.	0		0	10±2	Brought up to step pulling in as reluctance motor with wrong polarity.	

Two 1000 watt lamps were hung above the set and were used in taking movies of the starting. Mr. Kingsley posed for the picture at the starting panel.

The film speed was 16 frames per sec. the opening F 1.9. 16 mm film. Panchromatic. Eastman.

Dec 15 1930. These films came back last Thursday and they were fine. I put on a show Friday evening in the lab.

Notebook # 3

Filming and Separation Record

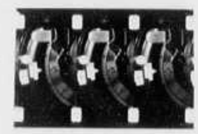
____ unmounted photograph(s)

1 negative strip(s) inside mounted envelope pg 51

____ unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 50 and 51.

Item(s) now housed in accompanying folder.



Evening on the table.

Moving Picture Photographs of 10 pole
Synchronous Motor. 1.9 lens 16 f.p.s.
Panchromatic film. by Henry Lane.

For all strobograms of Jan 7 1931 by Prof. Bowles.

33 KW shaft load 10 amps ± field current

Karl Wildes.
3-4616
Dec 4 1980.

[illegible] for the Cats.

Moving Picture Photographs of 10 pole Synchronous Motor. 1.9 lens 16 f.p.s. Panchromatic film. by Henry Lane.

For all strobograms of Jan 7 1931 by Prof. Bowles.

33 KW shaft load 10 amps ± field current

Kane Wilder.
3-4616
Dec 4 1980.

March 19 1931
A.E. Agents

Integraph Solutions of Salient Pole Motor.

$\frac{P_L}{P_M} = k \frac{d\theta}{dt}$

$= \frac{.150}{.02} = 7.5$

$K = .0167$

Reluctance = 0.3 × P_m. $b = 0$

Draft 7

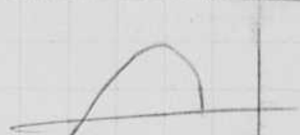
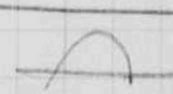
		$\frac{P_L}{P_m}$	k	θ_0	master $\frac{d\theta}{dt}$		
		0.15	~~.02~~		1.20 / 7.5		Initial Steady State.
		.15	.0167	0		●	55° – 160
		.20	~~.02~~	0	.93/10.0		
					.95	42° ●	75 – 150±
.176	.167		.0167		.95		
.263	.25	.30	~~.02~~	0	.55/15.	0	unstable, very!
.22	.208	.25	.02	0	.74/12.5	●	103 – 149 stable.
.238	.225	.27	.02	0	.60/13.5	●	122 –
.245	.233	.28	.02	0	.63/14	●	very close to critical. Just stable.
.250	.237	.285	.02	0		0	unstable.

k = .0167 instead of .02

wrong

March 20 1930 Ditto. K = .0177

Chart 2

	P_4/P_m	k	θ_0	$\frac{d\theta}{dt}$ var.	$\frac{d\theta}{dt}$ avg.	O	
.132	1.5	~~.02~~ .0167	180	1.2 / 1.34	7.5	⊙	
.123	1.4	~~.02~~	180	1.2	7.0	⊙	
.105	1.2	~~.02~~	180		6.0	●	
.114	1.3	~~.02~~	180	1.3	6.5	O	
	2.6					backwards 330°	32.
	2.3.			.12 / .82		332 backward	29.
	2.6	.02				"	
	2.0					" 	
	1.7					" 	
	1.4					" 	
	1.6.						
	1.5						

From Chart 2.

$\frac{P_i}{P_m}$	θ_0
.14	238 100
.17	264 84
.20	290 63

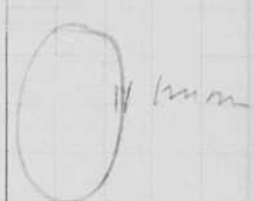
Chart 3. Mar 21 1931 Edgerton
Brown & Gemeshausen

$\frac{P_L}{P_m}$	θ_0	k		
.56	0	.045	●	
.565	0	"	○	
.38	180	"	○	
.375	180	"	○	
.370	180	"	○	
.360	180	"	●	critical at .365?

K = .0475

Chart 3a

Test of Integraph
$k = 0$
$\frac{P_L}{P_m} = 1$ slope through 90°
straight line for $f(\theta)$ force



.39	0	.03	●	
.40	0	.03	○	●
.39	0	.03		
.42	0	.03		○

K = .0318

Round Rotor $P_R = 0$.

4.5 about critical.

March 19 1931
H E Edgerton

Integraph Solutions of Salient Pole Motor

$\frac{P_L}{P_m} = k \frac{d\theta}{dt}$

$= \frac{.15}{.02} = 7.5$

K .177

Reluctance = 0.3 × P_m. b = 0

Drawn ✓

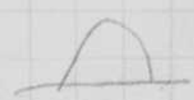
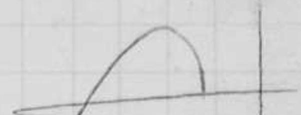
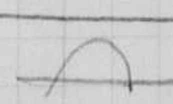
		$\frac{P_L}{P_m}$	k	θ_0	motor $\frac{d\theta}{dt}$		
		0.15	~~.02~~		1.20 / 7.5		Initial Steady State.
		.15	.0167	0		●	55° – 160
		.20	~~.02~~	0	.93/10.0		
.176	.167		.0167		.95 .95	42° ●	75 – 150±
.263	.25	.30	~~.02~~	0	.55/15.	0	~~unstable~~, very!
.22	.208	.25	~~.02~~	0	.74/12.5	●	103 – 149 ~~stable~~.
.238	.225	.27	~~.02~~	0	.60/13.5	●	122 –
.245	.233	.28	~~.02~~	0	.63/14	●	Very close to critical Just stable.
.250	.237	.285	~~.02~~	0		0	unstable.

k = .0167 instead of .02

wrong

March 20 1930 Ditto. K = .0177

chart 2

	P_2/P_m	k	θ_0	$\frac{d\theta}{dt}$ var.	$\frac{d\theta}{dt}$ avg.	
.132	1.5	~~.02~~ .0167	180	1.2 1.34	7.5	⊙
.123	1.4	~~.02~~	180	1.2	7.0	⊙
.105	1.2	~~.02~~	180		6.0	●
.114	1.3	~~.02~~	180	1.3	6.5	○
	2.6					backwards 330° 32.
	2.3.			.72 .82		332 29. backward
	2.6	.02				"
	2.0					" 
	1.7					" 
	1.4					" 
	1.6.					
	1.5					

From Chart 2.

$\frac{P_i}{P_m}$	θ_o
.14	238 100
.17	264 84
.20	290 63

Chart 3, Mar 21 1931 Edgerton
Brown & Gemeshausen

$\frac{P_L}{P_m}$	θ_0	k	
.56	0	.045	●
.565	0	"	o
.38	180	"	o
.375	180	"	o
.370	180	"	o
.360	180	"	●

critical at .365?

K = .0475

~~$\Delta f A$.~~

Chart 3a

Test of Integraph.

$k = 0$

$\frac{P_L}{P_m} = 1$ slope through 90°

straight line for $f(\theta)$ force

1 mm

.39	0	.03	●	
.40	0	.03	o	●
.39	0	.03		
.42	0	.03		o

Round Rotor $P_R = 0$.

4.5 about critical.

K = .0318

March 28 1931 Chart 4. R = 0635

New Integraph setup II

$\frac{P_L}{P_n}$	θ_0	$\frac{d\theta}{dt}_{var}$	k	$d\theta/dt$	inch $\frac{d\theta}{dt}$	
.68	0		0.06	11.33	6.01	●
.70	0		"		6.18	o
.69	0		"		6.09	●
.53	180				4.68	

Mar 29 1931.

Chart 5?

Integraph setup V.

on slip table
.375 in = 1 unit of slip.

P_L/P_M	θ_0	k	$d\theta/dt$ units	$d\theta/dt$ inches	
.715	0	0.06	11.59	4.46	o.
.690	0	.06	"	4.32	•
.53	180	.06		3.315	o
.52	"	.06		3.25	.o
.51	"	.06		3.19 very close	o

Reverse Solutions —

chart 6

k R=.06

P_L/P_M =	.67 ".119"	.63	.59	.55	.52	
θ						
	116 (final)	118 •	121.5 •	124. •	127 •	(final angle)
	36 •	52 •	62 •	72 •	77 •	
	340 •	310 •	281 •	254 •	227 •	
					112 •	

P_L/P_M	θ	k	$d\theta/dt$	$d\theta/dt$ inches	
.53	77°	.06		3.315	• barely
.53	80°	.06		"	o • second swing is just
.54	77.5	.06		3.38	o o o out on 3rd swing - just

Round Rotor Solution.

.70	0	.06	4.37

K = .085

Chart 7. Setup V.

P_c/P_m	k	θ_0	$\frac{d\theta}{dt}$ unit	$\frac{d\theta}{dt}$ inch		
.84	.085	0		3.95	●	?
.82	"	0		3.855	●	
.85	"	0		3.995	o	close.
.72	"	180		3.38	o	
.68		180		3.~~20~~	o	close
.67				3.15	●	

The scale on the θ table was in error for the solutions the k slope is correct on this page but needs to be corrected by

$$1/\sqrt{\frac{8.45}{9.39}} = \frac{\cancel{1}}{1.058} = 1.058$$

for all values up to now

that is k as noted should be 1.058 k.

April 1, 1931.

Determination of the conditions wherein a Salient-Pole Synchronous Motor pulls into step as a Reluctance Motor.

The equation stating the restrictions is

$$P_j \frac{d^2\theta}{dt^2} + P_d \frac{d\theta}{dt} + P_R \sin 2\theta = P_L$$

First change variables so that $2\theta = \alpha$

$$P_j \frac{d^2\left(\frac{\alpha}{2}\right)}{dt^2} + P_d \frac{d\left(\frac{\alpha}{2}\right)}{dt} + P_R \sin\alpha = P_L$$

$$P_j \frac{d^2\alpha}{dt^2} + P_d \frac{d\alpha}{dt} + 2P_R \sin\alpha = 2P_L$$

Change time variable so that $t = \frac{\lambda}{a}$.

$$a^2 P_j \frac{d^2\alpha}{d\lambda^2} + aP_d \frac{d\alpha}{d\lambda} + 2P_R \sin\alpha = 2P_L$$

$$\frac{d^2\alpha}{d\lambda^2} + \frac{aP_d}{a^2P_j} \frac{d\alpha}{dt} + \frac{2P_R}{a^2P_j}\sin\alpha = \frac{2P_L}{a^2P_j}$$

now let $\frac{2P_R}{a^2P_j} = 1$

$$a = \sqrt{\frac{2P_R}{P_j}}$$

$$\frac{d^2\alpha}{d\lambda^2} + \frac{P_d}{P_j\sqrt{\frac{2P_R}{P_j}}} \frac{d\alpha}{dt} + \frac{\cancel{2P_R}}{\cancel{P_j}\,\frac{\cancel{2P_R}}{\cancel{P_j}}}\sin\alpha = \frac{\cancel{2}P_L}{\cancel{P_j}\left(\frac{\cancel{2}P_R}{\cancel{P_j}}\right)}$$

$$\frac{d^2\alpha}{d\lambda^2} + \frac{P_d}{\sqrt{2P_jP_R}} \frac{d\alpha}{dt} + \sin\alpha = \frac{P_L}{P_R}.$$

From the cylindrical rotor case then with

$$k \text{ as } \frac{P_d}{\sqrt{2P_j P_R}} \text{ and } \frac{P_L}{P_R}$$

it can be found whether or not the conditions fall above or below the ultimate line for synchronization.

~~With $P_R = 0.3$~~

Unbalanced-Rotor Induction Motor

The torque equation for an induction motor with an unbalanced rotor is of this form

$$P_j \frac{d^2\theta}{dt^2} + P_d(1-b\cos 2\theta)\frac{d\theta}{dt} = P_L$$

$$\frac{d^2\theta}{dt^2} + \frac{P_d}{P_j}(1-b\cos 2\theta)\frac{d\theta}{dt} = \frac{P_L}{P_j}$$

or since $\frac{d\theta}{dt} = S.$

$$\frac{dS}{dt} + \frac{P_d}{P_j}\left(1-b\cos 2\int_0^t S\,dt\right)S = \frac{P_L}{P_j}$$

Or in another form by changing the units of time

let $t = \lambda/a$ a = constant.

$$a^2P_j\frac{d^2\theta}{d\lambda^2} + aP_d(1-b\cos 2\theta)\frac{d\theta}{d\lambda} = P_L$$

Let $a^2P_j = 1$

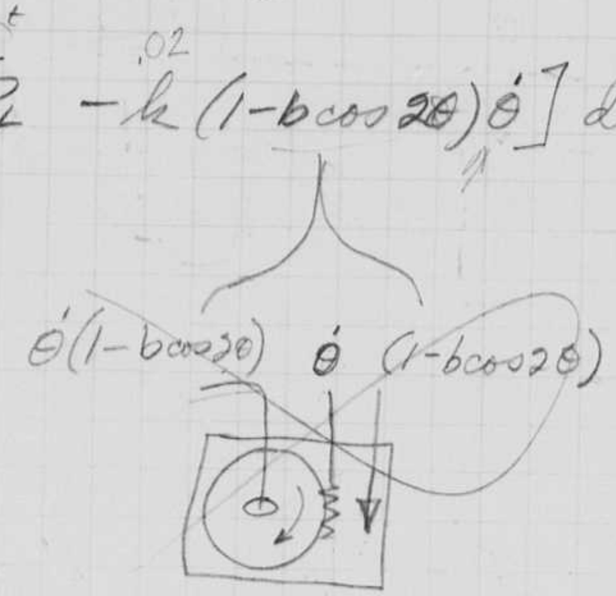
$$a = \frac{1}{\sqrt{P_j}}$$

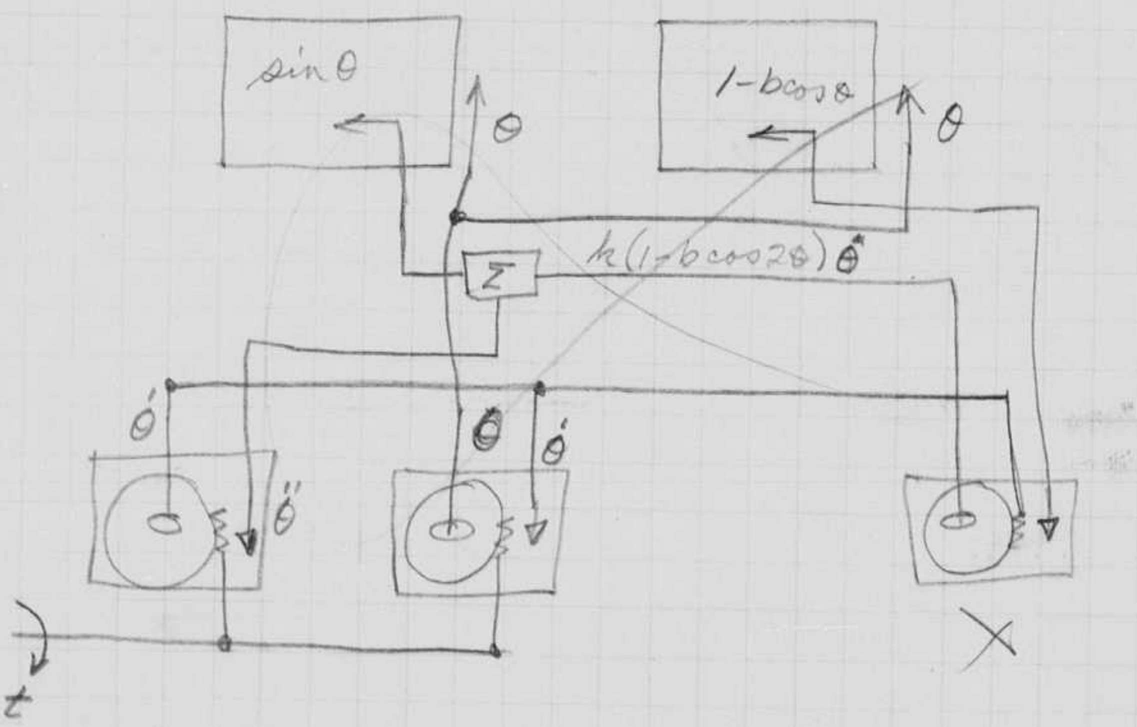
$$\frac{d^2\theta}{d\lambda^2} + \frac{P_d}{\sqrt{P_j}}(1-b\cos 2\theta)\frac{d\theta}{d\lambda} = P_L.$$

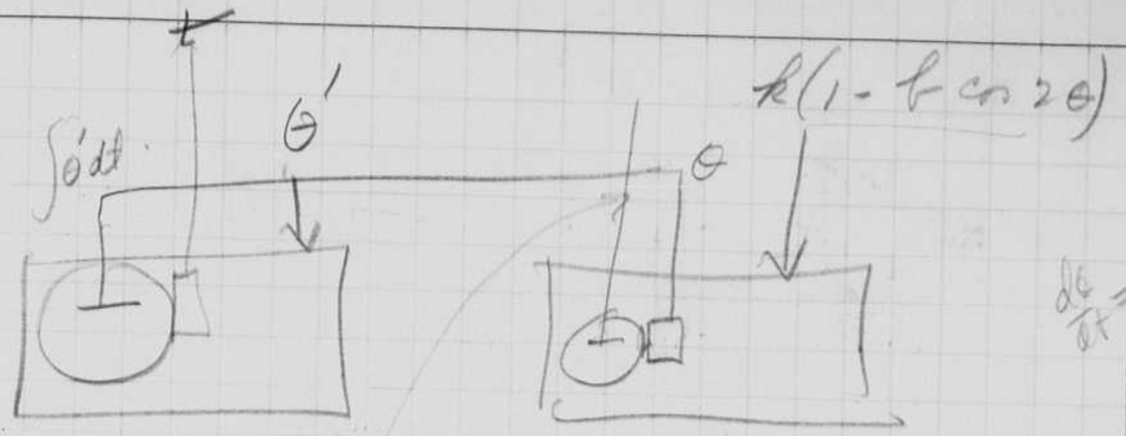
Steady state

$$\ddot{\theta} + k(1 - b\cos 2\theta)\dot{\theta} = P_L$$

$$\theta = \int_0^t\!\!\int_0^t \left[P_L - k\,(1 - b\cos 2\theta)\,\dot{\theta}\right] dt\,dt$$





