FJÖRTOFT, RAGNAR, 1949-1962.

ok

May 28, 1962

Professor Ragnar Fjörtoft
Det Norske Meteorologiske Institutt
Blindern, Norway

Dear Ragnar:

Please excuse my long delay in responding to your letters of January 29 and April 27 concerning the March meeting in Oslo. I think you have done an excellent job in choosing the topics to be covered, and I fully agree with you that there will be little conflict between the Oslo and Boulder meetings.

Would it not be worthwhile at the Oslo meeting to discuss the possibility of standard test data for numerical prediction models? Our IAMAP committee was for a while going to consider this, but we then heard that a subcommittee of the WMO Aerology Commission was going to work on this. Since this subject was of most interest to the national NWP centers, we were glad to give up our interest in it. It seems to me that the Oslo meeting would furnish a good opportunity to discuss the details of such a proposal <u>if the attendees come prepared to do so</u>, and if the members of this WMO subcommittee can present some preliminary plan or recommendations on this point.

With warmest personal wishes,

MI. Skjema 254 III. 1957. 3 000.

DET NORSKE
METEOROLOGISKE INSTITUTT
TELEGRAMADRESSE: METEOROLOGEN, OSLO
POSTGIRO NR. 5260

BLINDERN, 10-1-60.

MET.INST. AKT NR.
JOURNAL NR.

Ved svar bes oppgitt så vel dette brevs journalnummer som de påførte initialer.

Dear Jule:

Thank you for your letter. I am of course very thankfull for your gentle offer to find means of covering my travel expences. However, that I had arranged for quite alright here. The trouble is another one. It turned out that I had been too optimistic in judging the possibilities of getting away from the Institute for a period of a month around the time of the meeting. At no time I had really wanted to go just for the meeting. It was my plan to use the stay in U.S. to study the lastest devolpements in N.W. Prognosis for the benefit of our work here. This I still intend to do at some time after ~~when we have~~ the finishing of our work in preparing the budget, which is expected to ~~be~~ happen in the middle of February. This work is quite important this year because the budget shall cover a period around the installment of the new computer which is expected to influence the institute's organization to considerable extent. Jack Nadó, who is a good help for me in this work has been sick for 2 months. So you may see some of my difficulties in getting away just now.

I am quite sure we will have an opportunity to meet this year, and I am very much looking forward to see you again and discuss matters of common interest.

Yours sincerely

Ragnar.

AMS file

January 4, 1960

Dr. Ragnar Fjortoft, Director
Meteorological Institute
Blindern, Norway

Dear Ragnar:

I was greatly disappointed to learn that you are not able to present your paper at the 40th Anniversary Meeting of the AMS. We had very much looked forward to seeing and hearing you.

Please let me know if there is anything I can do to make it possible for you to come. I am confident that means can be found, even at short notice, to defer your travel costs if this is what stands in your way.

The meeting promises to be a good one, although it will suffer by your absence. You would see many of your friends, and we would have an opportunity to discuss the possibility of your future visit to MIT for an extended time which, as you know, I very greatly desire.

I hope that either this letter or other circumstances will permit you to change your plans.

Yours sincerely,

Jule Charney

jc:tg

January 4, 1960

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Meteorological Institute
Blindern, Norway

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I hope that either this letter or other circumstances will permit you to change your plans.

Yours sincerely,

Jule Charney

jc:tg

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

April 20, 1955

To Whom It May Concern:

This is to certify that Dr. Ragnar Fjørtoft was employed by the United States Office of Naval Research under Contract #N6 ori 139 T.O. 1 from September 1953 through June 1954. During this period he earned $8,888.88 serving as a meteorologist under this contract and on his departure from this country he paid the necessary United States Federal Income Tax on his salary.

COPY

Minot C. Morgan, Jr.
Comptroller

MCM:jp

cc Jule Charney

Ordrup april 14 th.-55.

Dear Jule:

I should of course have written to you before. I hope, however that you understand my special attitude towards letter writing. But anyway, let me thank you for my staying in Princeton, which was certainly a pleasure both for me and the family. I am sending you a manuscript which is going to be printed in "Tellus", hoping that you will find some interest in it.

There is still no decision with respect to the director position in Norway. There will probably be none until late summer. We are, however, prepared to move to Norway in any case, because the university here in Copenhagen has agreed to give me so much permission of leave that we can live in Norway.

Are you coming to Europe this summer? If so we certainly hope that you will find time to stay with us for some time. It is a pity that I am not in such a position that we can invite you for a longer period. This may, however be different if I get the position in Norway.

We are getting a lot of papers from the numerical prediction unit in Washington, personified by Thompson. I hope they are doing something else than drowning other people in paper.

My family is fine, and I hope that Ellinor and your children are the same.

I have a problem which you possibly can help me with. They wish to tax my income in U.S. here in Denmark. However, they cant do so if you can prove that the money you have earned is government money. I have thought that the salary I earned in U.S. was from O.N.R. Do you think that it will be possible for you to give me a statement showing what I have got in salary and which is auhorized in such a way that the tax officials here understand that it is U.S. government money?

I would be very thankful if you could do so, because the taxes here are terrible high on such an income I have had in U.S. Besides i have already paid taxes of the income in U.S.

Send your family our best greetings.