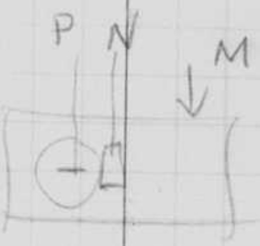


$$\frac{d\theta}{dt} = \int d\theta.$$

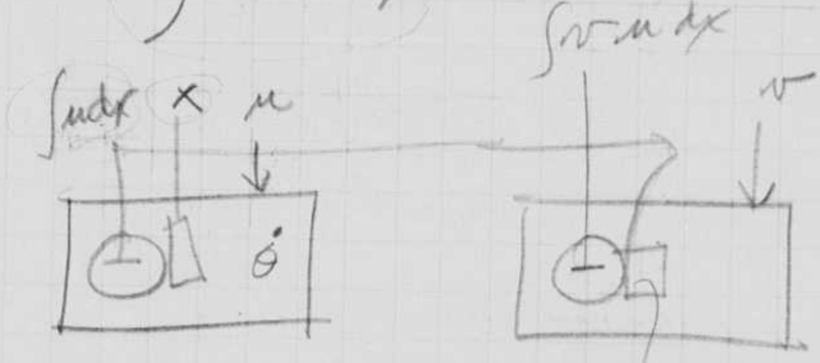
$$\dot{\theta} = \int \frac{d\theta}{dt}\, dt$$

$$\dot{\theta} = \int d\theta.$$

$$\int k(1-b\cos 2\theta)\,\theta' dt$$



$$\int u v\, dx$$



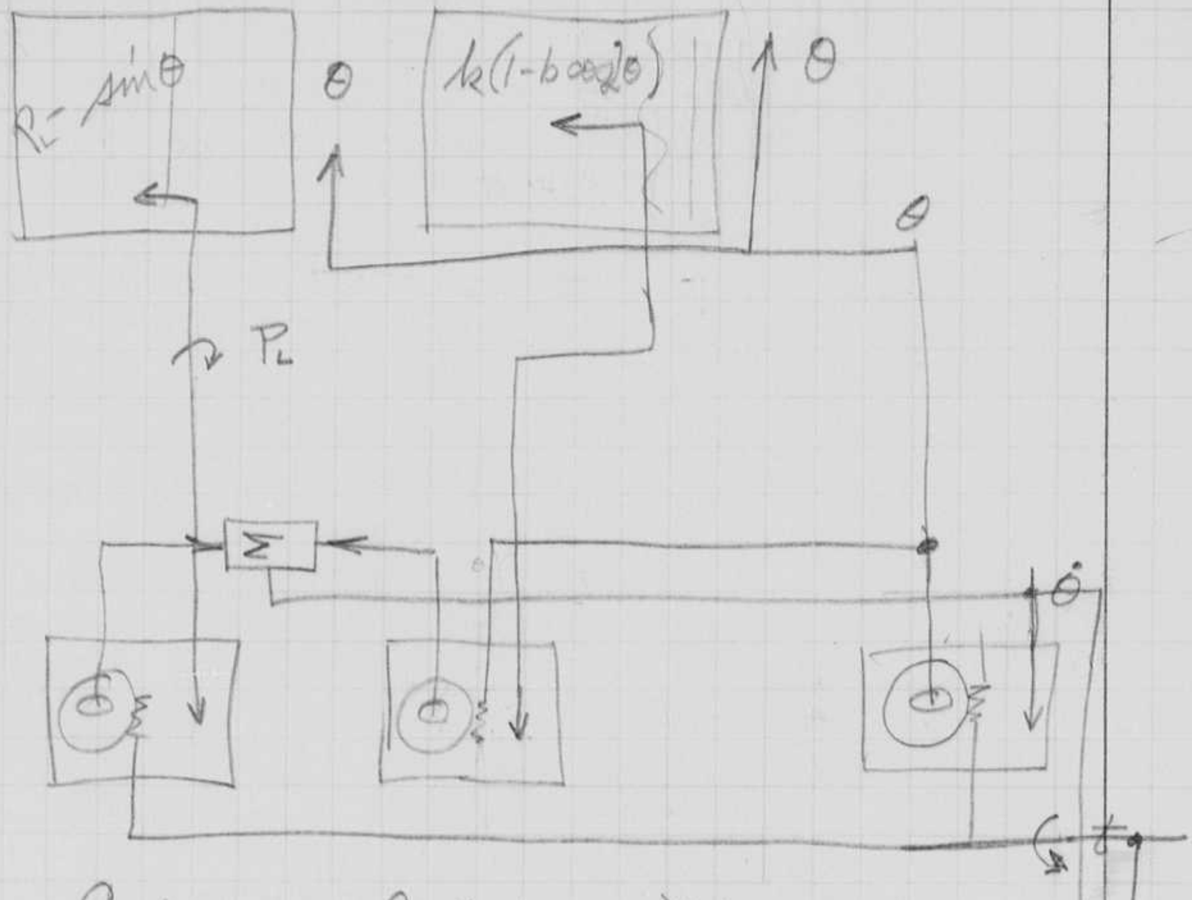
$$P = \int M\, dN$$

$$\int u dx$$

$$du \int u dx = u\, dx$$

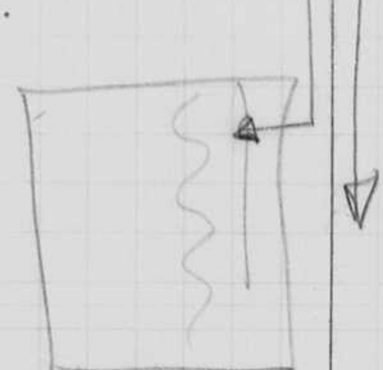
$$\theta = \iint P_L$$

$$\theta = \int \left[\int P_L dt - \left(\int k(1-b\cos 2\theta)\, \dot{\theta}\, dt \right) \right] dt.$$



$\int P_L\,dt \qquad \int k(1-\cancel{b}\cos 2\theta)\dot\theta\,dt.$

$$\int k(1-b\cos 2\theta)\,\cancel{dt}\,\frac{d\theta}{\cancel{dt}}\,\cancel{dt}$$



Notebook # 3

Filming and Separation Record

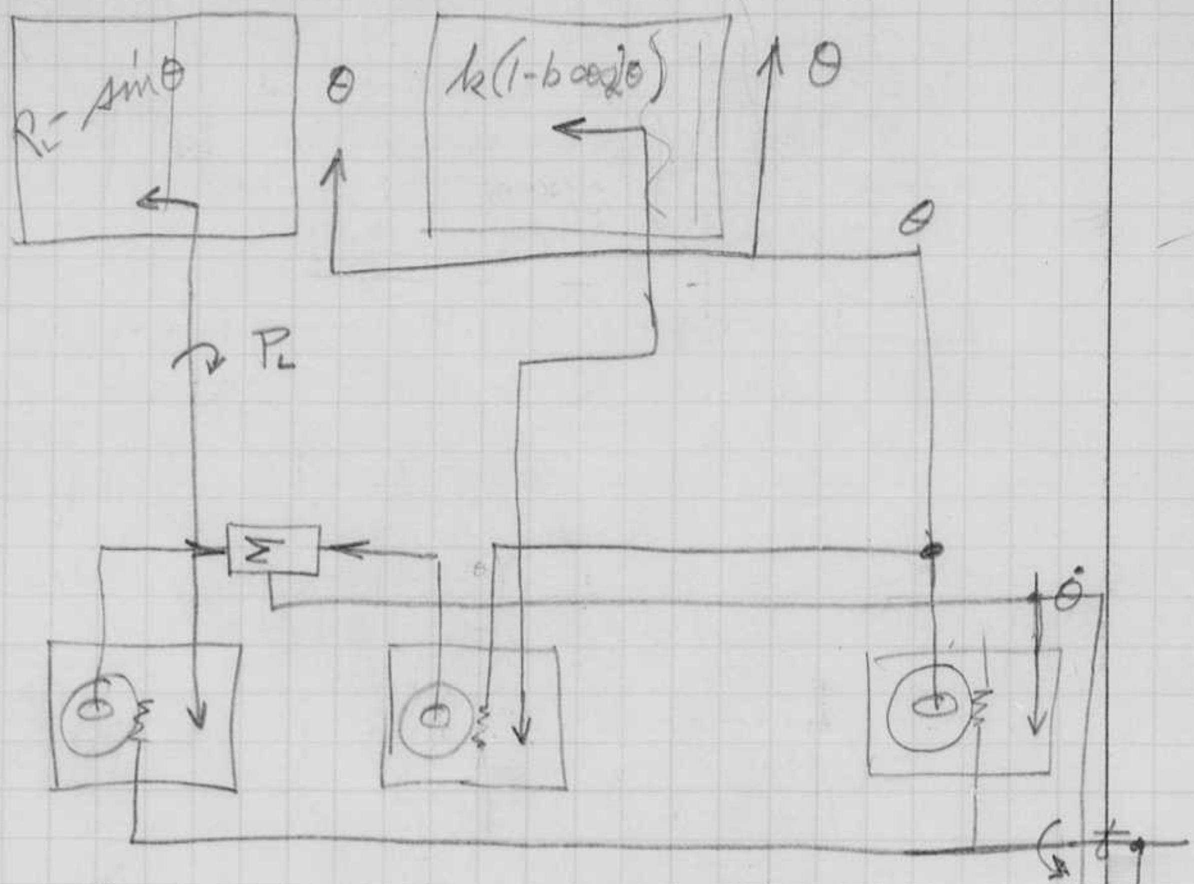
1 unmounted photograph(s)

____ negative strip(s)

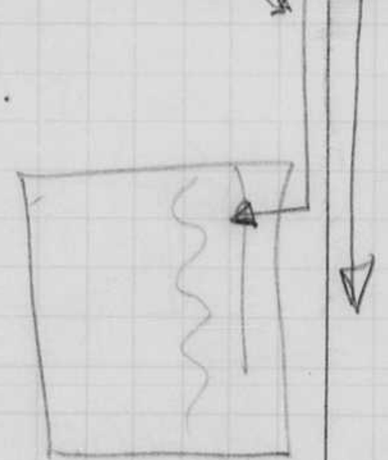
____ unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page 64 and 65.

Item(s) now housed in accompanying folder.



$\int P_L\,dt$ $\qquad$ $\int k(1-b\cos 2\theta)\dot{\theta}\,dt.$



$\int k(1-b\cos 2\theta)\cancel{dt}\,\frac{d\theta}{\cancel{dt}}\,\cancel{dt}$

Notebook # 3

Filming and Separation Record

1 unmounted photograph(s)

____ negative strip(s)

____ unmounted page(s)
(notes, drawings, letters, etc.)

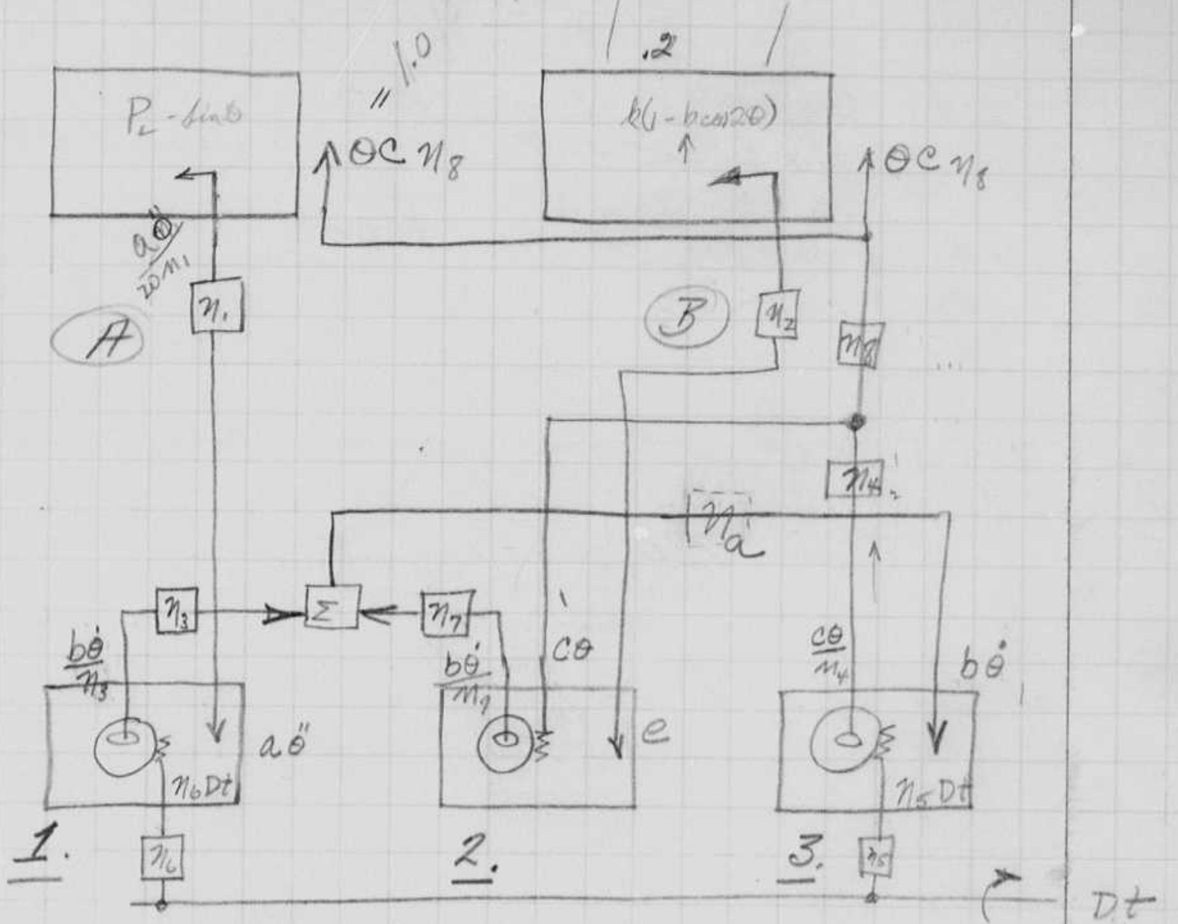
was/were filmed where originally located between page 64 and 65.

Item(s) now housed in accompanying folder.

Dt

Determination of Scales for non-linear damping Syn. Mach Prob.

April 5, 1931.



Integrator limitations

$a\ddot{\theta} < 40$ rev

$e < 40$ "

$b\dot{\theta} < 40$ rev.

$\boxed{c\theta < 400 \text{ rev. per min.}}$

Unit relationships

1 $\quad \dfrac{b\dot{\theta}}{n_3} = \dfrac{1}{32}\displaystyle\int a\ddot{\theta}\, n_6 D\, dt \quad \rightarrow \quad 1 = \dfrac{n_3 n_6\, a\, D}{32\, b}$

2. $\quad \dfrac{b\dot{\theta}}{n_7} = \dfrac{1}{32}\displaystyle\int e c\, d\theta \quad \rightarrow \quad 1 = \dfrac{n_7\, e c}{b\, 32}$

3. $\quad \dfrac{c\theta}{n_4} = \dfrac{1}{32}\displaystyle\int b\dot{\theta}\, n_5 D\, dt. \quad 1 = \dfrac{n_4 n_5\, b\, D}{32\, c}$

$\ddot{\theta}$ max is about 2 or less

$a=16$ — so $a < \frac{40}{2} = 20$ rev/unit Let $a = 16$

Max for e units $= 0.2$

$e = 128$ — $e \times 0.2 < 40$ $e < \frac{40}{0.2} = 200$ Let $e = 128$

$\dot{\theta}$ max is about 20

$b = 2$ — $b\dot{\theta} < 40$ $b < \frac{40}{20} = 2$ let $b = 2$.

Plot Q chart on the A table so

$c = 1.$ — that $c = 1.0$ also let $n_8 = 1.0$

$n_8 = 1.$ — then 180 elect deg. = 9.0 inches

Trial VI Unit equations become

1. $1 = \frac{n_3 n_6 a D}{32 b} = \frac{n_3 n_6 D}{4}$

$n_7 = 1/2$ — 2. $1 = \frac{n_7 e c}{b 32} = 2 n_7$

3. $1 = \frac{n_4 n_5 b D}{32 c} = \frac{n_4 n_5 D}{16}$

There are left 5 unknowns and two equations so that three quantities may be selected at random.

$n_3 = 1/2$

$n_4 = 1/2$ — Let $n_3 = 1/2$ $n_4 = 1/2$ $n_5 = 1.$

$n_5 = 1$

$D = 32$ — from 3. $1 = \frac{1/2 \cdot 1}{16} D$ $D = 32$

$n_6 = 1/4$ — from 1 $1 = \frac{\frac{1}{2} n_6 \cancel{32}^{8}}{4}$ $n_6 = 1/4$

Check.

1. $1 = \frac{n_3 n_6 a D}{32 b} = \frac{\frac{1}{2}\frac{1}{4} 16\, 32}{32 \times 2} = 1$ ✓

2. $1 = \frac{n_7 e c}{b 32} = \frac{\frac{1}{2}\, 128 \cdot 1}{2\, 32} = 1$ ✓

3 $1 = \frac{n_4 n_5 b D}{32 c} = \frac{1/2 \cdot 1 \cdot 2 \cdot 32}{32 \cdot 1} = 1$ ✓

Units (verticle) on B table).

$e = 128$ revs/unit.

$\eta_2 = 1/8$ Let $\eta_2 = 1/8$

$$\frac{e}{20\eta_2} = \frac{128}{20 \times \frac{1}{8}} = \underline{51.2 \text{ inches}} \text{ per unit.}$$

Units (verticle) on A table.

$a = 16$ revs/unit

$\eta_1 = 1/8$ Let $\eta_1 = 1/8$

$$\frac{a}{20\eta_1} = \frac{16}{20\frac{1}{8}} = \underline{6.4 \text{ inches/unit.}}$$

Horizontal scale on A and B tables is the same and is equal to

9" = 180 elect degrees

---

trial $\underline{VII}$ Incase $k = 0.05$ or less, the e scale can be doubled to 256 rev/unit.

The verticle scale would then become

$$\frac{e}{20\eta_2} = \frac{258}{20\frac{1}{8}} = 102.4 \text{ inches/unit.}$$

Since e only appears in unit equat 2 and since c and s are fixed, η_7 needs to be changed in the opposite sense to e

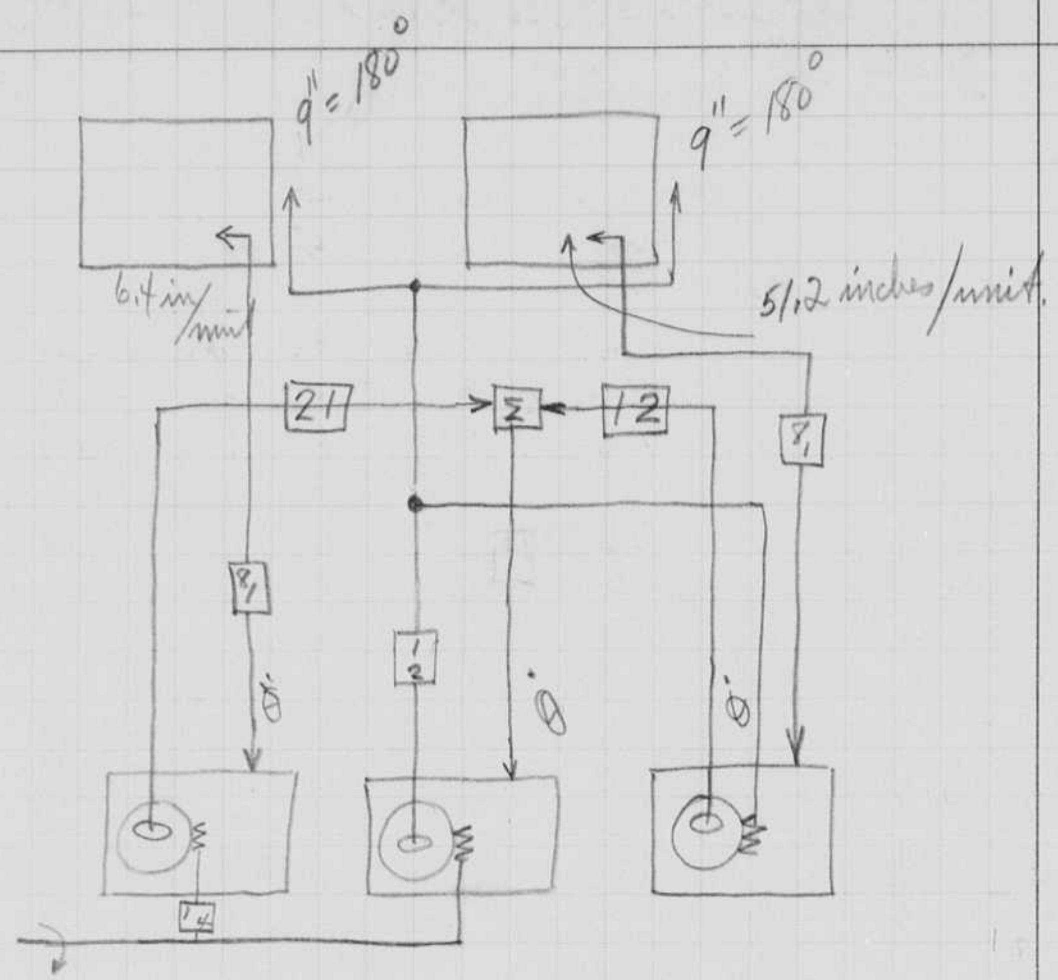
$\eta_7 = 1/4$ (2) $$1 = \frac{\eta_7 ec}{s32} = \frac{\eta_2 \overset{128.4}{256}\ 1.0}{32 \times 2} = \eta_7 4 \qquad \eta_7 = 1/4$$

or $e = \cancel{204.8}$ 512 for $\eta_7 = 1/8$ scales $\underline{VII}$

$e = 204.8$ " $\eta_7 = 1/4$

$e = 102.4$ for $\eta_7 = 1/2$

scales $\underline{I}$



Setup for scales VI

Effect of the time constant of the field circuit.

April 5, 1931.

The inductance of the field circuit will retard the build up of the synchronizing torque in approximately an exponential relationship. That is the synchronizing force is ~~actually~~ more closely represented by $[P_m(1-\varepsilon^{-\alpha t})\sin\theta]1$ than $(P_m \sin\theta)1$.

$$P_j \frac{d^2\theta}{dt^2} + P_d \frac{d\theta}{dt} + [P_m(1-\varepsilon^{-\alpha t})\sin\theta]1 = P_L,$$

$$\frac{d^2\theta}{dt^2} + \frac{P_d}{P_j}\frac{d\theta}{dt} + \left[\frac{P_m}{P_j}(1-\varepsilon^{-\alpha t})\sin\theta\right]1 = \frac{P_L}{P_j}.$$

Let $t = \frac{\lambda}{a}$ where λ = new variable.

$$a^2\frac{d^2\theta}{d\lambda^2} + \frac{aP_d}{P_j}\frac{d\theta}{d\lambda} + \left[\frac{P_m}{P_j}(1-\varepsilon^{-\alpha\frac{\lambda}{a}})\sin\theta\right]1 = \frac{P_L}{P_j}$$

$$\frac{d^2\theta}{d\lambda^2} + \frac{P_d}{aP_j}\frac{d\theta}{d\lambda} + \left[\frac{P_m}{a^2P_j}(1-\varepsilon^{-\frac{\alpha}{a}\lambda})\sin\theta\right]1 = \frac{P_L}{P_j a^2}$$

Now Let $\frac{P_m}{a^2P_j} = 1$

or $a = \sqrt{\frac{P_m}{P_j}}$

which gives

$$\frac{d^2\theta}{d\lambda^2} + \frac{P_d}{\sqrt{P_j P_m}}\frac{d\theta}{d\lambda} + \left[(1-\varepsilon^{-\frac{\alpha}{a}\lambda})\sin\theta\right]1 = \frac{P_L}{P_m}$$

$$\theta = \iint\left\{\frac{P_L}{P_m} - k\frac{d\theta}{d\lambda} - \left[(1-\varepsilon^{-\frac{\alpha}{a}\lambda})\sin\theta\right]1\right\}d\lambda\, d\lambda$$

$$= \int\left\{\int\frac{P_L}{P_m}d\lambda - \int k\frac{d\theta}{d\lambda}d\lambda - \int(1-\varepsilon^{-\frac{\alpha}{a}\lambda})\sin\theta\, 1\, d\lambda\right\}d\lambda$$

Slip variations due to a Reluctance torque equal to .3 Pm. Values taken from charts 1 to 9 inc.

Chart	L.R.	k.	$\dot\theta_{max}$	$\dot\theta_{min}$	$\dot\theta_{max}-\dot\theta_{min}$	avg $\dot\theta$ $\frac{\dot\theta_{max}+\dot\theta_{min}}{2}$	$\frac{\dot\theta-\dot\theta}{\dot\theta+\dot\theta}$ % var.	p. 52 % var
1. p. 52.	.15	.0177	1.62	1.4	.22	1.4	15.7	16.
	.20	"	2.05	1.88	.17	1.88	9.05	9.5
	.25	"	2.48	2.34	.14	2.34	5.98	3.92
	.27	"	2.68	2.54	.14	2.54	5.52	4.45
	.28	"	2.76	2.63	.13	2.63	4.95	4.5
	.30	"	2.93	2.80	.13	2.80	4.65	3.67
	.15	"	1.63		.22	1.41	15.6	16.0
	.14	"	1.55		.24	1.31	18.35	17.4
	.13	"	1.46		.25	1.21	20.65	20.0
	.12	"	1.38		.28	1.10	25.40	—
					$\theta_{max}-\theta_{avg.}$	avg.		
2.	.14	"	3.48	2.61	.40	3.08		13.0
	.17	"	4.09	3.38	.34	3.75		9.07
	.20	"	4.70	4.09	.30	4.40		6.82
	.23	"	5.30	4.77	.24	5.06		4.75

April 8 1931 Solutions with set ups of page 68. scales VI

51.2
4
204.8

Cylindrical rotor check.

P_L/P_m	k	$\dot\theta_0$	$\dot\theta_0$	K				
0.6	.05	12.0	0				● close.	Perfect check of Brown & Zimmerhausen
0.605	.05	12.1	0				○ close.	
		$\dot\theta$	$2\times\dot\theta$	51.2k	θ_0			
.89	.12	7.42	14.84	6.14	180		● close	
.90	.12	7.50	15.0	"	180		● closer.	
.915	.12	7.62	15.24	"	180		critical.	
						$\dot\theta_{max}$		
.97	.14	6.93	13.86	7.2	180	24.75	● close.	
.98	.14	7.0	14.0	7.2	180		● closer.	

T_j changed to 1/8 from 1/2 k scale thereby modified (see page 67). 204.8 inches per radian Scales V

				204.8				
Chart 8	.14	.01	14.0	28.0	2.064	0		● 30°
	.15	.01	15.0	30.0	"	○		○
	.145	.01	14.5	29.0	2.064	0		○ close.
							$\dot\theta_{max}$	
	.06	.01	6.0	12.0	2.064	180	31.3	○
	.05	.01	5.0	10.	2.064	180		● 10°
	.055	.01	5.5	11.	2.064	180	31.75	● 1° close

Salient Pole case $P_R = 0.3\,P_m$

	.055	.01	5.5	11.0	2.064	180	32.3	○ Initial slip = 14.25
	.05	.01	5.0	10.0	2.064	180		○, second ●
	.04	.01	4.0	8.0	"	180	31.5	● 158°
	.14	.01	14	28	"	0		○
	.13	.01	13	26	"	0		● 45°
	.135	.01	13.5	27	"	0		●
Chart 9	.38	.03	12.66	25.32	6.195	0		○
	.37	.03	12.32	24.64	6.190	0		● 15°
	.21	.03	7.0	14.0	"	180	31	● 13°
	.22	.03	7.3	14.6	"	180		○
	.38	.05	7.60	15.2	10.32	180		●
	.39	.05	7.8	15.4	"	"		○? Shifted final pt.

Changes of scales. Diagram on page 65

Given constants.

$a = 16$

$e = \cancel{128}$ (Depends upon scales).

$b = 2$

$c = 2$ $\quad n_8 = 1/2$ (These were $c = 1$ $n_8 = 1$).

the angle scales and plots remain the same as scales VI and VII.

Let $n_a = 1/4$

this gives a 4:1 scale relationship between the output slip on the resultant table of this VIII against that of VI and VII.

Unit equations

$$1 = \frac{n_3 n_a n_6 a D}{32 b} = \frac{n_3}{4} \frac{n_6 16 D}{32 \times 2} = \frac{n_3 n_6 D}{16}.$$

$$1 = \frac{n_7 n_a e c}{b \times 32} = \frac{n_7 \cancel{e} \cancel{2}}{4 \times \cancel{2} \times 32} = \frac{n_7 e}{128}$$

(Let $e = \overset{512}{\cancel{204.8}}$ as in VIII) or 204.8 in/unit.

Then $n_7 = \frac{128}{512.0} = \frac{1}{4}$

$$1 = \frac{n_4 n_5 b D}{32 c} = \frac{n_4 n_5 2 D}{32 \times 2} = \frac{n_4 n_5 D}{32} =$$

Let $n_5 = 1$ $\quad n_4 = 1/2$ $\quad n_3 = 1/2$

$n_4 n_5 D = 32$ $\qquad D = 32 \times 2 = 64$

$$n_6 = \frac{16}{n_3 D} = \frac{16}{64 \cdot 1/2} = \frac{1}{2}.$$

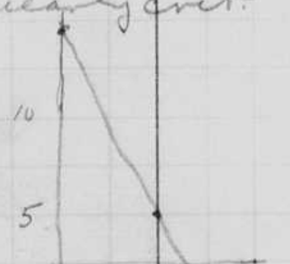
April 11, 1931. (b = 1). Slip variations.

	k	P_1/P_m	$\frac{d\theta}{dt}$	$2\frac{d\theta}{dt}$ tempor. stable		$\dot\theta$ variation in revs. $\dot\theta_{max}\ \theta_{avg}$		$-(\dot\theta_{min} - \dot\theta_{avg})$		% θ_{max}	% θ_{min}
9	.03	.1	3.33	6.67	→ 241°	1. 350/360		1. 140/360	1.935 1.389	2.30	2.08
	.03	.2	6.66	13.32 240 120		1 285/360		1. 233/360	1.825 1.647	13.69	12.86
	.03	.3	10.	20.0		1 282/360	1.778	1 220/360	1.611	2.88	.705
	.03	.4	13.33	26.66		1 287/360	1.797	1 222/360	1.617	.614	.606
	.03	.5	16.67	33.34		1 248/360	1.689	1 240/360	1.666	5.065	5.0
10	.015	.15	10.0	20.0		310/360	0.861	300/360	0.833	4.30	4.16
	.015	.075	5	10.0		28/360	.09	284/360	.789	8.17	7.89

Slip variation due to Resistance Torque. 3 P.m.

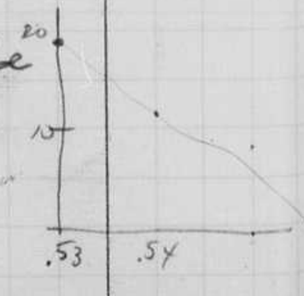
~~11.~~	~~.03~~	~~.15~~	~~5.0~~	~~10~~	Cylindrical rotor		Unbal. Rotor
11	.03	.23	7.67	15.34 122	θ_0	●	Very stable.
	.03	.39	13	26.	0	●	
	.03	.42	14	28.	0	○	
	.03	.40	13.33	26.66	0	●	
	.03	.41	13.66	27.3	0	○	
12.	.03	.23	7.67	15.333	180	●	very.
	.03	.24	8.0	16.0	180	●	close. 40
	.03	.25	8.33	16.66	180	●	close 7° nearly crit.
	.03	.26	8.66	17.33	180	○	

No zero load.



change $e = \frac{517}{2}$ $n_7 = 1/2$

13	.06	.70	11.67	12.33	0°	●	14°
	.06	.71	11.83	23.65	0°	●	5°
	.06	.72	12.0	24.0	0°	●	2° close.
	~~.06~~	.73	12.18	24.33	0°	○	
	.06	.53	8.83	17.66	180°	●	18° to spare
..	.06	.54	9.0	18.0	180	●	12
	.06	.55	9.17	18.33	180°	●	8
	.06	.56		18.66	180	○	




50
40
%
30
20
10
0
K=.03
K=.015
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
L.R.

April 12, 1931.
G. E. Edgerton.

Reverse solutions.

$R = 0.06 \quad P_R = 0.3 P_m.$

Chart.	k	L.R.	θ FINAL.	θ start		$\dot{\theta}$ avg.	turns
14	0.06	.48	129.5	128.	✓	8.	16
	"	.51	128.0	126.5	✓		17
	"	.54	125.5	124.0	✓		18.
	"	.57	123.5	122.0	✓		19
	"	.60	121	119.5	✓		20.
	"	.63	119	117.5	✓		21.
	"	.66	117	115.5	✓		22
	"	.69	114.5	113.0	✓		23.

Forward solution
" .63 $\theta = \theta_0$ ⊙ very stable? !!

Forward check for cylindrical rotor
.06 .69 nearly critical 0 just barely Good check!

Some trouble was experienced with the integraph due to too much load on the angle shaft. This may explain some of the discrepancy of the days results.

April 14, 1931. Data from Integraph Solutions.

Chart no.	k.	θ_0	L.R.	$\dot{\theta}_0$ avg. inches	$\dot{\theta}_0$ inst. inches	$\dot{\theta}$ max. transient	$\theta=180$ $\frac{\dot{\theta}_{max}}{\dot{\theta}_{avg}}$	$\theta=0$ $\frac{\theta_{max}}{\theta_{avg}}$
1	.0177	180	.12	1.13	1.38	3.1	2.75.	1.22
3	.0475	180	.36	3.0	3.38	6.05	2.02.	1.13
5	.0635	180	.51	3.22	3.59	6.0	1.87	1.12
7	.085	180	.61	3.16	3.52	5.7	1.80	1.11
8	.01	180	.04+	.47	.66	1.56	3.32.	1.40
9	.03	180	.21	.71	.805	1.56	2.20	1.14
9.	.05	180	.38	.76	.88	1.52	2.0	1.16
20	.10	180	.796	.32	.36	5.61	1.77	
19	.02	180	.13	.047	—	6.3	1.55	

Chart 14. intersections. $k = 0.06$
$P_R = 0.3 P_m$.

L.R.	Steady State $\dot{\theta}$ average	$\dot{\theta}$ max	$\dot{\theta}$ min	$\frac{\dot{\theta}_{max}}{\dot{\theta}_{min}}$ SS	transient $\dot{\theta}$ max	θ_0	θ_0.
.48	3.2		3.6			83	
.51	3.4		3.8			76°	240
.54	3.6		3.95			70	262
.57	3.8		4.18			60.5	281
.60	4.0		4.35			51.5	298.5
.63	4.2		4.51			37.5	314
.66	4.4		4.72.			16.5	343
.69	~~4.6~~						

April 18-1931
H.E. Edgerton

Integraph Solutions.

Setup as outlined on page ~~66~~ 73

Damping scale = ~~204.8~~ 102.4 inches / unit on table # 2. $N_7 = 1/2$.

Reverse solutions.

.06
.1075
.06
0.1??

16 x .49 = .783

Chart	k.	L.R.	θ FINAL	θ Start	$\dot\theta$ avg.	$\dot\theta$ turns
	0					
~~14~~ 15	0.06	.49	129	127.5	8.17	16.1/3
						~~16. 2/3~~
	,06	,50	127.5	126.0		16 2/3
	,06	.51	~~128~~ 127	~~126.5~~ 125.6		
	,06	.50	127.5	126.0		

Speed variations due to reluctance torque.

16	0.06	.3	5	10
		.6	10	20
		.9	15	30.

April 19, 1931. Cont of speed variations

0.04	~~.3~~ .2	5 ~~7.5~~	~~15~~	10
	~~.6~~ .4	10		20
	~~.9~~ .6	15		30

Pulling into step equa

			$\theta°$	$\dot\theta$	turns	
17.	0.06	.48	180°	8	16.	o just! .475 probably stable.
	,06	,65	0	10.84	21.65 230	• quite stable
	.06	.67	0		22.30 1102	o ? belt off.

Reverse Solutions $P_R = 0.3\ P_m$ $b = 0$

Chart	k	LR	θ_{final}	θ_{start}	$\dot{\theta}_{avg}$	$\dot{\theta}_{turns}$	θ_0	θ_0
17	0.04	.32	141.5	140	8	16	98°	225
		.36	139.	138.5	9	18	79.0	256
		.40	135	133.5	10	20	67.0	283
		.44	132	130.5	11	22	48.5	307
		.48	128.5	127.5	12	24	13°	345
		.34	140.5	139.0	8.5	17.	86.0	243

Forward ~~tests~~.

8.18 inch

			θ_0	$\dot{\theta}$	turns	
18	0.08	.64	180°	8	16	very close 2 or 3°
	~~08~~	~~.645~~	~~180~~	~~8.07~~	~~16.14~~ 47°	
	08	.81	0	10.13	20.26	5°
		.82	0		20.50	0
		.82	0			

Apr. 20. 1931.

Reverse Solutions

$6.14 \frac{0.06}{0.02} = 2.045$

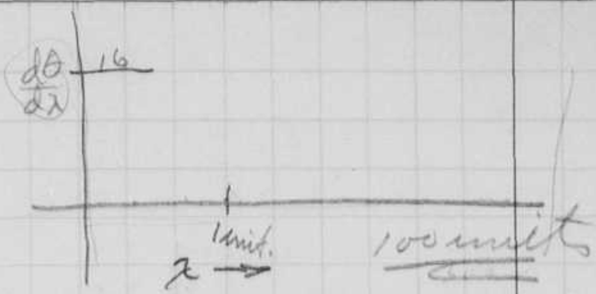
Chart	k.	L.R.	θ final	θ start	$\dot{\theta}$ average	$\dot{\theta}$ turns	θ_0	θ_0.
	0.02.	~~.270~~						
19.		.26 ✓	147.5	146	13	26 ✓	22.0	339
		.24 ✓	149.5	148	12	24 ✓	48.6	310
		.22 ✓	151.5	150	11	22 ✓	65	290
		.20 ✓	153.+	151.5	10	20 ✓	77.5	274
		.18 ✓	155.5	154.0	9	18 ✓	84	258
		.17 ✓		155.0		17 ✓	94	249
		.16 ✓	158	156.5	8	16 ✓	99	240
		~~.15~~	~~159~~	~~157.5~~	~~7.5~~	~~15~~		
		.14 ✓	160	158.5	7	14 ✓	108	220
		.13	161.5	160.0	6.5	13.		

Forward solutions.

$6.14 \frac{1}{.6} = 10.23$

			$\dot{\theta}$	turns	θ_0		
20.	0.10	0.91	9.1	18.2	0	●	12°
		.92	9.2	18.4	0	●	8°
						critical	93.5
	.10	.94	9.4	18.8	0	○	just
apr 21.	.10	.8	8.0	16.0	180	○	just.
	.10	.79	7.9	15.8	180	●	just! 1° to go.

$\frac{d\theta}{dx} = 16$ units



$D = 64$ rev/unit.

$\frac{64}{400}$.

$100 \times 64 = 6400$ rev.

$n \; \frac{6400}{\cancel{20}} =$ inch $= \cancel{20}^2$

$\frac{64 \text{ rev unit}}{16} = 4$ rev/unit.

$= \frac{4}{20}$ inch/unit, ?

$= \frac{1}{5}$ in/unit

or 5 units/inch

$n = \frac{400}{6400} = \frac{1}{16}.$

2(.

Results against time.

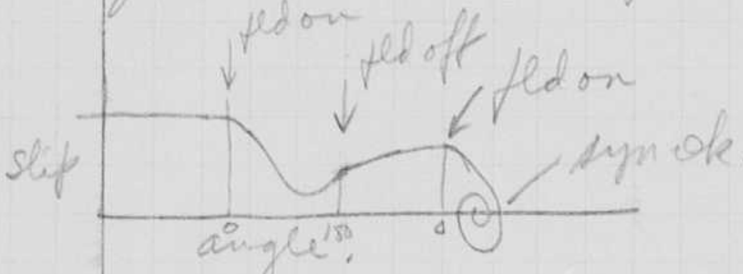
$6.14 \times \frac{4}{6} = 4.095$

k	L.R.	θ_0	$\dot{\theta}_{avg}$	turns		
.04	.31	180	7.75	15.5	o	
.04	.30	180	7.5	15.0	o	
	.29	180	7.25	14.5	•	14°
.04	.48	0	12	24	•	8°

~~April~~ May 1, 1931. Pulling into step with intermittent field current.

W. Edgerton

If the exciter is connected only when the field is such as to give motor action it might be possible to pull into step heavier loads. The stroboscopic relay can be arranged so that the field circuit is energized only when the angle lies between 0 and 180 electrical degrees. Then if a motor fails to pull in on the first swing, it ~~may~~ will not be driven further from synchronism when the angle is such as to give generator action since the field will be opened then.