Yours

Ragnar

RAGNAR FJØRTOFT
PROFESSOR OF METEOROLOGY
UNIVERSITY OF COPENHAGEN
COPENHAGEN, DENMARK

Copenhagen
June 26.

Dear Jule:

This to tell you that the visa are now allright so that only the american immigration officials in New York can now stop us. We have ship accomodations on "Oslofjord" which sails from Oslo August 12th and arrives in New York August 20th or 21th. We hope it will bepossible for you to get an apartment for us at the project. – I have looked into the paper of yours and Philips. I feel rather strongly that the possibility exists for the errors to be explained as integration errors. – Our address ~~will n~~ from now until we leave will be: Svenseid st. Telemark, Norway.

We are all looking forward to see the Charneys. In particular am I looking forward to our cooperation in the scientific field which I hope will be fruitfull for your group and myself.

Best greetings yours
Ragnar.

LUFTPOST
PAR AVION

AEROGRAM

Dr. Jule Charney
The Institute for Advanced Study
~~Princeton, New Jersey~~
~~U.S.A.~~
c/o Dept. Meteorology
University of Chicago
Chicago 37, Illinois

HVIS DER LÆGGES NOGET IND I AEROGRAMMET
VIL DETTE BLIVE FREMSENDT AD LUFTVEJEN

AFSENDER
Ragnar Fjørtoft
København universitet
København.

TO OPEN TEAR OFF HERE

COPY

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, N. J.

December 11, 1952

Dear Dr. Fjørtoft:

Dr. Charney has told me that he has spoken to you concerning the possibility of your visiting us for a period of one or two years beginning September 1953 and that you reacted favorably. I would like to officially confirm his invitation and to express my pleasure at the prospect of collaborating with you once again.

I have proposed to the School of Mathematics of the Institute for Advanced Study that you be granted the privileges of membership during your visit, and have received its approval. I am enclosing a bulletin of information describing the background of the Institute and the facilities available to members.

At the same time we are able to offer you a position as a member of the Meteorology Group at a salary of $8,000 per nine-month academic year for the two year period.

I hope you will find this invitation satisfactory and that we can expect to hear from you before long.

Sincerely yours,

John von Neumann

Enclosure

Dr. Ragnar Fjørtoft
Department of Meteorology
University of Copenhagen
Copenhagen, Denmark

cc: Dr. Oppenheimer
Miss Trinterud
Mr. Levy (ONR)
Miss Underwood
Dr. Charney

COPY

Charlottenlund, January 28, 1953.

Dr. Jule Charney,
The Institute for Advanced Study
Princeton, New Jersey
U. S. A.

Dear Jule:

./. I have enclosed a letter to von Neumann answering his invitation which I did not receive until two weeks ago because it had been mailed as ordinary mail.

I have been in contact with the American Embassy here to get a visa. Because of the new immigration law it is certain that it is quite impossible not to escape to bother you with some formal requests. I found discomfort in writing to von Neumann about this, so I am writing you instead, but you may have to contact him on this matter.

The first thing I made sure is that we need immigration visa. To get such a visa it takes ordinarily twelve to eighteen months. However, one may get a "non quota" immigration visa if one belongs to one of several preference groups. As a matter of fact I was made sure that we belong to the first preference group which consists of high qualified workers, technicals, scientists and their families. The important point of the new immigration law is that this also has to be found out by the Attorney General in Washington D. C. after a formal request has been made from the immigrant's host organization (in this case your institute) on forms to be obtained at the closest Immigration and Naturalization office in your district. While making this request it should not be forgotten also to be made for Ragnhild, while I should think it unnecessary to mention the children specifically.

The next thing which has to be done is to confirm the invitation in a letter direct to the American Embassy in Copenhagen whose address is:

American Embassy
Immigration section
Borgergade 16
Copenhagen.

(Page 2)

I think it is important that you take action on these matters as soon as possible.

We got the absolutely last accomodations on "Oslofjord" for the sailing in the second half of August.

Best greetings to Ellinor and the children

/s/ Ragnar

COPY

Copenhagen, January 28, 1953.

Professor Dr. John von Neumann,
The Institute for Advanced Study,
Princeton, New Jersey,
U. S. A.

Dear professor von Neumann:

I was very happy to receive your invitation to come to Princeton. I am grateful for the conditions you are offering me and believe that the amount of salary offered to me is very satisfactory indeed.

As I did not receive your letter until two weeks ago it has not as yet been possible to get a definite answer from the officials of the university here how they will response on an appliance for absence in leave. Very likely there will be little or no difficulty in getting an absence in leave for a period of one year while I think it doubtful whether it will be given for as much as two years, unless perhaps it be given as an extension later on.

I will inform you closer as soon as I know more definitely from the university here.

Sincerely yours,

Ragnar Fjörtoft

COPY

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, N. J.

February 2, 1953

Dear Professor Fjörtoft:

Thank you for your letter of January 28. I am very gratified to learn from it that we can expect to have you with us next year. I appreciate your difficulty in securing immediate leave for two years, but I am very glad that you will make an effort to obtain an extension later on.