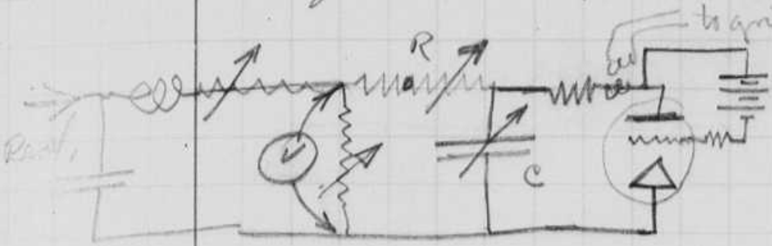


As shown in the figure the field first goes on, then if the motor fails to pull in and reaches 180° elect. degrees the field will be taken off and the motor will start to gain its initial slip. However again at 0 degrees the field is connected to the exciter and the motor again tries to pull in from a smaller slip.

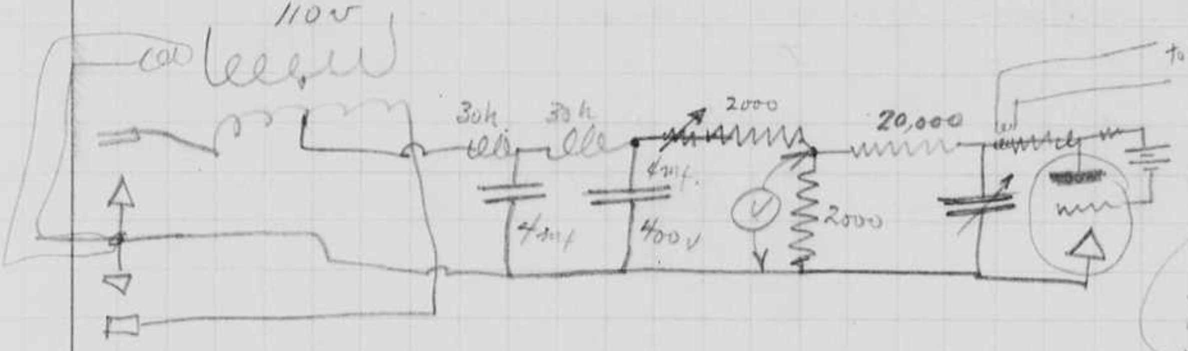
Thyratron Oscillator

May 3, 1931.
H E Edgerton

for driving stroboscope grids



the frequency is determined by the R and the C of the charging circuit and by the bias voltage on the grid.



e_g = 10 – 150 volts negative with respect to the plate.

$200 \times .63 =$ 126. volts

$RC = 1/60 \text{ sec} = 0.0167 = 20{,}000\,C$

$$C = \frac{.0167}{20{,}000} = \frac{1.67\times10^{-2}}{2.\times10^{4}} = .8\times10^{-6} = .8 \text{ mf.}$$

Or make R = 100,000 ohms.

$$C = \frac{.0167\times10^{-2}}{10.\times10^{4}} = .167\times10^{-6} = .167 \text{ mf.}$$

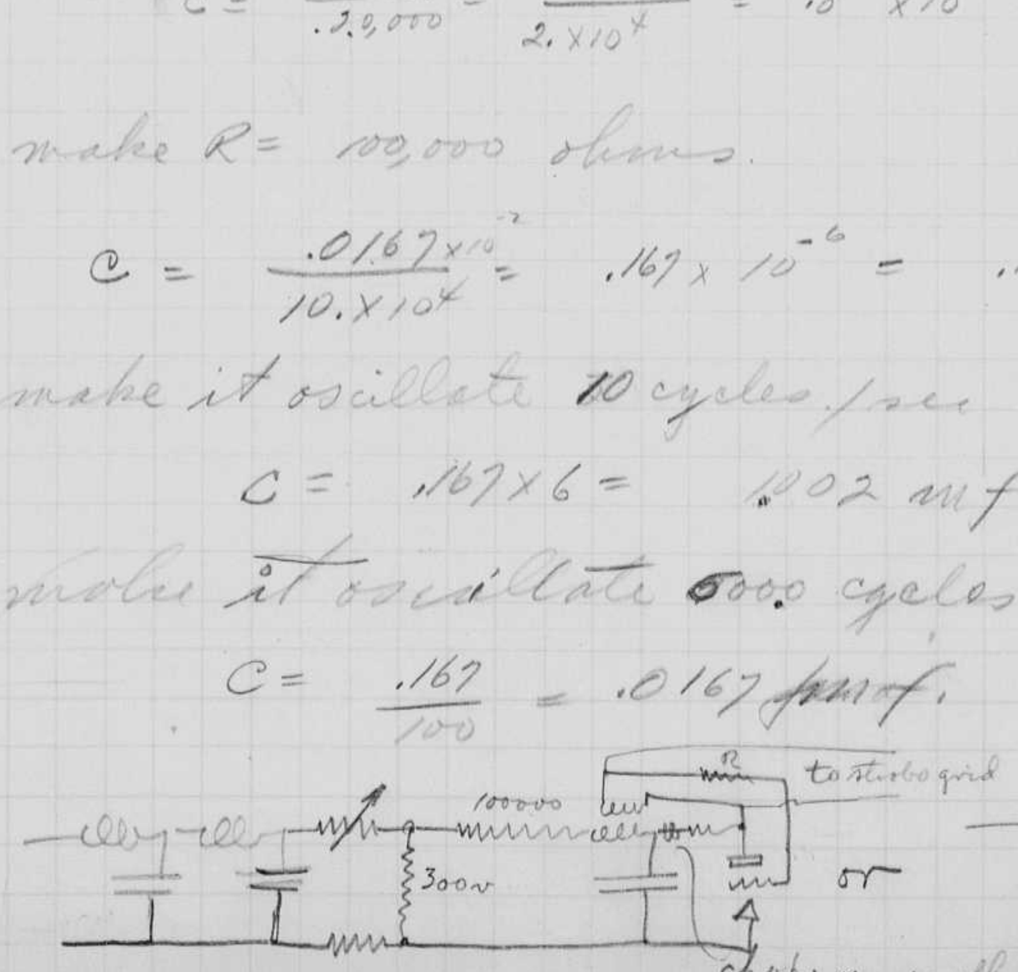
to make it oscillate 10 cycles/sec

$C = .167\times6 = 1.002$ mf.

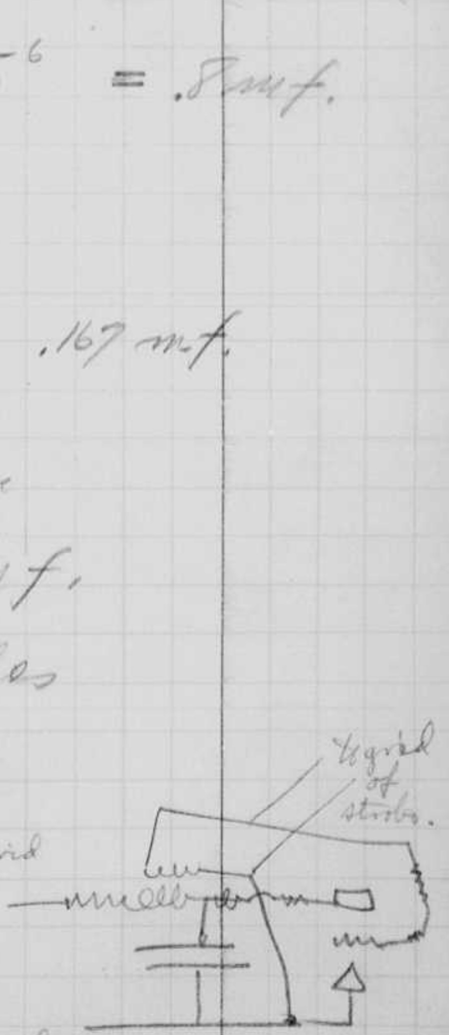
to make it oscillate 5000 cycles

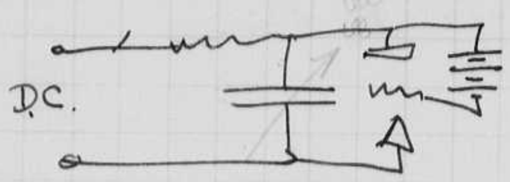
$$C = \frac{.167}{100} = .0167 \text{ mf.}$$

Witnessed May 3, 1931



or



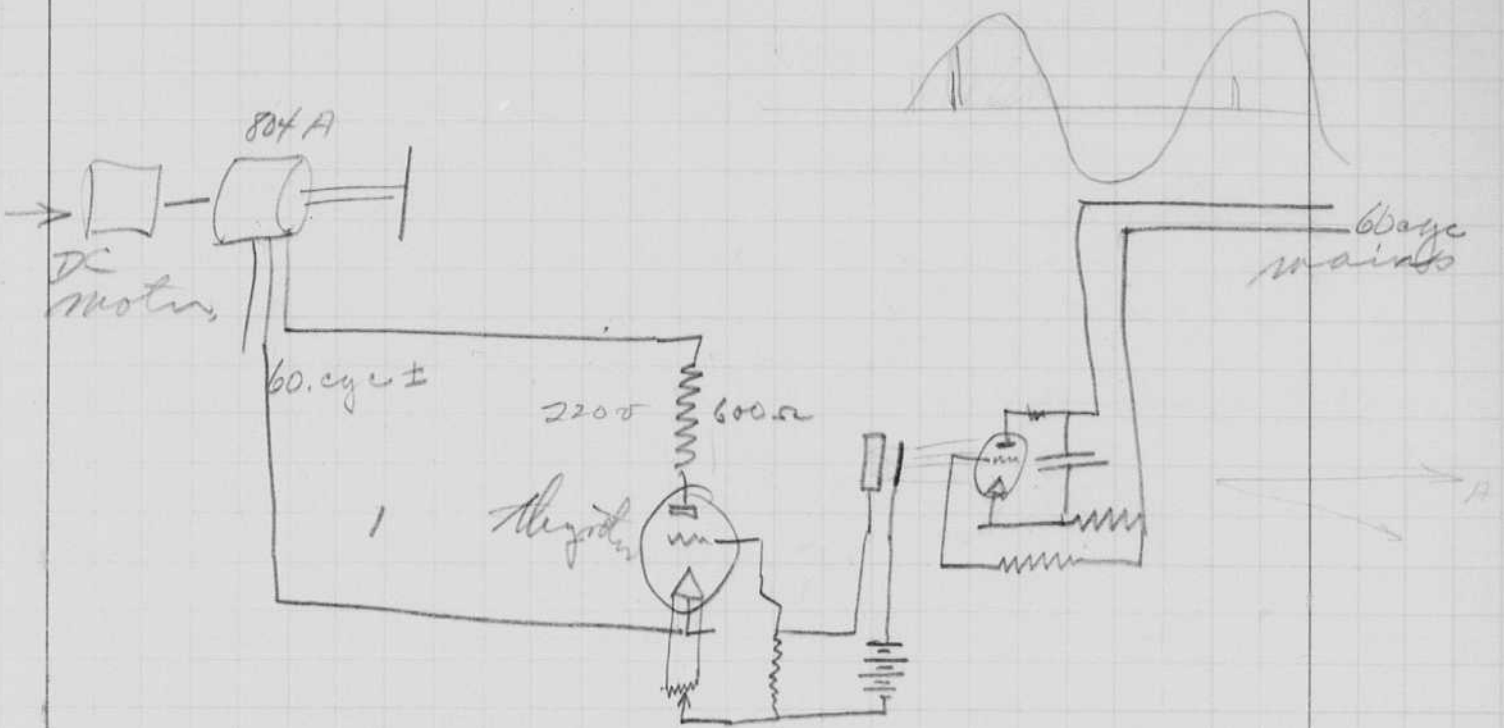

D.C.


e_c

May 5 1931

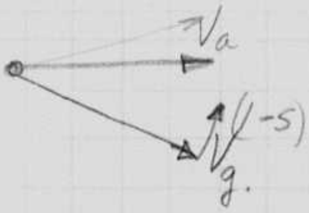
H.E. Edgerton
T.S. Gray.

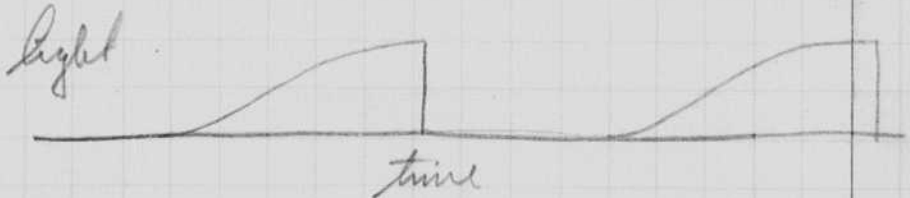
Photocell operation with Stroboscopic Light.



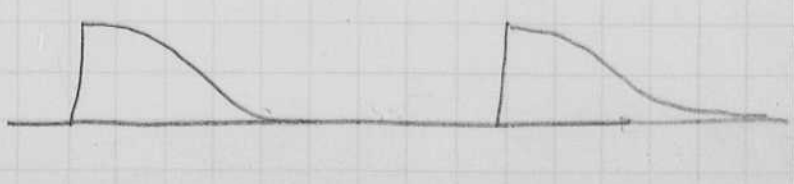
The light from the thyratron operated at a frequency which was the beat of the two frequencies

When the frequency of 804A (alternator) was less than the mains the ~~frequen~~ light gradually increased to a maximum and then snapped off.





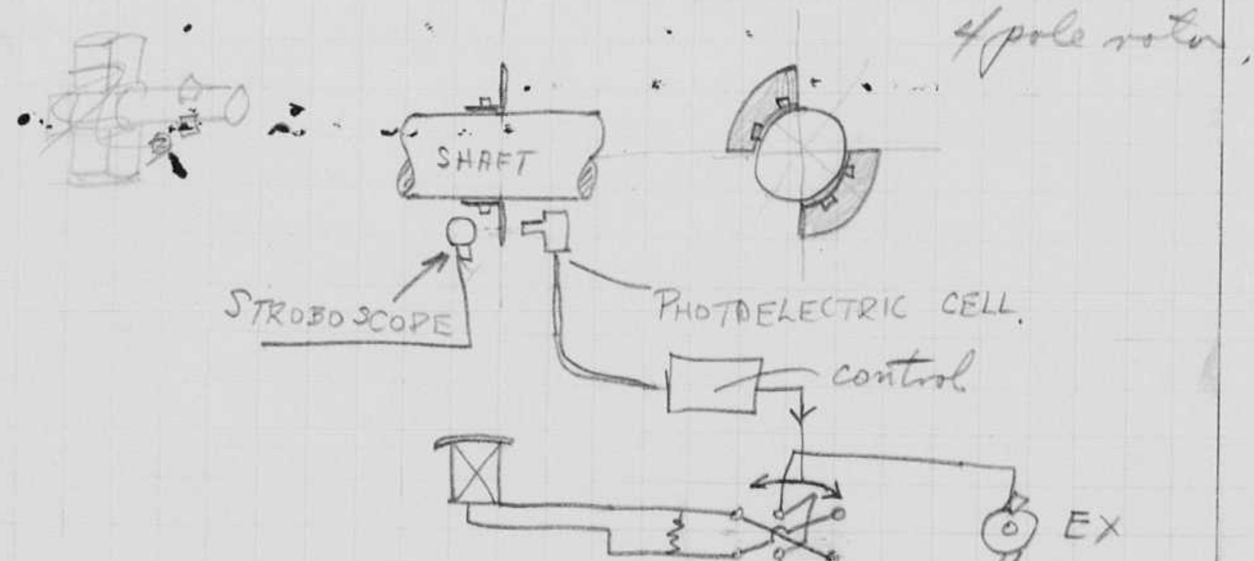
With the frequency of 804A greater than the mains the variation of the light is the opposite



Field Switching Scheme.

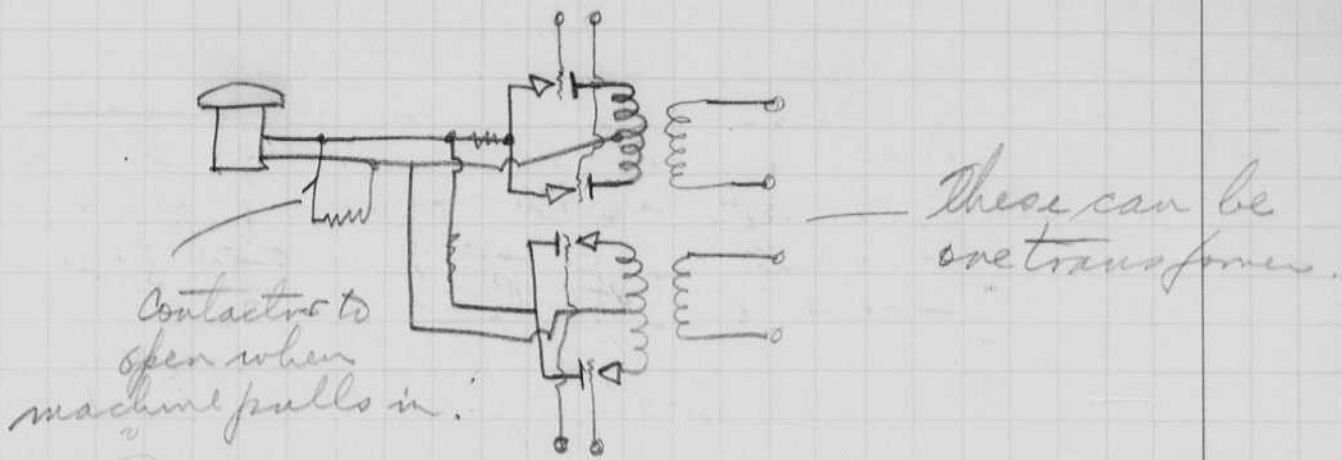
May 5/1931
H E Edgerton

I propose to switch a synchronous motor field so that ~~it is always in such~~ the current in it is always in a sense as to give motor action. This can be accomplished by arranging a stroboscopic light opposite an obstruction driven by the rotor which is ~~so half of the time~~ in the path of the stroboscopic light from ~~the~~ to a photoelectric cell. The phase of the obstruction will be so arranged that it will allow the stroboscopic light to strike the photocell only when the angular displacement is between zero and 180 electrical degrees.

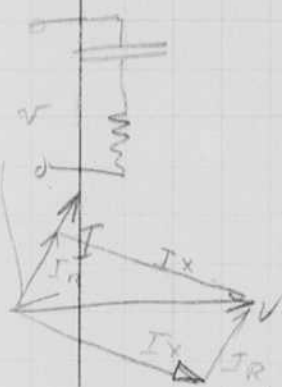


The field is connected one way when the stroboscopic light hits the ~~relay~~ photocell and in the other when the light is obstructed.

Cont.



Photocell and amplifiers are arranged to ~~to~~ connect the proper rectifier to the field to make it give motor action.



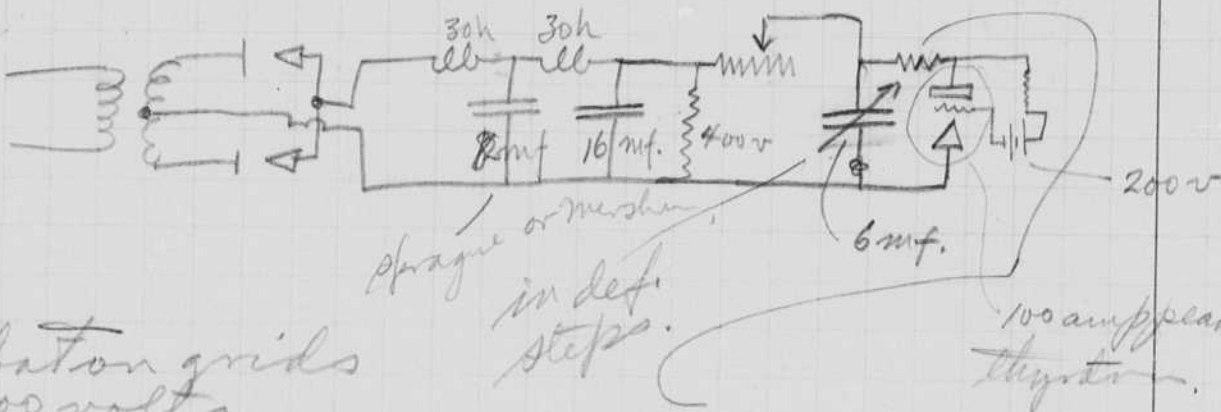
This circuit was shown to me on May 5, 1931 by H. E. Edgerton at Mass. Inst. of Technology, Cambridge, Mass.