Looking forward to seeing you here, and with best personal regards, I am,

Sincerely yours,

John von Neumann

JvN:eg

Professor Ragnar Fjörtoft
Department of Meteorology
University of Copenhagen
Copenhagen, DENMARK

CC: Dr. Charney

July 9, 1953

Dear Ragnar:

Thank you for your letter. It ~~has been~~ was so long since I ~~have~~ heard from you that I had begun to worry about your coming at all (of course, I know I owed you a letter). Needless to say, I was delighted that you were able to accept our invitation, and am looking forward eagerly to your arrival. It is now definite that Eady and Bolin will be in Princeton also for the fall semester, and we should have a good time.

Elinor and Nicky also await your arrival with great anticipation. Nicky has been practicing his tennis, and is anxious to play Kari, although I am afraid he does not realize what a young lady she has become.

I am now at the University of Chicago and shall remain until the end of July. Elinor and the children are staying with her parents in St. Paul. They will join me at the end of the month and we shall drive back to Princeton, remain there for a few days, and then go to Woods Hole, Massachusetts on vacation. We hope to see Rossby there, but I have heard via Petterssen that his doctor advised him to go easy and that he has postponed his trip to the U. S. I certainly hope it is nothing serious. In any case, we expect to stay in Woods Hole until Sept. 1. When you arrive on August 20 or 21, you can go immediately to Princeton for your apartment is all arranged, but it occurred to me that you might like to spend a few days with us at Woods Hole. It is a kind of vacation resort with swimming and sailing, and at the same time a number of people with whom one can talk science. Also, the weather is likely to be superior to that in Princeton. Alternatively, you might want to go first to Princeton to get settled and then come to Woods Hole. If you are at all interested please let me know as soon as possible, and I will try to get accommodations for you and family at Woods Hole.

Our work in Princeton has progressed quite satisfactorily. As you know we have been using the Great Storm of Nov. 1950 as a test situation. We wanted to see to what extent it was possible to predict the rapid and intense cyclogenesis that began about 1500 GMT on the 24th. Both the 2 and $2\frac{1}{2}$ dimensional models failed to predict it, the 2 2/3 dimensional model did predict the storm, and with its full intensity. We used the simplified model given by equation (34) in the Phillip-

Charney article, just with the levels 850-500-200 mb, and then with the levels 900-700-400 mb to give better definition in the low levels where the storm first began. I am sending you a series of diagrams which I think are self-explanatory. The graph gives the correlation coefficients between the observed and predicted 24-hour height changes.

I have saved the best news for the last. We have found that it is possible to pay your travel expenses for yourself and family up to the amount of $1,000. This is in addition to your salary. You may receive the money upon arrival in Princeton or you may have it sent to you in Norway. If you decide on the latter course, please write to Oppenheimer's secretary--not Oppenheimer, he is in Brazil--and tell her when, where, and how you want the money sent.

Please give my warmest greetings to Ragnhild, Kari, and Inge.

With best regards,

Jule

JC/cs

COPY

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, N. J.

December 11, 1952

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Sincerely yours,

John von Neumann

Enclosure

Dr. Ragnar Fjørtoft
Department of Meteorology
University of Copenhagen
Copenhagen, Denmark

cc: Dr. Oppenheimer
Miss Trinterud
Mr. Levy (ONR)
Miss Underwood
Dr. Charney

COPY

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COPY

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, N. J.

February 2, 1953

Dear Professor Fjörtoft:

Thank you for your letter of January 28. I am very gratified to learn from it that we can expect to have you with us next year. I appreciate your difficulty in securing immediate leave for two years, but I am very glad that you will make an effort to obtain an extension later on.

Looking forward to seeing you here, and with best personal regards, I am,

Sincerely yours,

John von Neumann

JvN:eg

Professor Ragnar Fjörtoft
Department of Meteorology
University of Copenhagen
Copenhagen, DENMARK

CC: Dr. Charney

February 9, 1953

Dear Dr. Fjortoft:

This is to notify you that the Institute for Advanced Study has been designated by the Department of State as a sponsor of the Exchange-Visitor Program. The serial number assigned to the Program of the Institute is No. P-156.

There is now a special type visa under the Exchange-Visitor Program. If you have already made satisfactory visa arrangements, there is no reason to use this Program. But if you have not made visa application, or if it should seem adviaable to do so, you may obtaih a visa under the Program; you should present this notification to the United States Consul in your country, and to the Immigration and Naturalization Service at the port of entry into the United States.

Through this notification you are designated as a participant in the exchange Visitor Program. Designation does not ensure the issuance of a visa, but it should prove helpful.

Sincerely yours,

(Mrs. Ruth W. Barnett)
Assistant to the General Manager

Dr. Ragnar Fjortoft
Department of Meteorology
University of Copenhagen
Copenhagen, Denmark

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

February 9, 1953

Dear Dr. Fjortoft:

Your letter of January 28, addressed to Dr. Charney, has been referred to me for answer.

I want to let you know that I have consulted the Immigration and Naturalization office in our district and have been advised to inform you that you should apply for an Exchange*Visitor permit, which is referred to as Section 402(f) of the Immigration and Naturalization Act. This visa is easily obtained and may be renewed every six months, upon the recommendation of your host institution or sponsor, so long as your passport remains valiid six months longer than the termination date of your application for renewal. This extension is easily botained, and is in fact sent directly from this office with no inconvenience to you.