Charles Kingsley Jr.

Witnessed May 13, 1931 E. L. Bowles

May 6, 1931.
H. E. Edgerton.

A-C operated variable frequency Stroboscope.

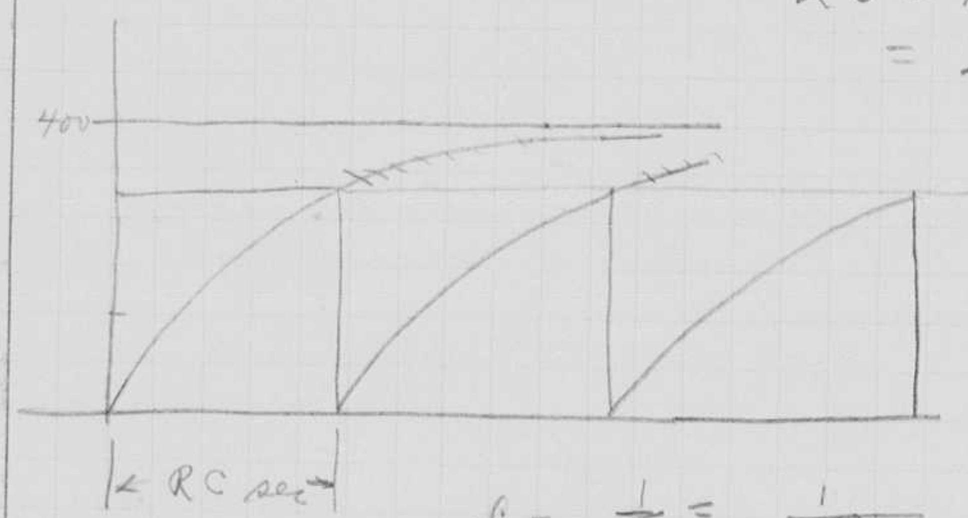


C baton grids 300 volts.

$\frac{400}{100} = 4$ ohms.

$RC = 4 \times 6 \times 10^{-6}$

$= 24$ ms.



$f = \frac{1}{RC} = \frac{1}{R \times 10^{-6} \times 6}$ $\quad 300 \overline{)1{,}000{,}000.}$ 3000.

$60 = \frac{1}{R \times 6 \times 10^{-6}}$

$R = \frac{60}{6 \times 10^{-6}} = 10 \times 10^{6}$ ohms.

~~$RC = 6 \times 10^{-6} \times 10 \times 10^{6} = 60$ sec~~

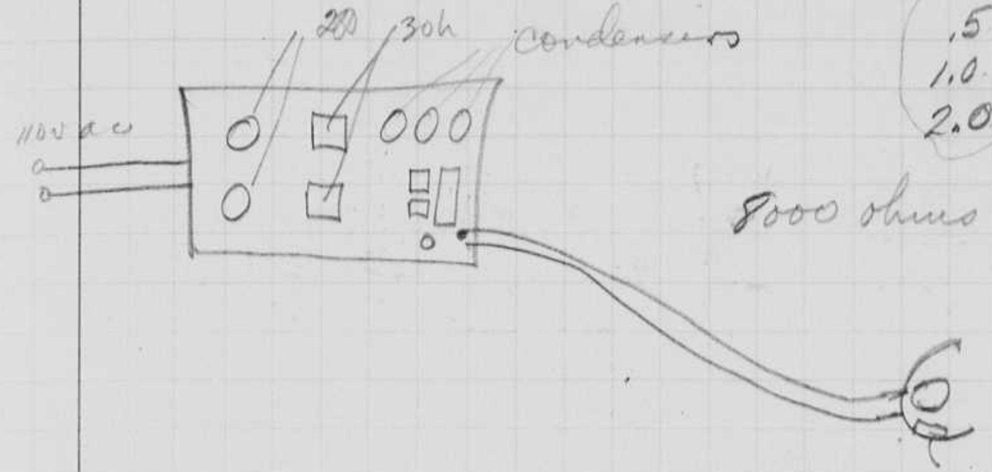
$\frac{1}{60} = RC \quad R = \frac{1}{60 \times 6 \times 10^{-6}} = \frac{10^{6}}{360} = 3000$ ohms.

try 2 mf. for discharge and 4 ohms in disc. circuit

$R = \frac{1}{60 \times 2 \times 10^{-6}} = \frac{10^{6}}{120} = 10{,}000$ ohms.

5,000 – 30,000 ohms. chg current
0.5 to 4 mf. condenser

Cont



.25
.5
1.0
2.0

8000 ohms — 20,000.

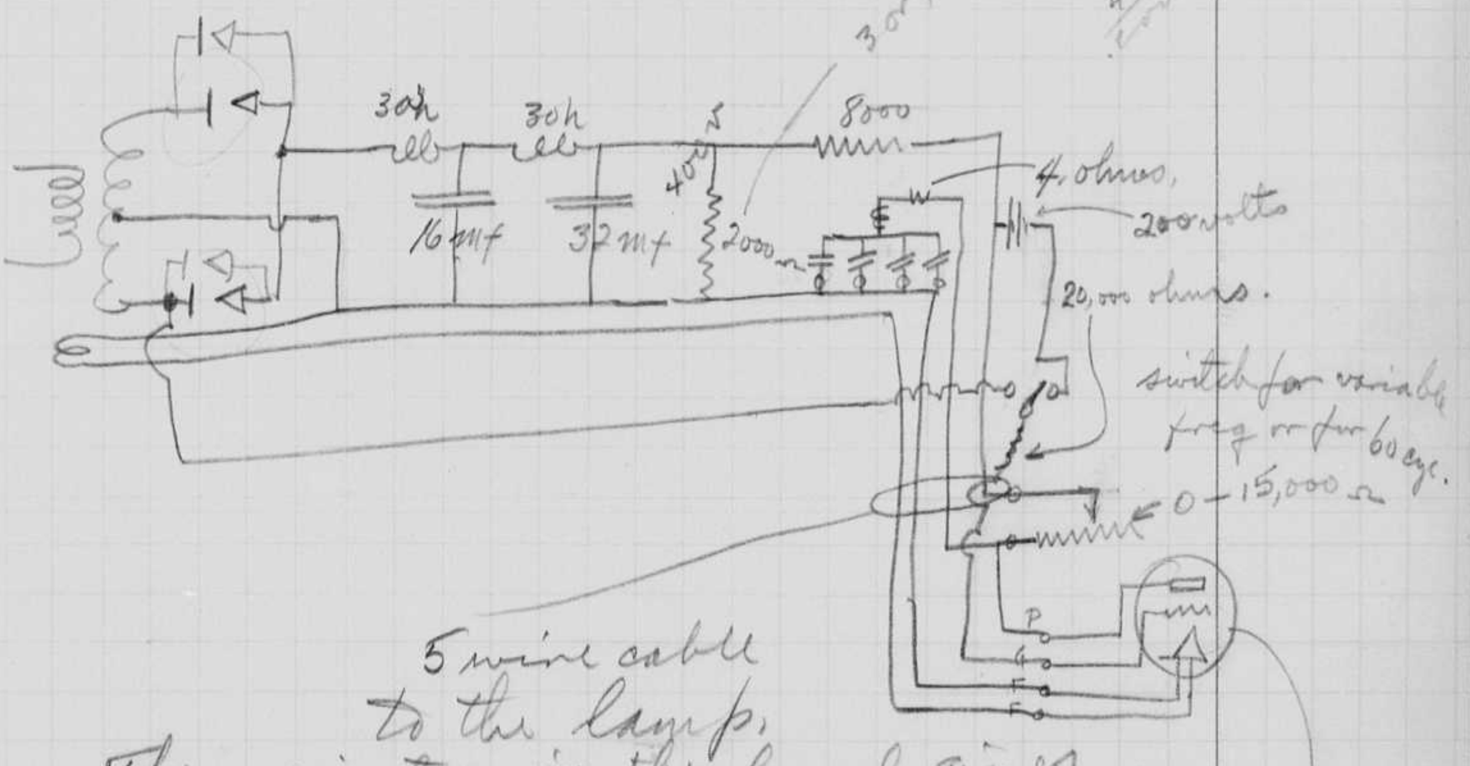
80/400

15000 ohm resistance.

$25 = \frac{50}{2}$ ma. rating

200 ma.

3 or 4000.



5 wire cable to the lamp.

The resistance in the head gives freq control right there where the light is.

$.25 \times 10^{-6} \times 8000 = .002000 \times 10^{-6} = .002$ sec

$\frac{1}{.002} =$ 500 cycles. max freq. $500 \times 60 = 30,000$

100 amp peak.

$3.75 \times 23,000 \times 10^{-6} = .090,000$

$\frac{1}{.09} =$ ~~10~~ 11 cyc. 660

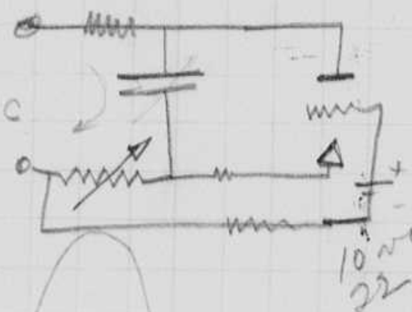
r.p.m.

range

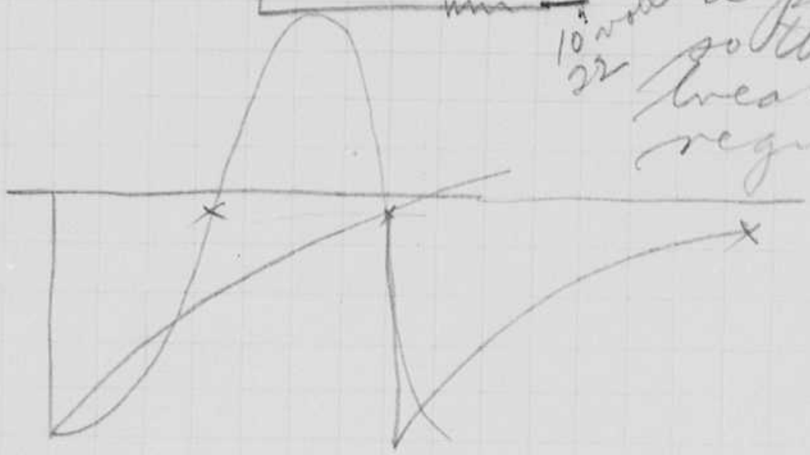
.20 / .40

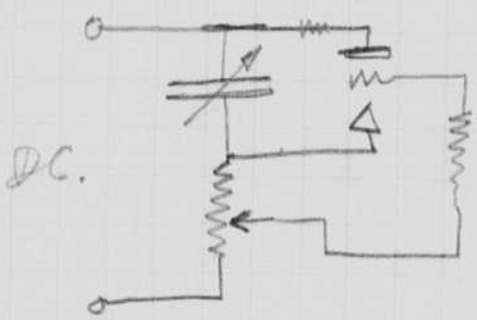
70

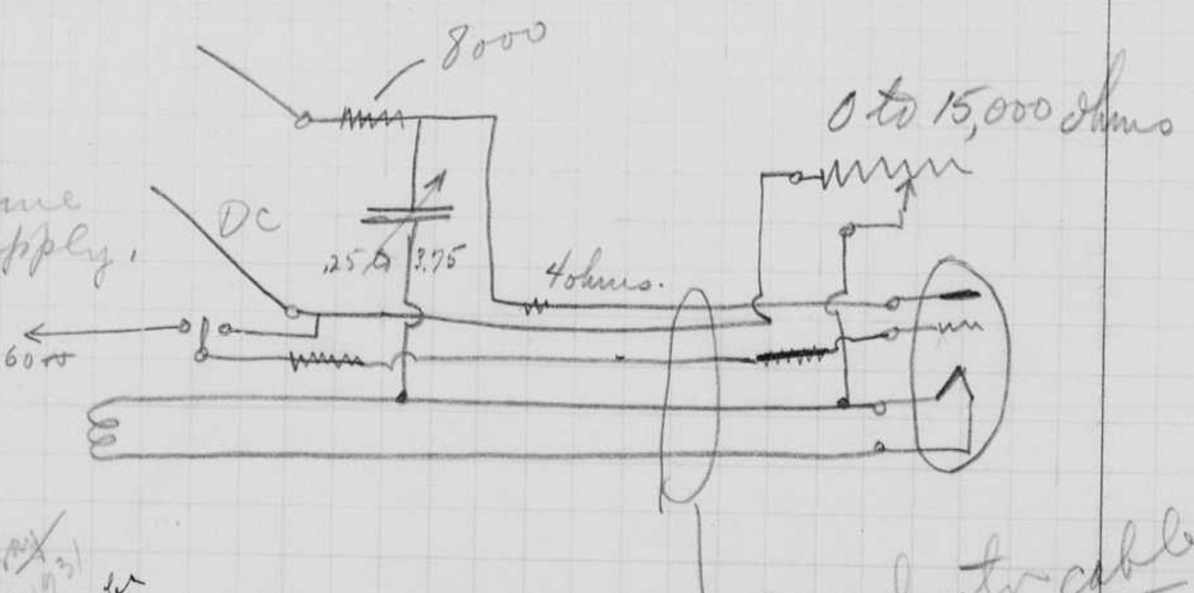
Cont.



This eliminate the ~~c battery~~ large c battery but will require a positive c bias so the tube will break down regularily.

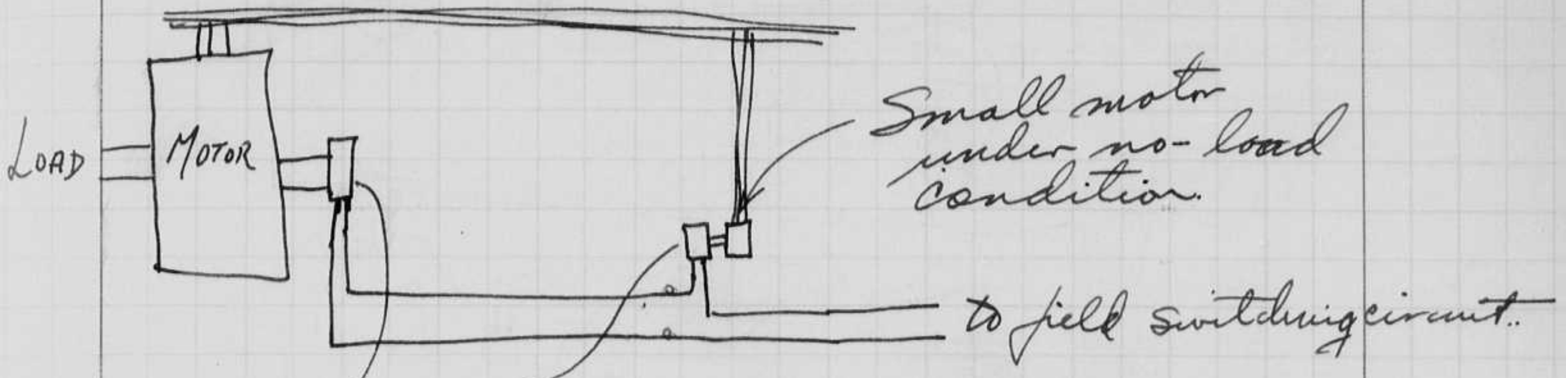




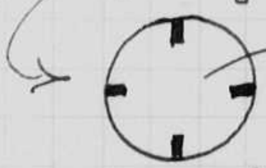


Witnessed
T. C. Gray
May 6, 1931
J. L. Brooker
May 12 31

May 14 1931 Field Switching Scheme for a
H. E. Edgerton Synchronous Motor.



Commutator to have one ~~seg~~ segment for each pair of poles.

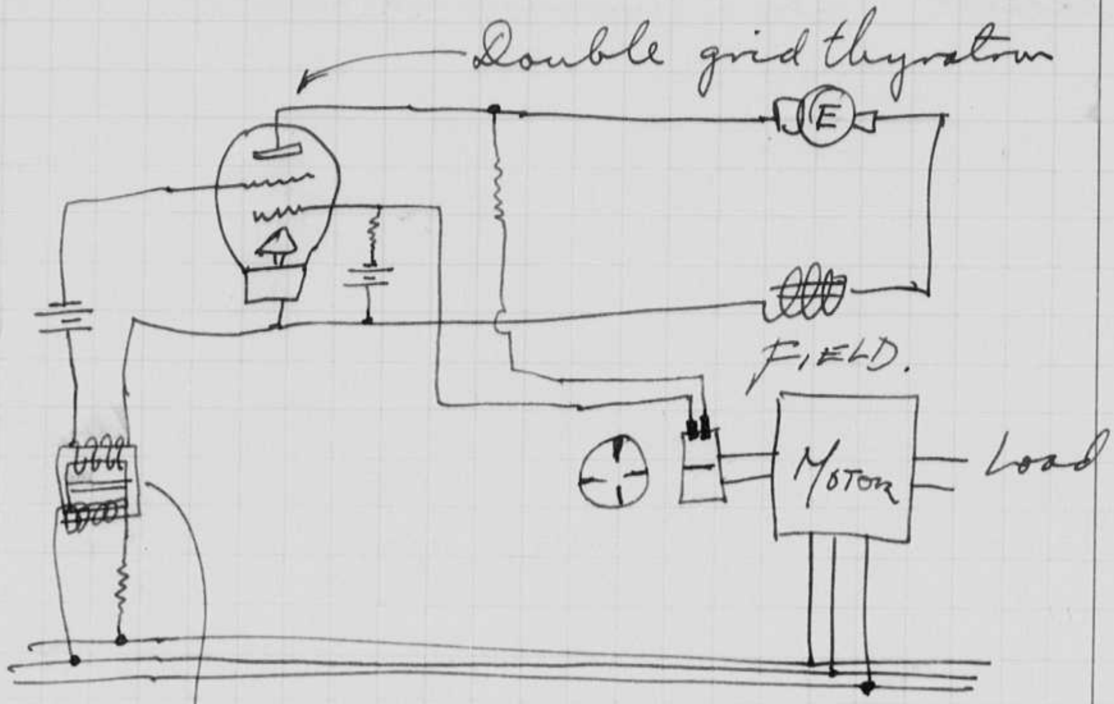


The electrical circuit through the field switching arrangement is only made when the two sets of brushes ~~commutators~~ are ~~on their~~ both on their respective commutators segments at the same time.

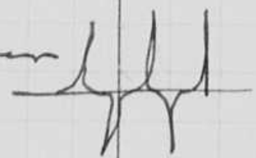
One disadvantage of this scheme is that the electrical contact is not made very long and so the ~~impulse~~ impulse to the trip circuit needs to be amplified by some sort of sensitive relay.

An advantage of this scheme is that the small synchronous motor ~~need only to~~ does not need to be lined up with the big motor and can be placed anywhere on the control board, etc.

Cont. Field Switching Schemes.



Saturated core transformer to give a peaked voltage.