The visa to which you refer in your letter to Dr. Charney is indeed complicated and often takes an interminable time to obtain since various application must be filed with the Attorney General's office in Washington along with letters of recommendation from sponsoring Institutions and individuals and copies of your publications. The legal advisor in the local Immigration and Naturalization office strongly advised against this visa (known as 15-H(sub 1)) largely due to the fact that it is often a prolonged affair and is of no real advantage, being used more often by persons who have been refused an Exchange-Visitor visa. The other thing against it is that your wife must apply for an Exchange- Visitor visa since the 15-H(sub 1) is used only for highly skilled workers and may not be used by their dependants.

I am attaching two letters regarding your application, one in reference to the Exchange-Visitor Program, and the other to the application of your wife.

Please do not hesitate to let me know of anything I can do to be of help with your plans to come to Princeton.

Sincerely yours,

(Mrs. Ruth W. Barnett)

Dr. Ragnar Fjortoft
Department of Meteorology
University of Copenhagen
Copenhagen, Denmark

Enc:

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

February 9, 1953

COPY

To Whom It May Concern:

This is to certify that Dr. Ragnor Fjortoft has bee invited to come as a Member of the Meteorology Group of the Institute for Advanced Study for the academic year 1953-1954. Dr. Fjortoft has been granted a salary of $8,000 for the academic year and will live in an apartment in the Institute Housing Project. Dr. Fjortoft would like to be accompanied by his wife, Mrs. Ragnhild Fjortoft and his two minor children.

The Institute for Advanced Study will greatly appreciate every consideration shown Dr. and Mrs. Fjortoft's application for a visa.

Sincerely yours,

(Mrs. Ruth W. Barnett)
Assistant to the General Manager

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

February 2, 1953

Dear Professor Fjörtoft:

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Looking forward to seeing you here, and with best personal regards, I am,

Sincerely yours,

John von Neumann

JvN:eg

Professor Ragnar Fjörtoft
Department of Meteorology
University of Copenhagen
Copenhagen, DENMARK

cc: Dr. Charney

Copenhagen, January 28 1953.

Professor Dr. John von Neumann,
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Princeton, New Jersey,
U. S. A.

Dear professor von Neumann:

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Sincerely yours

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Dr. Jule Charney,
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U. S. A.

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Immigration section
Borgergade 16
Copenhagen.

I think it is important that you take action on these matters as soon as possible.

We got the absolutely last accomodations on "Oslofjord" for the sailing in the second half of August.

Best greetings to Ellinor and the children

Ragnar.

1/8/53

Second original letter and another IAS brochure mailed today, via Airmil.

ESG

THE INSTITUTE FOR ADVANCED STUDY

PRINCETON, NEW JERSEY

December 11, 1952

Dear Dr. Fjørtoft:

Dr. Charney has told me that he has spoken to you concerning the possibility of your visiting us for a period of one or two years beginning September 1953 and that you reacted favorably. I would like to officially confirm his invitation and to express my pleasure at the prospect of collaborating with you once again.

I have proposed to the School of Mathematics of the Institute for Advanced Study that you be granted the privileges of membership during your visit, and have received its approval. I am enclosing a bulletin of information describing the background of the Institute and the facilities available to members.

At the same time we are able to offer you a position as a member of the Meteorology Group at a salary of $8,000 per nine-month academic year for the two year period.

I hope you will find this invitation satisfactory and that we can expect to hear from you before long.

Sincerely yours,

Johnnos Neumann

John von Neumann

Enclosure

Dr. Ragnar Fjørtoft
Department of Meteorology
University of Copenhagen
Copenhagen, Denmark

cc: Dr. Oppenheimer
Miss Trinterud
Mr. Levy (ONR)
Miss Underwood
Dr. Charney

COPY

Copenhagen January 1,1953.

Dear Jule:

I am writing to tell you that if am going to get an invitation to c me to Princeton from summer it will be very important for me to know this as early as possible: Partly it is the question what to do with our apartment here in case we leave for U.S. This question should be settled at least four months before our departure, Then it is the negotiations with the director of the university. And finally it one may have to consider that it may take quite a long time to get a visum. I think I mentioned these things to you while you were here. When I am reminding you about it now it is not so much because I think you have forgotten it as because of the constant pressure exserted upon me from the side of Ragnhild to get everything clear at the earliest possible time in case we are going to U.S.

I take the opportunity to thank you and Ellinor for the very nice chair you gave us as a present.

Kari is spending Christmas in Norway together with a danish girlfriend. The rest of us are looking forward to a wintervacation lateron.

I hope you are all by good health and wish you a pleasant and profitable new year.

Yours very truly

Ragnar.

THE INSTITUTE FOR ADVANCED STUDY
ELECTRONIC COMPUTER PROJECT
PRINCETON, NEW JERSEY

not sent

December 1, 1952

Dr. Ragnar Fjörtoft
Department of Meteorology
University of Copenhagen
Copenhagen, Denmark

Dear Dr. Fjörtoft:

Dr. Charney has told me that he has spoken to you concerning the possibility of your visiting us next year for a period of one or two years and that you reacted favorably. I would like to officially confirm his invitation and to express my pleasure at the prospect of collaborating with you once again.

I have proposed to the School of Mathematics of the Institute for Advanced Study that you be invited as a member of the Institute for two years beginning September 1953 and have received its offical approval. I am enclosing a bulletin of information describing the background of the Institute and the functions of members.

At the same time we are able to offer you a position as a member of the Meteorology Group at a salary of $8,000 per nine-month academic year for the two year period.