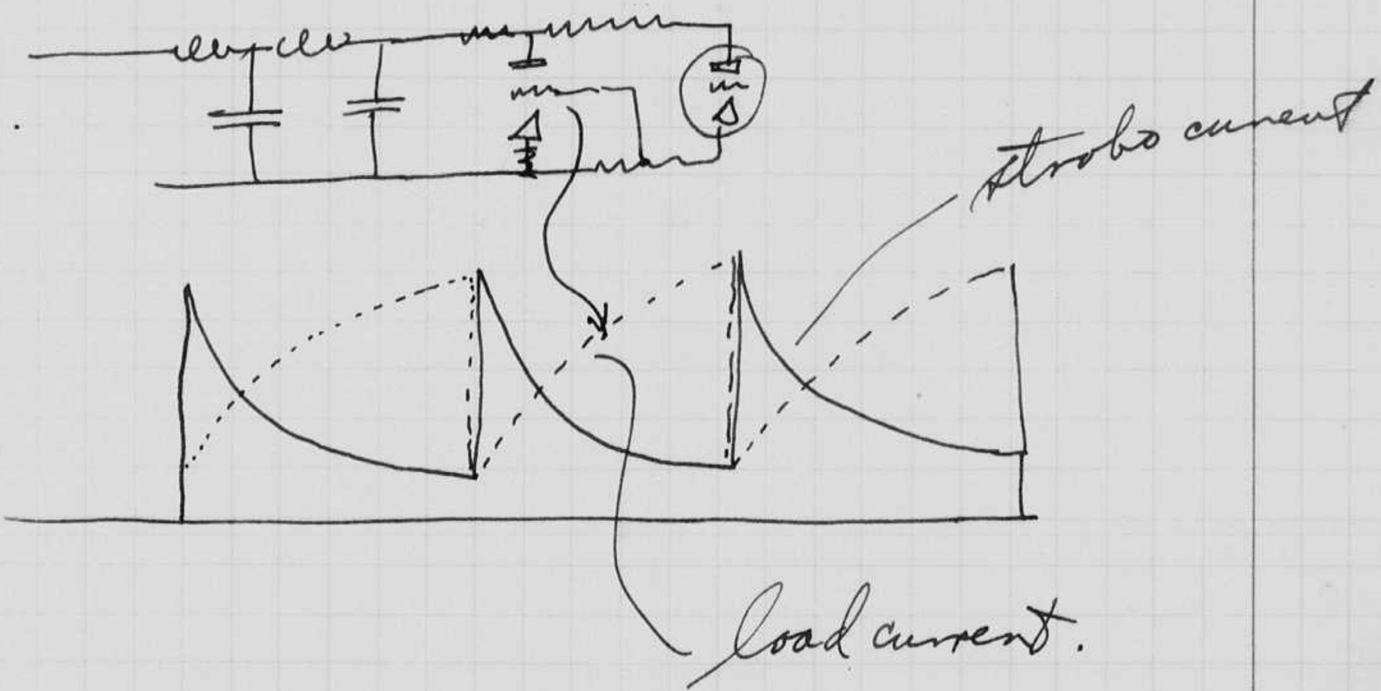
The thyratron will not conduct until the two grids are positive. This only occurs when the angular displacement has a certain phase relationship ~~to the~~ which is adjusted so that it occurs at the most favorable switching angle, that is, zero degrees.

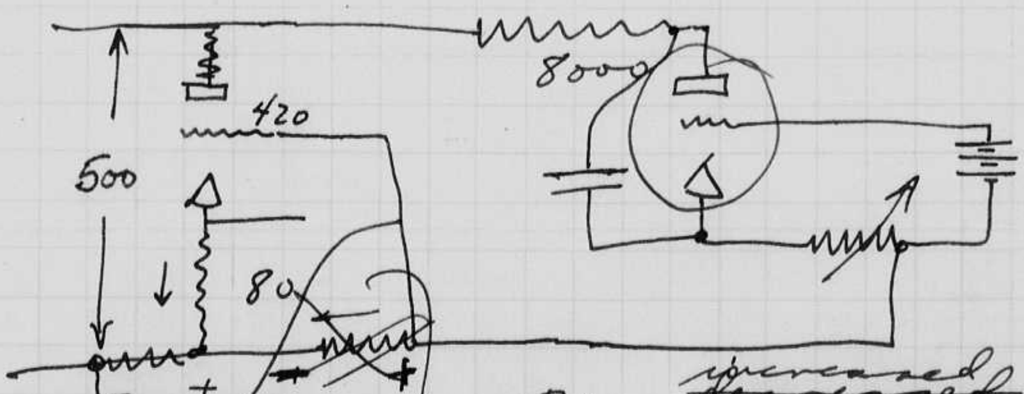
It would be more accurate to say that the thyratron does not conduct until there is a certain relationship between the potentials on the two grids. ~~It~~ the necessary voltages to cause breakdown may be either positive or slightly negative.

May 16 1931
H. E. Edgerton

Constancy of the frequency of the stroboscope shown on pages 88-90 depend upon a constant d-c voltage. One method to get this is to have a large enough "B" eliminator so that the load current does not appreciably change the voltage.

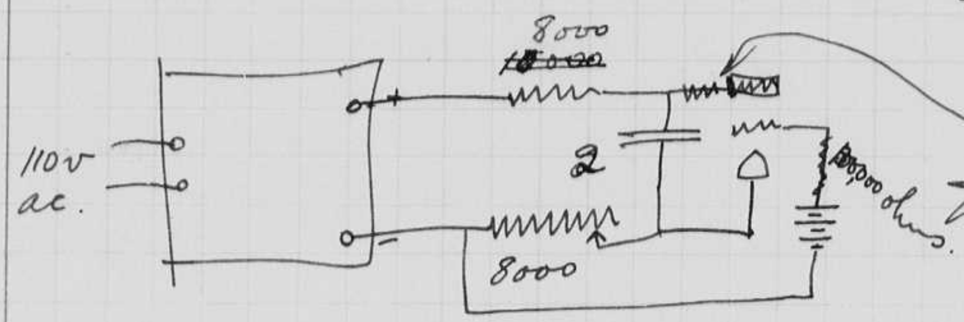
A load just like the stroboscope but arranged so that the sum of the current is constant may help to keep the frequency constant.





Bias ~~increased~~ ~~decreased~~ increased by charging current to the condenser.

Cont. Scheme to be tried.



4×5

$\frac{500}{x} = 25$ $\quad x = \frac{\cancel{500}}{25}$ 20

dgr. = 50 ohms. to limit current.

$RC = \frac{1}{60}$ sec.

$C = 2 \times 10^{-6}$ $\quad R = \frac{10^6}{60 \times 2} = \frac{10^{4}}{1.2 \times 10^{2}}$

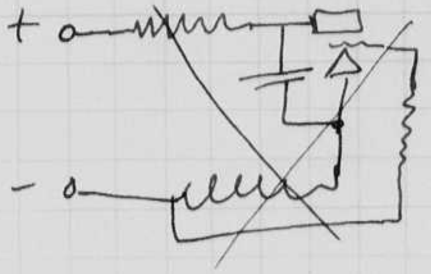
$= 10,000$ ohms.

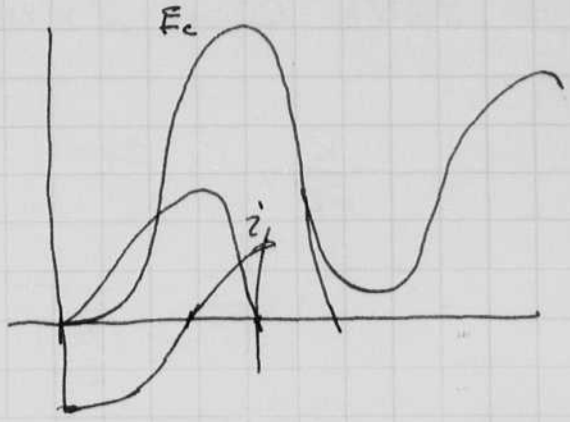
~~Material list for strobo.~~

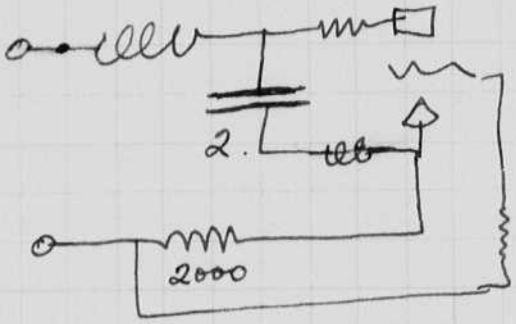
2 mf good for 100 v. d.c. .25 to 2 mf steps.
1 - 8000 ohms
1 - 8000 ohms slide.
1 - 100,000 ohms
1 - "B" battery 22 volts.
1 - 50 ohm slide wire.

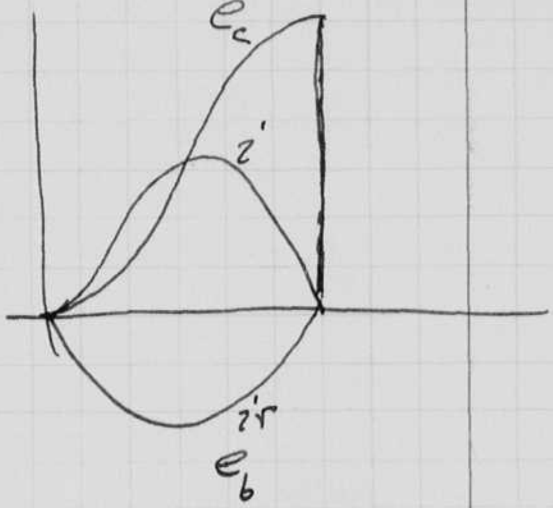
FG-27 and fil. transformer.

Cont.









$$\frac{1}{2\pi\sqrt{LC}} = f$$

$$T = 2\pi\sqrt{LC}$$

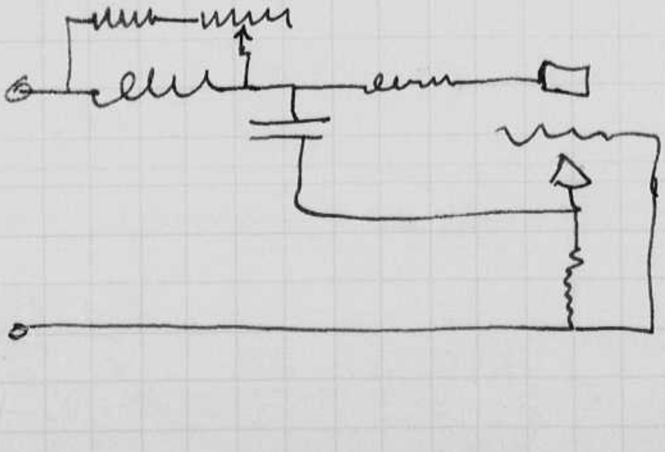
$$\frac{1}{60} = 2\pi\sqrt{2 \times L}$$

$$\frac{1}{3600} = 4\pi^2\, 2 \times L.$$

$$L = \frac{1}{\underset{10}{36{,}00_{\circ}}\, \underset{8}{4\pi^2}\, 2\times 10^{-6}} = \frac{1}{.288\times 10^{?}}$$

$.288{,}000.$

$$= \frac{1}{.288} = .347\,h.$$



$$\alpha = \frac{R}{2L} \qquad \frac{L}{R} = \frac{.347}{2000} =$$

$$\alpha = \frac{2000}{2 \cdot .347} = 3000$$

$L = .347h \quad C = 2\times10^{-6} f \quad R = 2000$

R.

$$\frac{R}{2L} = \frac{2000}{2\times.347} \cong 3000$$

$$\cancel{\frac{R}{\sqrt{\frac{L}{C}}} = .268.}$$

$$\cancel{\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}} = \cancel{\sqrt{1.43\times10^6 - 10\times10^6}}$$

$$\frac{1}{LC} = \frac{1}{.347}\frac{1}{2\times10^{-6}} = \frac{10^6}{.7} = 1.43\times10^6$$

$$\begin{array}{r} 3000 \\ 3000 \\ \hline 10,000,000 \end{array}$$

3.000
10×10^6

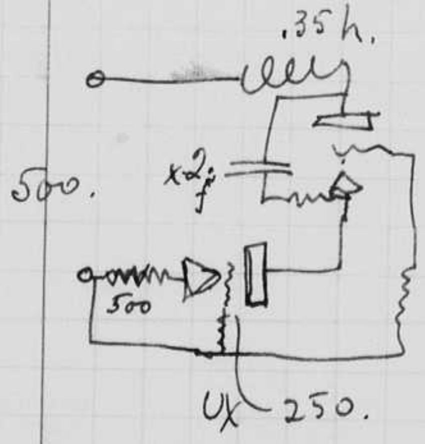
the circuit is ~~over~~ damped.

R must be less than

$$\left(\frac{R}{2L}\right)^2 < 1.43\times10^6$$

$$\frac{R}{2L} < 1.2\times10^3 = 1200$$

$R = 2400 .35 =$ 840 ohms.



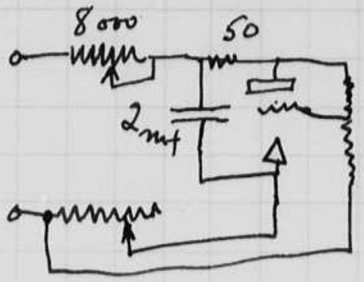
too complicated!?

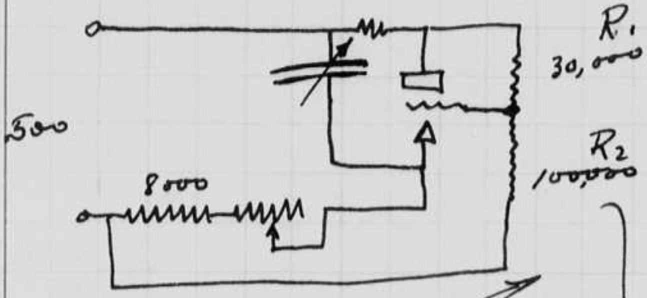
$10\,mf \times 20 = 200\,m.s.$

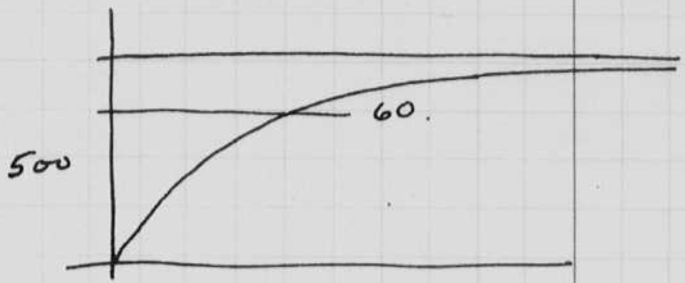
$$\frac{0.800}{\cancel{10^3}\sqrt{\frac{.35}{2.}}} = 2.$$

$\sqrt{.17} = .41$

May 19 1931
H E Edgerton.

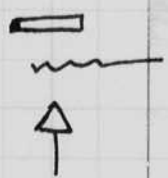






make $\frac{R_2}{R_1}$ = 3. approx.

This makes the grid go positive ~~before the con~~ as soon as the condenser has built up to 60 % of ~~its final va~~ the final value it would have if the tube did not strike.



$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = e = E1.$$

Let $t = a\lambda$.

$$\frac{a^2 d^2q}{a^2 d\lambda^2} + \frac{a^2 R}{aL}\frac{dq}{d\lambda} + \frac{a^2}{LC}q = a^2\frac{E}{L}1.$$

$$\left(p^2 + a\frac{R}{L}p + \frac{a^2}{LC}\right)q = a^2\frac{E}{L}1. \qquad p = \frac{d}{d\lambda}.$$

Let $\frac{a^2}{LC} = 1 \qquad a = \sqrt{LC}$

$$\left(p^2 + \frac{R\sqrt{LC}}{L} + \frac{(\sqrt{LC})^2}{LC}\right)q = \cancel{L}C\frac{E}{\cancel{L}}1.$$

$$\left(p^2 + \frac{R}{\sqrt{\frac{L}{C}}}p + 1\right)q = EC1.$$

$$\left(p^2 + \frac{R}{\sqrt{\frac{L}{C}}} p + 1\right) = 0$$ ~~for~~ roots.

$$p = -\frac{R}{2\sqrt{\frac{L}{C}}} \pm j\sqrt{1 - \left(\frac{R}{2\sqrt{\frac{L}{C}}}\right)^2}$$

critically damped case

$$\frac{R}{2\sqrt{\frac{L}{C}}} = 1. \quad R = 2\sqrt{\frac{L}{C}}. \quad \sqrt{.17} = .41$$

$$= 2\sqrt{\frac{.35}{2\times10^{-6}}}$$

$$= 2\times10^3\times\sqrt{\frac{.35}{2}} = .8\times10^3 = 800 \text{ ohms}$$

~~In a similar manner the current instead of the charge.~~

$$\left(\frac{dL}{dt} + R + \frac{1}{C}\int dt\right) i = E1.$$

Let $t = a\lambda$.

$$\left(\frac{dL}{a\,d\lambda} + R + \frac{a}{C}\int_0 d\lambda\right) i = E1.$$

$$\left(\frac{d}{d\lambda} + \frac{Ra}{L} + \frac{a^2}{LC}\int d\lambda\right) i = a\frac{E}{L}1.$$

$$a^2 = LC. \quad p = \frac{d}{d\lambda}.$$

$$\left(p + \frac{R}{\sqrt{\frac{L}{C}}} + \frac{1}{p}\right) i = \frac{E}{\sqrt{\frac{L}{C}}}1.$$

$$\left(p^2 + p\frac{R}{\sqrt{\frac{L}{C}}} + 1\right) i = p\frac{E}{\sqrt{\frac{L}{C}}}1.$$

$$i = \frac{p\,E/\sqrt{\frac{L}{C}}\,1}{\left(p^2 + p\,R/\sqrt{\frac{L}{C}} + 1\right)}$$

$$\frac{p}{(p^2+2\alpha p+\omega_0^2)}\,1 = \frac{\varepsilon^{-\alpha t}}{\omega}\sin\omega t\,1$$

$$\omega_0^2 > \alpha^2 = \omega_0^2-\alpha^2$$

$$\tan\phi = \omega/\alpha$$

$$\cancel{\alpha=\frac{R}{2L}}$$

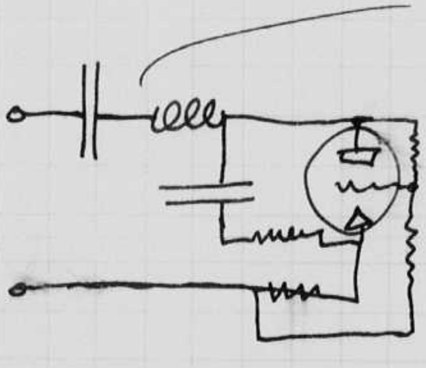
$$\alpha = \frac{R}{2\sqrt{\frac{L}{C}}}$$

$$i = \frac{E}{\sqrt{\frac{L}{C}}}\;\underline{\varepsilon^{-\alpha t}}\sin(\lambda)\;1$$

$i_{max} = 2E\sqrt{\frac{C}{L}}$ for $\alpha = 0$ no damping.

$L = .35 \quad C = 2\times10^{-6} \quad E = 500$

$$i_{max} = 2\times500\sqrt{\frac{2\times10^{-6}}{.35}} = 1000\times10^{-3}\overset{2.4}{\sqrt{5.7}} = 2.4\text{ amps.}$$



~~to~~ prevent shunting the tube if the grid fails.

$$\frac{Li^2}{2} = \frac{1225}{2} = 112 \text{ joules.}$$

$$\frac{CE^2}{2} = 112$$

$$E = \frac{225}{10\times10^{-6}} = 20.0\times10^6 \text{ volts}^2$$

10,000 cycles per sec.