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Sincerely yours,

John von Neumann

John von Neumann

JGC/cpb

Enclosure

CC: Dr. Oppenheimer
Miss Trinterud
Mr. Levy (ONR)

TO OPEN CUT HERE

TO OPEN CUT AT TOP

NORGE 45 ØRE·POST

AEROGRAM

PAR AVION
LUFTPOST

Dr. Jule Charney
Institute for Advanced Study
Princeton, New Jersey
U.S.A.

BRETT HER

BRETT HER TIL SLUTT

AVSENDER R. Fjørtoft
Langerudsvingen 16, Abildsø, Oslo.

the laws of propagation may be quite different ($\bar{v}_{t=0}$ essentially different from $v_{t=0}$) and $\frac{\delta^2}{4}\nabla\zeta$ not negligible). The above mentioned tool is based upon the formula

$$\frac{\partial\zeta}{\partial t} = -v\cdot\nabla\zeta - v\cdot\nabla f = -\frac{1}{\delta^2}\bar{v}\times\overline{\bar{v}}\cdot k - v\cdot\nabla f$$

where $\bar{v}$ again is a mean wind on a circle, now with infinitesimal radius δ. But more about this on other time —

Kari and Trygve have now an exciting time with skiing and skating practically outside our windows. I will go til København in the middle of January while the rest of the family will stay here to over the summer. I had my last forecasting duty two days ago. — Best wishes for Christmas and the new year to Elinor, you, Nicky and Nora from all of us.

Yours Ragnar.

MERK! DET ER FORBUDT Å LEGGE NOE INN I AEROGRAMMET. LEGGES NOE INN I AEROGRAMMET, GÅR SENDINGEN IKKE SOM LUFTPOST, MEN MED VANLIGE BEFORDRINGSMIDLER (SKIP, JERNBANE).

SALGSPRIS 50 ØRE

Oslo 13-12-51.

Dear Jule!

I recently visited Rossby and Bert in Stockholm. There is, as you know, a large group there and rather inspiring. For quite a time they have worked with "barotropic" forecasts just like you did in Princeton last spring.

After returning from Stockholm I was able to construct a tool by means of which one single man in about a minute can compute $J = \frac{\partial \zeta}{\partial t} = -\mathbb{V}\cdot\nabla\zeta_{abs}$ in a point. This tool is already being used in the forecasting section in Oslo and is a big aid. J is computed for selected points and isolines J = const are drawn on the maps (complete time necessary 30 - 40 minutes). For time being I work with approximate formulae for the propagation of the system J = const. The following formula seems to work quite well for a period of about 24 hr.:

$$\frac{\partial J}{\partial t} = -\overline{\mathbb{V}}_{t=0}\cdot\nabla J - \frac{\rho^2}{4}\,\mathbb{k}\times\nabla f\cdot\nabla J$$

ρ is the radius of a circle around the reference point and taken to be 1/4 of the typical dimension of the system J = const (1/4 of a wavelength). $\overline{\mathbb{V}}_{t=0}$ is a mean velocity on this circle. As you see small scale systems have a tendency to move with the wind ($\overline{\mathbb{V}}_{t=0} \approx \mathbb{V}_{t=0}$), but for large scale systems

Headquarters Air Weather Service.
Andrews Air Base
Washington 25, D.C.

Dear Jule!

It is difficult to imagine two places where meteorological research work is done which are so entirely different as Princeton and the place here. In the first place you have here this huge main building with about 1500 people, not all meteorologists, but certainly a great fraction. Many of them are nice young girls, a fact which indeed must be very disturbing for any kind of scientific work. - I had to undergo a 1½ hr thorough physical examination, with X-ray, bloodtype test, undressing, and so on. (dressed). Then to swear several kinds of loyalty oaths, ... Well, anyway, I do think I won't regret to have seen this place. - What I am actually writing you for is to tell you that I forgot to inform you before I left that I brought with me the Aberdeen maps. The people here are now copying them for me and making slides. Not ~~even~~ less than 8 men are working on them and they will propably be finished by tomorrow. Then I will send the maps back to you, and later on also the slides.

– On page 22a, line 10, in my compendium article I have forgotten to fill in $\int \zeta'^2 dF$, and on page 22, line 7, ζ_0 should be replaced by ζ_0'. –

Best greetings to Ellen and your children, and the Smagorinsky's and Davies's, and Norma.

Yours
Rossby.

THE UNIVERSITY OF CHICAGO

CHICAGO 37 · ILLINOIS

DEPARTMENT OF METEOROLOGY

July 20, 1950.

Dear Jule!

I had forgotten to label the η-map for Febr. 13 which we send to Chicago. Will you be sure to bring that map when you are coming at the end of the month. The other maps have now been drafted and slides have been made for the first three dates. I used them for a lecture Rossby wished me to give. Wexler will be here at the same time as you, and propably also Höiland if he decides to stop here some days on their way to Los Angeles. It seems as a conference is "sailing up". — Well, this was all. Best greetings to Ellinor and Vicky and Nora, and wellcome to Chicago.

Yours

Ragnar.

FJØRTOFT

FILE SENT TO
OPPENHEIMERS OFFICE.
8 - SEPT. 1949 -

February 14, 1950

To whom it may concern:

Mr. Ragnar Fjortoft is on leave of absence from the Norwegian Meteorological Institute for the purpose of studying the methods of numerical weather forecasting which are being developed by the Theoretical Meteorology Project of the Institute for Advanced Study, Princeton, New Jersey. He has been associated with this project since September 1949.

Sincerely,

Jule Charney,
Group Leader
Theoretical Meteorology Project

cp

April 21, 1949

Mr. Ragnar Fjørtoft
Meteorologisk Institutt
Oslo (Blindern), Norway

Dear Fjørtoft:

Eliassen will be leaving us in July or August and we will have room for another person on the meteorology project. Recalling our conversation in Stockholm, I spoke to Professor von Neumann concerning the possibility of inviting you to take Eliassen's place for an academic year. Upon learning of your qualifications he at once approved the idea and authorized me to extend an invitation for the next academic year. I am therefore pleased to be able to offer you a position on the theoretical meteorology project at the Institute for Advanced Study for ten months beginning September, 1949. The salary will be $5000 for the period, and there will be a travel allowance for yourself of $500.

Your duties would be primarily to sit and think and have good ideas. As you probably have heard through Eliassen, the project, being in the developmental stage, is but loosely organized and is dependent more on ideas than on techniques. It is true that the ultimate purpose is to devise a method for numerical forecasting, but the term "numerical" embraces virtually all quantitative dynamical procedures and so imposes no real restriction. We simply have the advantage that we shall soon have an extremely powerful means of verifying theories in the electronic computing machine now being constructed here. The machine will be an assurance that we need no longer be guided so much by the mathematical exigencies in the choice of physical models.

Princeton, with the Institute and the University, is one of the foremost intellectual centers of the U. S. You would find it a very stimulating place, living in pleasant suburban surroundings and yet within but an hours train ride from New York. If our plans materialize we shall also have Eady with us.

If you do decide to come, and with your wife and children, you will probably want to know about housing. The housing situation is difficult all over the United States, but we are fortunate in having a group of houses especially reserved for temporary members of the Institute, and while we cannot promise a house outright I can say that in all probability you would have one of these. The houses are surrounded by spacious grass lawns and woods which are ideal for children.

I hope I have said enough to make the offer sound attractive for I should personally like very much to have you with us.

Please let us know of your decision as soon as possible. This is important if you wish us to reserve an apartment.

Sincerely,

Jule Charney

JC:mh

cc: Dr. von Neumann

Blindern, Mai 3, 1949.

Dr. Jule Charney,
Institute for Advanced Study,
Princeton,
New Jersey,
U.S.A.

Dear Charney,

At first I would like to express my best thanks for the invitation to work at the meteorological project at the Institute for Advanced Study. Provided permission will be given from the Meteorological Institute, which is most likely, and provided there will be opportunity of travelling, I will come to Princeton in the last of August together with my family. As soon as I know certainly that it will be possible to go, I shall write you again.

My paper is now ready for printing. I will soon have a copy of the manuscript ready which I shall send you.

My best greatingsto your wife.

Sincerely

Ragnar Fjørtoft

Ragnar Fjørtoft.

Blindern 20-6-49.

Dr. Jule Charney,
Institute for Advanced Study,
Princeton,
New Jersey,
U.S.A.

Dear Charney!

Concerning my request for permission I yesterday had a conferance with the proper authorities in our Department of Finance. I was told that it was a certainty that the permission would be given in due time.

So far I have not got a convenient place on a ship. I can get one on a liner leaving Southampton 15th. July. This I think is too early for me if not special arrangements are made. However, seats are reserved for me and my family on a plane leaving Oslo 23ed. August. But as you know travelling by air is rather expensive and therefore if I not in the meantime get an opportunity for travelling on a ship, it most probably willbe necessary for me to leave my family behind until October, because at that time places are available on a ship leaving Oslo.

Mr. Eliassen and also a representative in the U.S.Embassy told me that a so-called "Professorvisum" would be most useful for me rather than other forms of visa. For this purpose I most urgently want you to certify that I am going to give lectures in Princetown. Will you therefore please forward this certification as soon as possible.

Recently dr. Namias visited us. We had then some very interesting map discussions concerning the longe-range forecasting problems.

My paper will be printed in November. I will be glad to have discussions with you on some of its problems. On 1st. July I start my lectures for the american students at the Summer School in Oslo. Afterwards I will have a few weeks vacation, to which I am looking very much forward after these months of rather hard work: My paper, forecast duty, support of Høilands lecture duties (collokvia), and the terrible load of red tape connected with the U.S.A. journey.

My best greatings to you and your wife.

Yours sincerely

Ragnar Fjørtoft.

Ragnar Fjørtoft.

27 June 1949

Mr. Ragnar Fjörtoft
Norwegian Meteorological Institute
Blindern, Norway

Dear Mr. Fjörtoft:

I have received your letter of June 20, in which you inform me that you would like to have a statement from us outlining the exact nature of the invitation which we have extended to you to join the Meteorological Group for the next academic year and of the duties during your stay here.

These duties will consist in participation in the research work of the group at the Institute in theoretical meteorology, and in giving some lectures to the members of the group and others who may be interested in your results in relevant parts of the subject. Your compensation will be $5000 for the academic year 1949-50 with an additional allowance of $500 for traveling expenses.