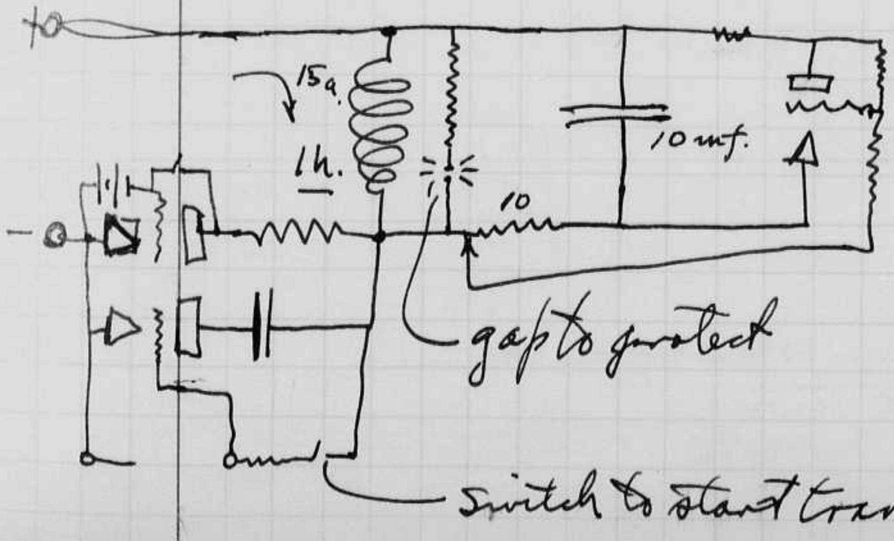
$\frac{1}{10000}$ sec per cycle.

1×10^{-4} sec. = time const

$1\times10^{-4} = 10\times10^{-6}R.$

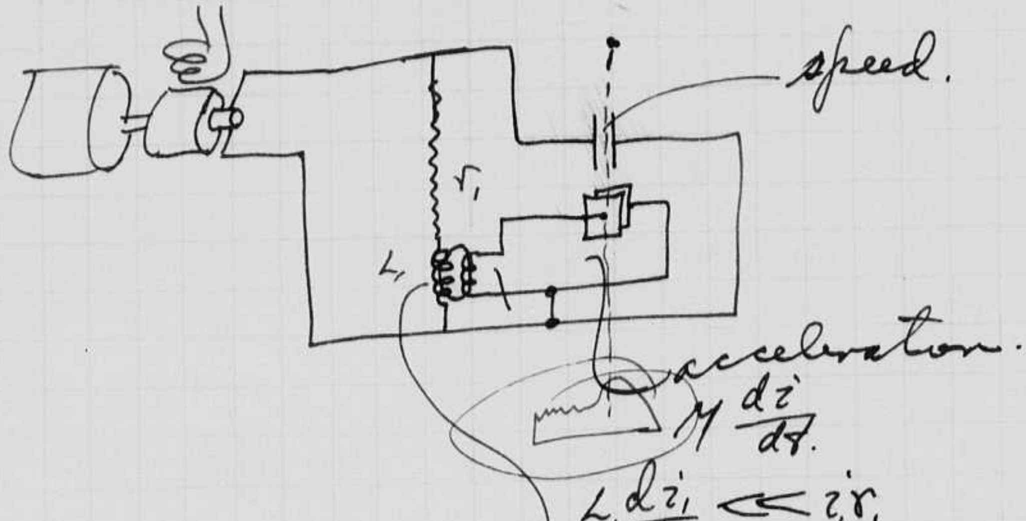
$$R = \frac{1\times10^{-4}}{10\times10^{-6}} = \frac{10^2}{10} = 10 \text{ ohms}$$

if tube has not blocking R.



May 17 1931
H. E. Edgerton.

Speed - torque (acceleration)
measurement by means of the cathode ray oscillograph.



$L_1\frac{di_1}{dt} \ll i_1 r_1$

E ripple must be less than .5%
$M\frac{di}{dt}$ must give voltage enough to deflect beam.

say $\frac{.5 \text{ amp.}}{.1 \text{ sec}} = 5.\text{ am./sec} = \frac{di}{dt}$.

$100 \text{ volts} = M\frac{di}{dt}$

$M = \frac{100}{5} = 20.\text{ h.}$ $\quad \frac{L_1}{L_2} : \left(\frac{N_1}{N_2}\right)^2$

$\frac{100}{.5} = 200 \text{ ohms.}$

$L\frac{di}{dt} \ll 100$ say 1 volt.

$L = \frac{1}{5} = 0.2\text{ h.}$ $\qquad M = \sqrt{L_1 L_2}$

$L_2 = \frac{M^2}{L_1} = \frac{20 \cdot 20}{0.2} = \underline{4000}\text{ h}$

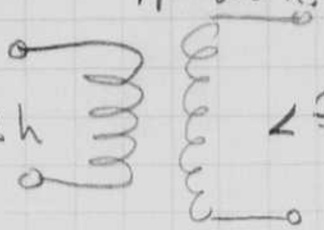
This transformer needs a very large ratio of turns.

$\left(\frac{N_2}{N_1}\right)^2 = \frac{4000}{0.2} = 20000$

$\left(\frac{N_2}{N_1}\right) = 140. \text{ to } 1 \text{ ratio.}$

50 turns.
7500 turns.

$M = 20\,h.$



Ratio of trans

150 to 1.

$L\frac{di}{dt} = j\omega L i$ for ac:
at 60 cycles. $377L \ll 200$ ohms.
say 2 ohms.

$$L = \frac{2}{377} = 0.053\ h.$$

if M needs to be from 15 to 100 h.

$\left(\frac{N_2}{N_1}\right)^{2} = \sqrt{\frac{L_2}{L_1}}$ $\quad M = \sqrt{L_1 L_2}$ unity coupling.

$$L_1 = \frac{M^2}{L_2}$$

$$\frac{N_2}{N_1} = \sqrt{\frac{L_2 L_2}{M^2}} = \sqrt{\frac{L_2^2}{M^2}} = \frac{L_2}{M} \qquad L_2 = \frac{M^2}{L_1}$$

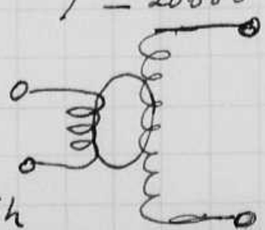
$$\frac{N_2}{N_1} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{M^2}{L_1^2}} = \frac{M}{L_1}$$

Let $M = 15 \quad L = 0.053.$

$$\frac{N_2}{N_1} = \frac{15 \times 372}{2} = 2800 \text{ to } 1.$$

$M = 100 \quad L = 0.053.$

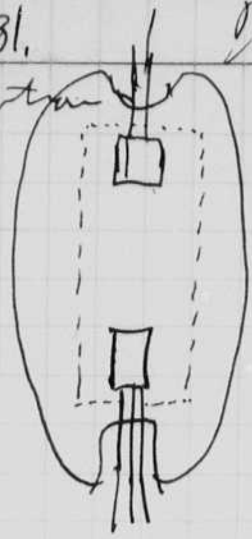
$$\frac{N_2}{N_1} = 100 \times \frac{372}{2} = 18{,}600 \text{ to } 1.$$

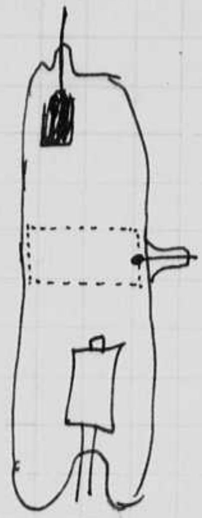


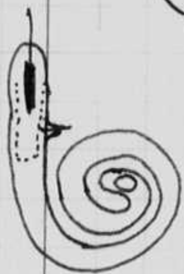
(0.5 amp dc.) no non-linear effects.

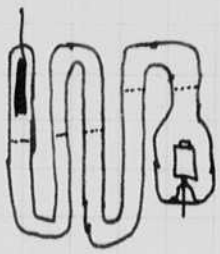
May 17 1931. Thyratron for light

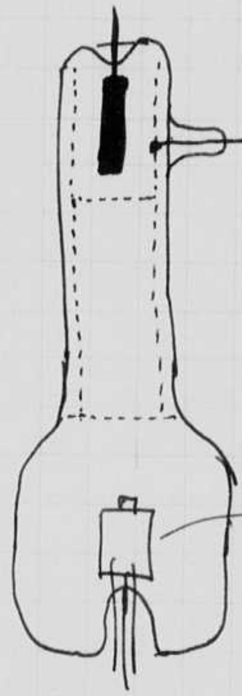
H. E. Edgerton



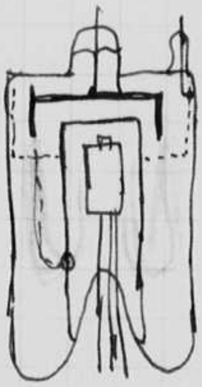


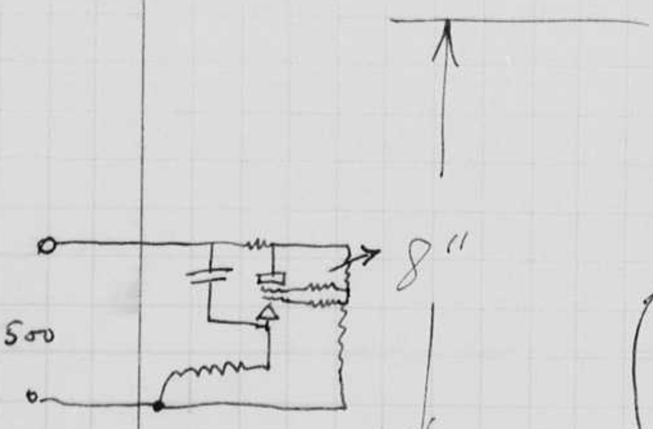






150 amp cathode (peak).





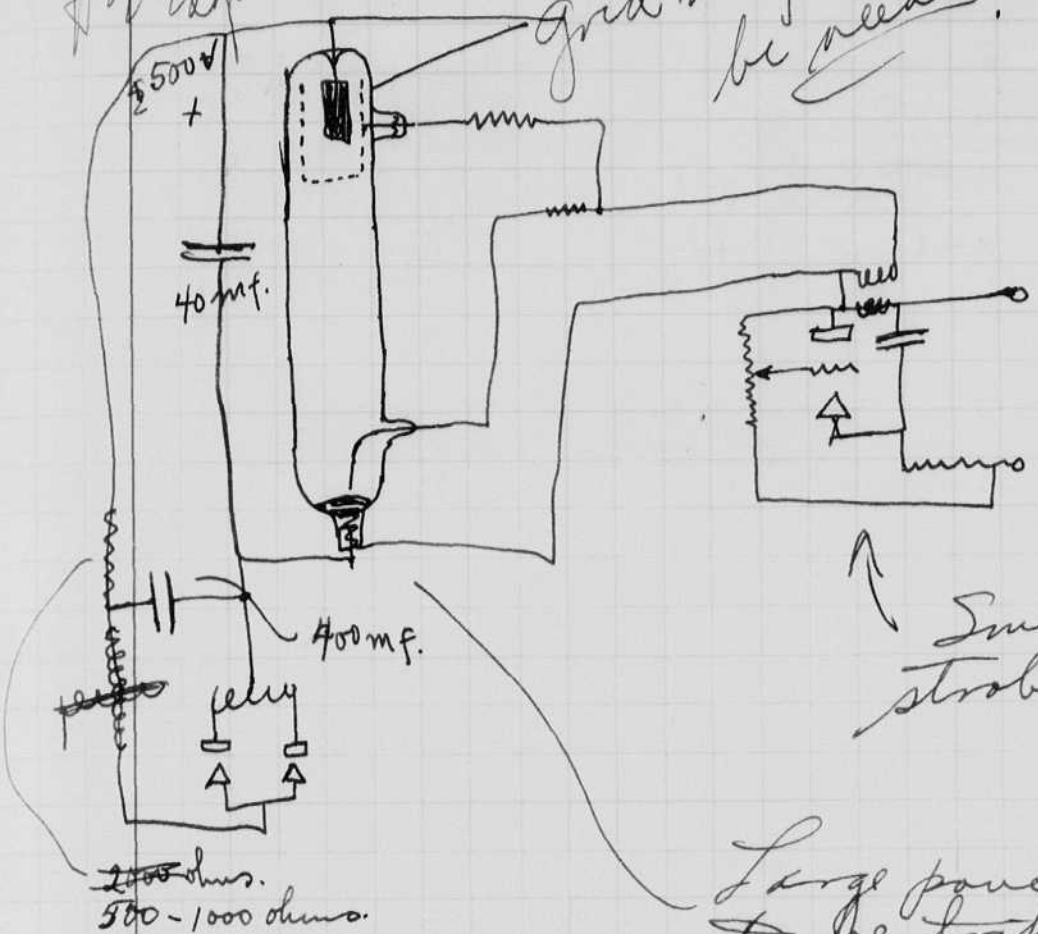
grid for de-ionizing and cutting off the arc.

grid for starting

150 amp peak cathode

May 17 1937
H.E. Edgerton

Grid may not be needed.



Small visual stroboscope.

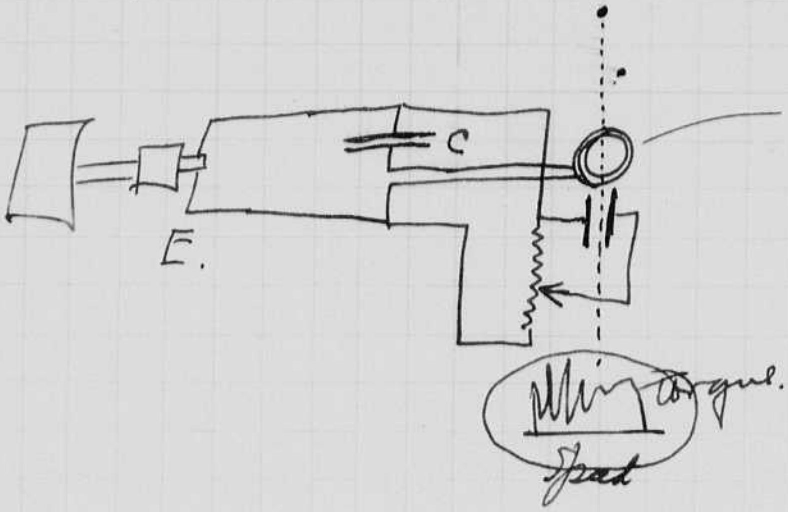
Large powerful strobo to be tripped by the small one.

No holding spot needed. The discharge of the small strobo is put through a spark coil which forms a spot on the big tube's pool.

Wrap externally with wire and connect it to the starting anode.

May 18, 1931. Speed-torque Cathode ray tests.

J. G. Edgerton.

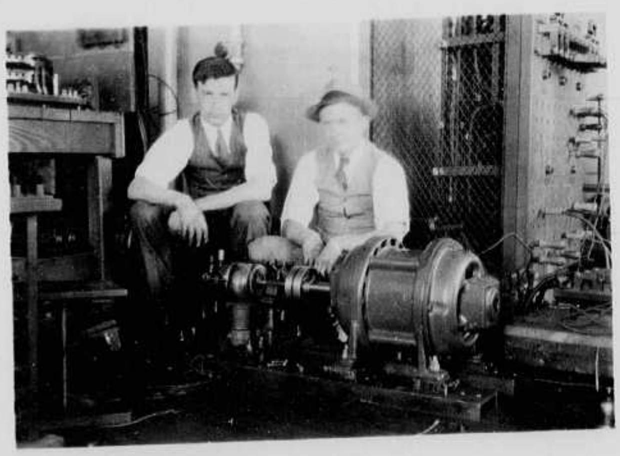


$L\frac{di}{dt} \ll C\frac{de}{dt}$

$Ri \ll C\frac{de}{dt}$ E.

$L\frac{di}{dt} \ll$ E.

Jim Byrne



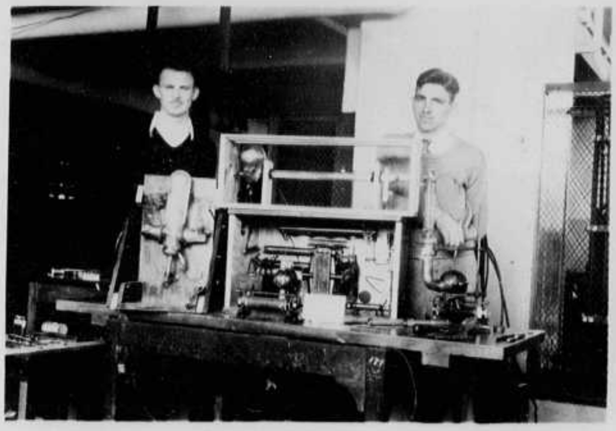
Dick Mason.

with pulsationless-free generator.

May 19 1931

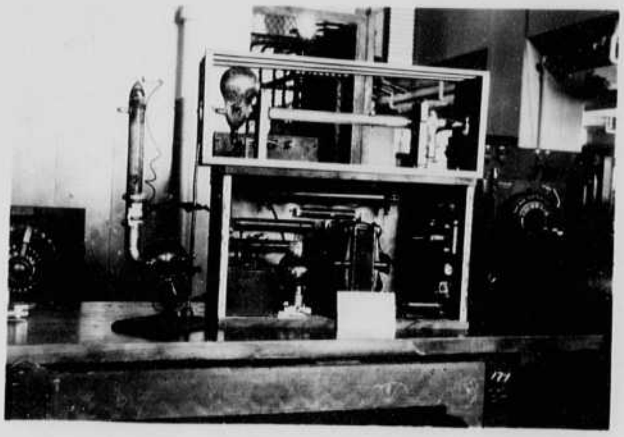
Photographs taken
May 19, 1931.

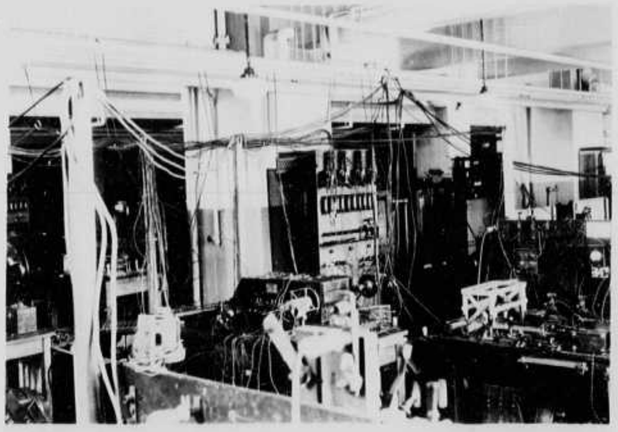
J. B. McClure



Lawrence
M. Stauder

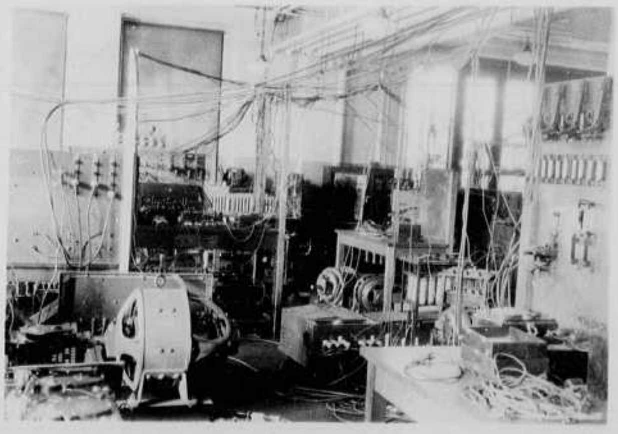
Stroboscope equipment





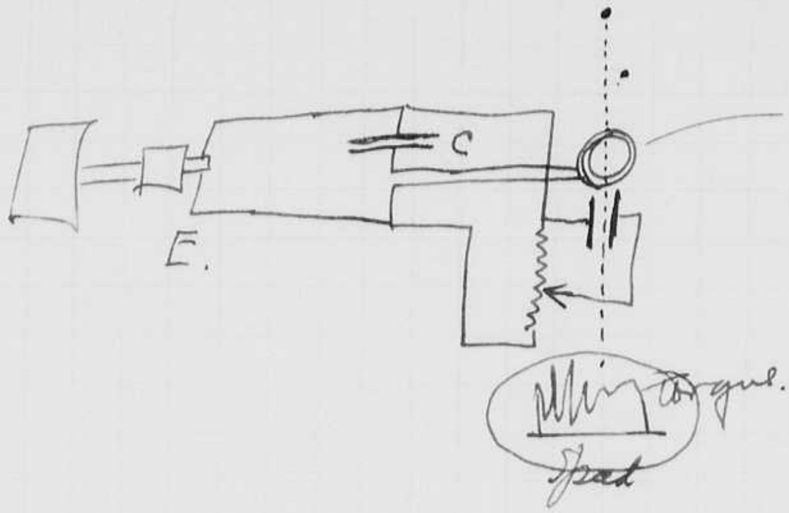
Machine transients
Laboratory.