We are, of course, looking forward with the greatest expectation to having you here. However, you must not regard yourself as obligated to be in Princeton exactly by September 1. If it will help you to conclude more satisfactory travel ~~accomo-dations~~ arrangements please allow yourself as much leeway in time as you ~~feel~~ find necessary. There would be some advantage in your being ~~hear~~ here earlier in order to obtain an apartment for your family. The Institute apartments are scarce, and it is advantageous to be on the spot when they are being assigned. We shall, of course, do everything we can to see that you are adequately housed.

I hope that this letter will be of help to you in connection with your application for a visa at the United States consulate in Oslo.

Sincerely,

Jule G. Charney

JGC:AK

original retyped.

28 June 1949

Mr. Ragnar Fjörtoft
Norwegian Meteorological Institute
Blindern, Norway

Dear Mr. Fjörtoft:

I have received your letter of June 20, in which you inform me that you would like to have a statement from us outlining the exact nature of the invitation which we have extended to you to join the Meteorological Group for the next academic year and of the duties during your stay here.

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Sincerely,

Jule G. Charney

JGC:AK

Jule G. Charney

FORCHTGOTT

The Airflow Round a Conical Hill

by Dr. Jiri Förchtgott.

(Translated by the author from *Meteorologické Zprávy*, No. 3-4, March 1951)

Reprinted from

GLIDING

Vol. 2 No. 4 Winter 1951–52

The Airflow Round a Conical Hill

by Dr. Jiri Forchtgott.

(Translated by the author from *Meteorologické Zprávy*, No. 3-4, March 1951)

THE effect of a short mountain ridge, or even of a conical hill, on the streamfield of a stable air current is a form of mechanical disturbance. The opinions of our practical observers—sailplane pilots—on this question have gradually changed in the years since the war. Initially there was the conviction that only a weak vertical component of no practical importance was produced by conical obstacles, the region of up-currents being too small. Later some of the more experienced glider pilots began to look for new possible soaring places in the vicinity of the well-used ridge of Rana when the air above the Rana slope became crowded with school gliders. They were successful in the space above the tops of the isolated hills Oblik and Mily—the smaller space with weaker up-currents became more interesting, and real soaring in it was considered as a sign of high pilot quality.

Thanks to these pilots, prejudices about the smallness of the effect of isolated hills on the air current were overcome. Thanks also to glider pilots, many further prejudices concerning the real nature of some atmospheric phenomena will be overcome.

Experience has shown that there is sufficient space with enough uplift for soaring in front of isolated hills. As in the case of long ridges, it can be supposed that the velocity and depth of the streaming layer are of equal importance, and one must expect to find certain types of flow corresponding to different kinds of airstream. This means that, just as soarable regions are found in the lee of long ridges, they may also be found in the lee of isolated hills.

Single observations made in the lee of conical hills have in the meantime no direct attestation by means of gliders (in our country), but they indicate some surprising facts that are very marked under certain airstream and humidity conditions. The shape, structure and motion of clouds can show clearly the type of flow; and this can also be followed occasionally or systematically by soaring flight.

When the air is stably stratified there are several possible types of flow that can be recognised by the pattern of the flow in the lee of the hill. To classify these regimes precisely a series of systematic observations is needed. At present it is possible to describe, though only schematically, one of these flow patterns, perhaps the most fundamental one, which by its characteristics has attracted the attention of authors in various countries (e.g. R. S. Scorer, London, explains in a similar way the origin of some typical clouds in the lee of Gibraltar; also one must mention observations of cloud caps over conical mountains, volcanoes, etc.).

The experience of glider pilots shows that on the windward side of isolated hills it is possible to reach the same height as in front of a ridge of similar section, the only condition being a greater wind velocity. It seems that the same type of flow as for mountain ridges is produced by isolated hills in winds of higher velocity. For instance, Figure 1 shows the analogy of the vortex type of flow behind a ridge, but the same airstream blowing over a ridge would produce a "higher" type of flow—the "wave type." The relation between the depth of the air-current and the type of flow produced by an isolated hill is not at present known.

Flow over a conical obstacle presents a problem in three dimensions, whereas the flow over a mountain ridge is two-dimensional. Most of the airstream in the layers below the top (e.g. below the level "a") flows round the sides, air in the upper levels only going over the top. One must expect great horizontal deflections of the streamlines in the low levels near the foot of the hill, while in higher levels the vertical deflection gradually predominates. Near to the front foot of the obstacle the streamlines diverge and cause a weakening of the stream, with the result that the vertical component of the wind is very small or even negative in front of the lower half of the