Spring 1931.



May 18, 1931. Speed-torque Cathode ray tests.

H. E. Edgerton

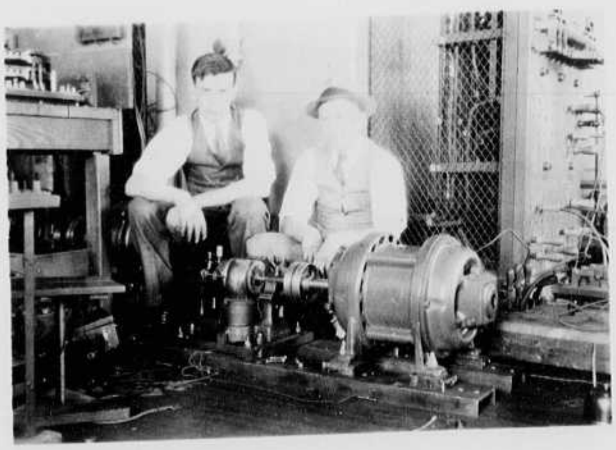


$L\frac{di}{dt} \ll C\frac{de}{dt}$

$Ri \ll C\frac{de}{dt}\ E.$

$L\frac{di}{dt} \ll E.$

Jim Byrne



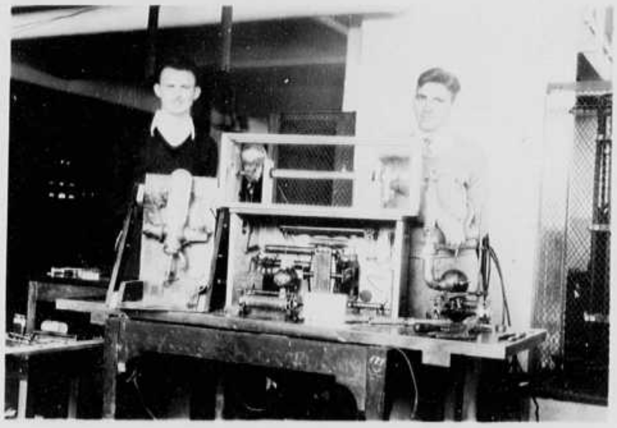
Dick Mason.

with pulsationless-free generator.

May 19 1931

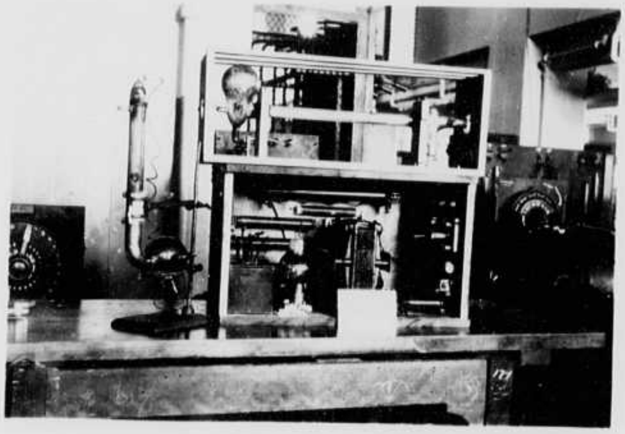
Photographs taken
May 19, 1931.

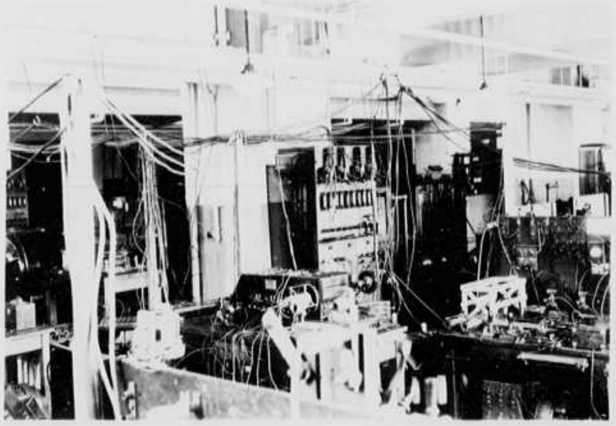
J. B. McClure



Lawrence
M. Stauder

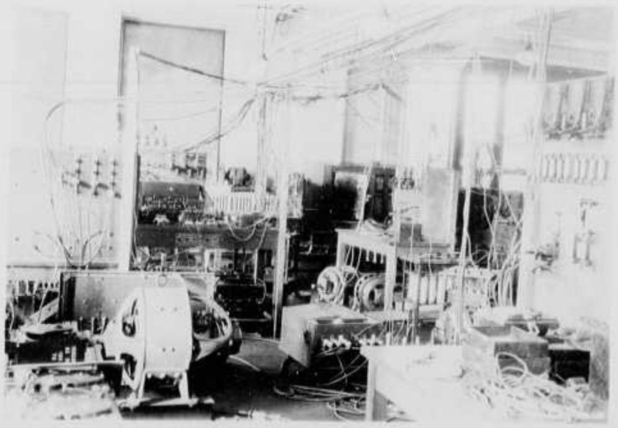
Stroboscope equipment





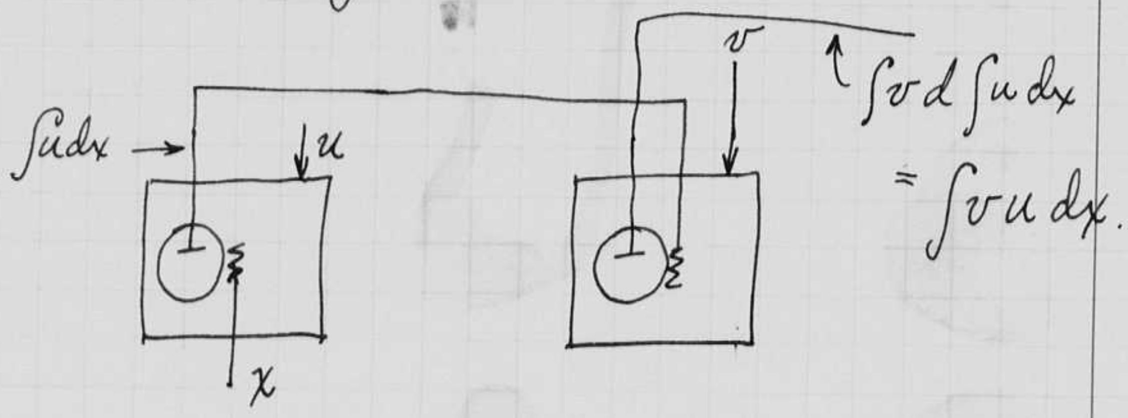
Machine transients
Laboratory.

Spring 1931.

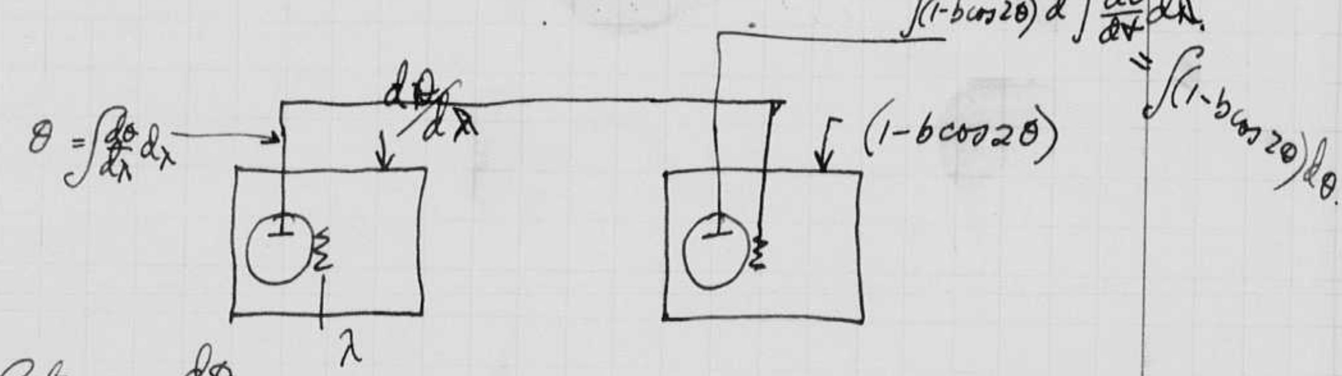


Method of Getting $\int uv\,dx$ on the Integraph.

$$\int uv\,dx.$$



In the ~~syn~~chronous machine problem the term $\int (1-b\cos 2\theta)\frac{d\theta}{d\lambda}\,d\lambda$ can be solved in this manner.



Let $u = \frac{d\theta}{d\lambda}$.

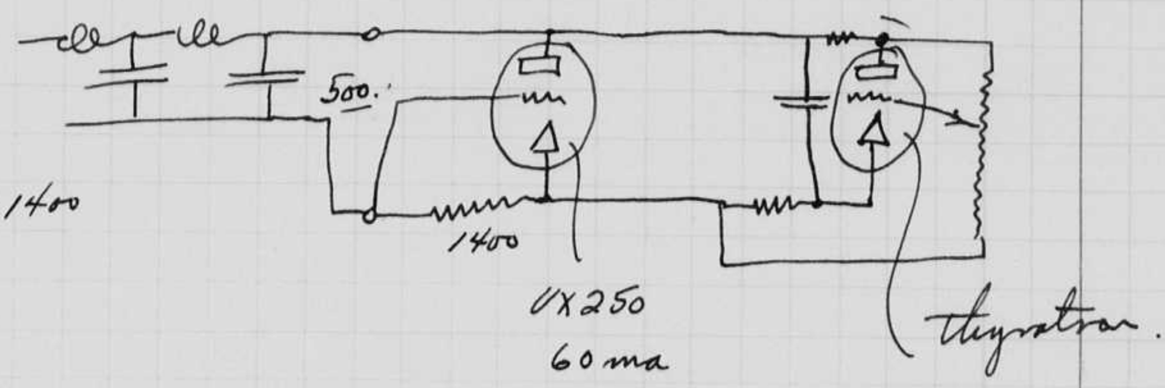
$v = (1-b\cos 2\theta)$.

May 22/1931
J. E. Edgerton.

Cont of Strobo of page 93.

V.T. adjusted so that it will draw a current such that the current from the filter is constant.

$\frac{80}{.055\,ma.} = 1400$



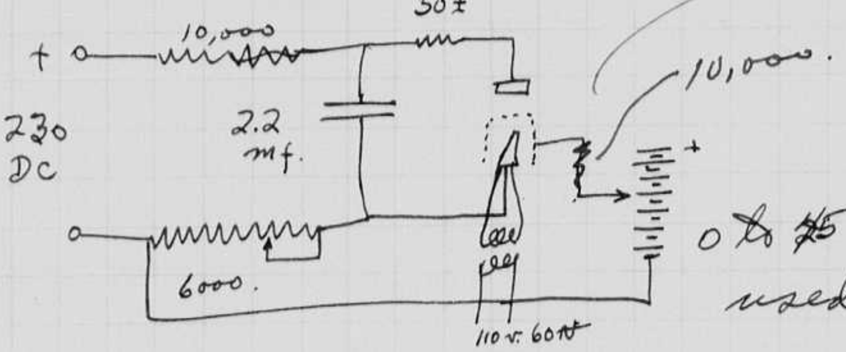
Design Resistances etc ~~..~~ so that the total current is a constant. The charging current increases the bias and reduces the current drawn by the 250. tube.

May 26, 1931.

Stroboscope.

Circuit described on page 94 connected up and tried.

F.G 27.



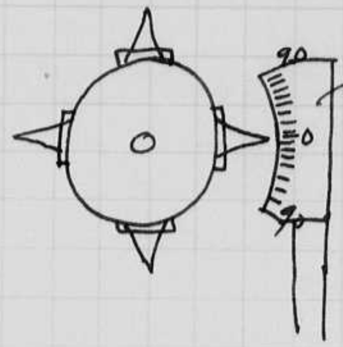
0 to ~~45~~ volts bias used.

There is a periodic variation of the flashes which is apparently due to the interference of the voltage on the filament. The 60 cycles beat with the frequency of the flash circuit.

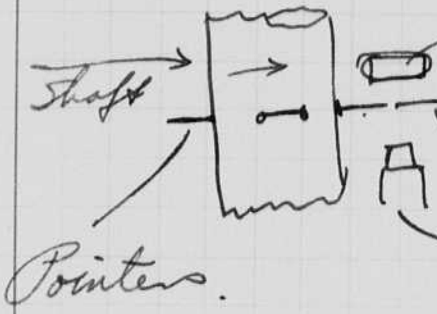
An F.G. 33 did not work on this circuit for some reason. It would flash when the grid was connected to the anode but would not operate with the grid + ~~23~~ 45 volts!

Stroboscopic Movies.

May 28, 1931
H E Edgerton



Scale with slots for the light to shine through.



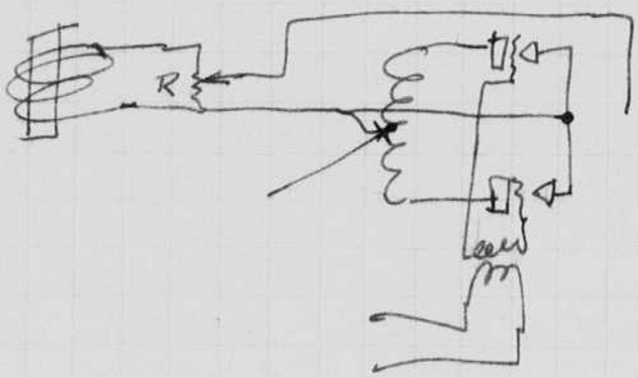
light. 60 cyc sec flashes.

Scale

camera Example 16 mm film driven at const speed by a small synchronous motor so that there would be 60 frames per second. A relay system would start the camera and the stroboscope simultaneously.

May 29/1931. Field-Switching for Ph. Generator. Synchronous Motors

A thyratron supply on the field of a synchronous motor can be made to supply intermittant field current so that the field is on from 0 to 180 degrees and is off from 180 to 360 (generator angles).



X M.G.

May 30 1931
H. E. Edgerton

Phase angle control of thyratrons.

Ref. W. B. Nottingham. March 1931 p. 271.
"Characteristics of small grid-controlled hot-cathode mercury arcs or thyratrons

He shows a 24 degree (60 cycle) phase shift in curves for an F.G. 27 and a FG 17. the circuit that he uses depends upon a condenser ~~and~~ a resistance for the phase shifting. The angle is calculated since R and C are known. this shift in phase corresponds to 1×10^{-3} seconds or 1000 micro sec. which is a very long time.

I am sure that this question needs further investigation. It may be possible that the current in the grid circuit makes his calculation of phase angle invalid especially since the R and C must be in a steady state in order to compute the angle by the impedance method.

if $\alpha = 45°$ $R = 1 \times 10^{6}$ ohms.

$X_c = 1/\omega C = 1 \times 10^{6}$ ohms.

$C = \dfrac{1}{\omega X_c}$

time const of phase shifting circuit $= T = RC = R\dfrac{1}{\omega X_c} = \dfrac{1 \times 10^{+6}}{377 \; 1 \times 10^{6}} = \dfrac{1}{377}$ sec.

$= .00265$

Another factor which may make his angle calculations invalid are currents between the anode and grid before conduction. I have noticed such a glow between anode and grid in the long arm glass thyratrons before the main arc strikes.

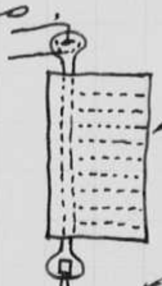
June 16, 1931. ~~Photograph~~

H.E. Edgerton

Movie Projection by means of intermittant light.

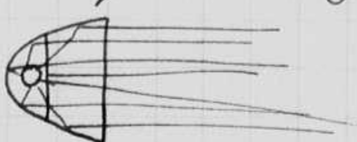
Practically all present projection schemes use a concentrated light source which approaches a point source.

Such a point source appears at the present to be impossible with the mercury arc lamp. It may be possible however to use a line source of light to advantage. I believe that an intense capilliarly can be built which will be small in diameter but very intense. A parabolic mirror and parallel light shields may make it feasible to get a parallel beam of light which may be directed and focused by means of cylindrical lenses.



light baffles to make the light parallel.

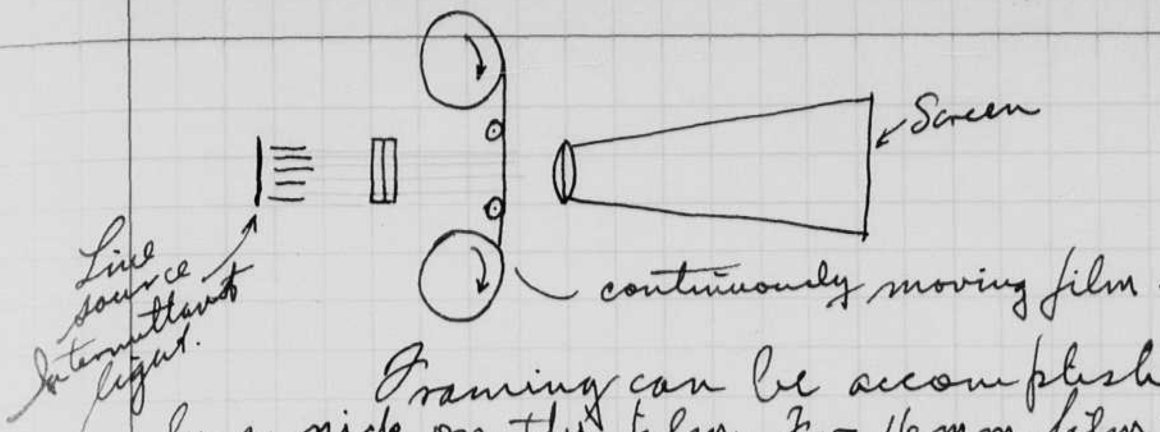
Parabolic reflector to make light directive.



Plane light source.

light grid to make beams parallel.

During the last week I worked at the Sprague Specialties company at Quincy on the motor driving the Visivox. These intermittant light schemes were discussed at length with Bill Dunn. constant speed film seems to have advantag



Framing can be accomplished by a nick on the film. For 16 mm film the sprocket holes will accomplish this purpose.

Notebook # 3

Filming and Separation Record

____ unmounted photograph(s)

____ negative strip(s)

1 unmounted page(s)
(notes, drawings, letters, etc.)

was/were filmed where originally located between page ____ and ____.
in side back cover

Item(s) now housed in accompanying folder.

Nov 15

Oscillation data for determination of WR^2

I_f 804^A	V_ϕ	$E_a\phi$	X_s	time in secs	Cycles of oscill.	WR^2	
4.0	$120/\sqrt{3}$	68.5	3.26	34 23	15 10	323	from 187 KVA machine. iron core reactors
8.8	$126.5/\sqrt{3}$	140	"				
8.7	$124/\sqrt{3}$	139	"	32 ? 29.75	20		
8.7	$123/\sqrt{3}$	139	"	30.0		294	
5.92	$120/\sqrt{3}$	101	"	36.5	20	309	
4.15	$118/\sqrt{3}$	63.5	"	16 ? 17.5	15		
"	"	"	"	18.25	15	278	

Iron core removed from reactors.

4.15	$115/\sqrt{3}$	73	1.41	19.5	15	252	from 187 KVA machine
"	"	73	1.41	20	15	266	
8.3	$231/\sqrt{3}$	134	1.41	20.25	30	250	

machine operating from bus.

2.9	$226/\sqrt{3}$	50	1.09	13.5	15	210

H. E. E.