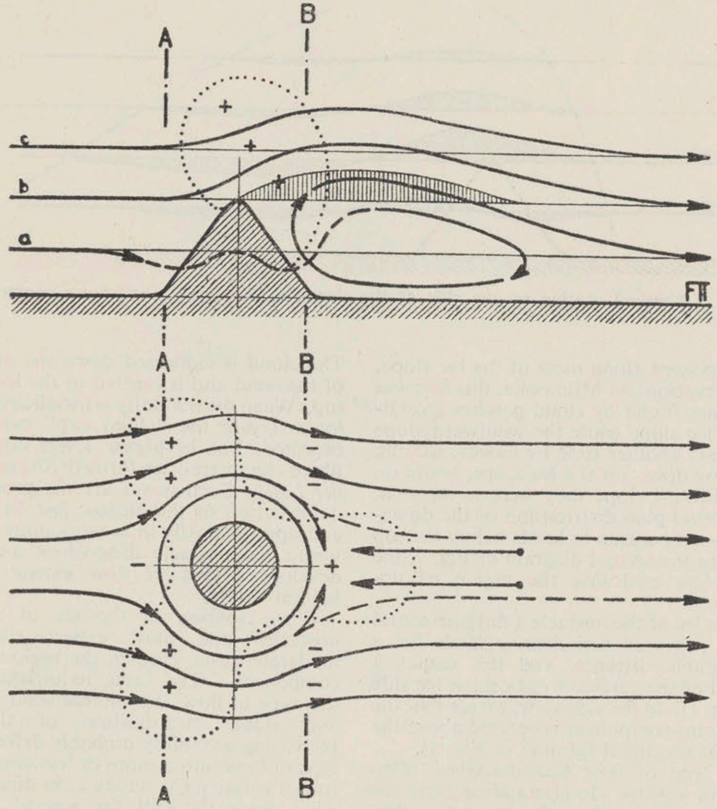

Fig. 1.—Side view and ground plan of the streamlines over a conical hill. The lower diagram shows the flow in the level "a." Up- and down-components are denoted by + and — respectively.

slope—no use for soaring. Only in front of and above the top is there enough lift for soaring. Since the streamlines in level "a" converge on the right and left of the front side of the obstacle, the lift is greater there than immediately in front of it. In Fig. 1, upward wind components are indicated by + and downward ones by—. The cross-section A, in Fig. 3, shows the lift and sink in front of the hill. The sink in front of the hill explains why everyone who flies in a glider below the top of the hill is forced to land. According to experience of soaring in the slope wind in front of a ridge most pilots tend to stay in front of an obstacle; but now, in front of a conical hill, one finds the unsoarable downward component, which is proportional to the intensity of the sideways divergence.

In the lee of the obstacle, the streamlines return to their original distribution. Below the top level, on the right and left sides behind the obstacle is found divergence of the streamlines and a down component. Behind the lee foot there must be expected intense convergence and a corresponding

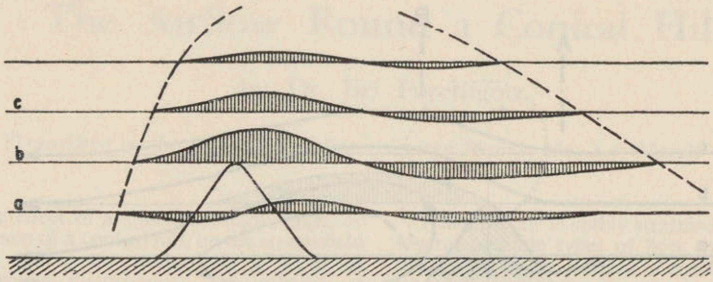

Fig. 2.—Vertical section to one side of the obstacle, showing up- and down-components.

up-component along most of the lee slope. By observations on Milesooka, this fact was sometimes found by cloud patches ascending the lee slope while the windward slope was clear; another time by hawks, soaring from low down on the lee slope, while on the windward slope they were never seen. The ground plan distribution of the down- and up-components in levels below the top is seen in the second diagram of Fig. 1, the dotted line enclosing the region of up-component.

In the lee of the obstacle a flat horizontal vortex, not very turbulent, extends for a considerable distance, and this causes a reversal of the surface wind on the lee side (see Fig. 1). In the side view, we see that the greatest up-component is centred above the lee slope (enclosed by dots in Fig. 1).

The type of flow just described often produces a wave cloud standing near the top of isolated hills and mountains when they rise above the surrounding obstacles. The cloud is elongated down the direction of the wind and is centred to the lee of the top. When the humidity is insufficient in the lower layers the "cloud cap" cannot be expected, but in higher levels cirrus-like fibred clouds are often formed (*Stromschnelle der Luft*). Such waves are the product of deformation of the airflow just described, and appear mostly in mountainous regions where the obstacle dimensions and well-developed types of flow extend to the highest levels.

When soaring in the lee of isolated obstacles one must expect slight or moderate gusts, even in the region of up-component. The basic requirements for this type of flow are suitable wind velocity and stable stratification of the air. Increasing instability probably deforms this type of flow into a more or less continuous row of rotating Cu clouds extending downwind from the hill for several tens of kilometres.

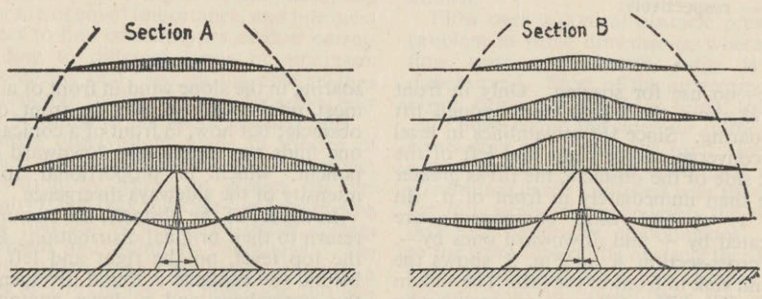

Fig. 3.—Vertical sections through lines A and B of Fig 1, showing up- and down-components of the wind.

TO OPEN CUT AT TOP

NORGE

60

ØRE-POST

ASKER 23.7.49

ASKER 23.7.49

AEROGRAM

PAR AVION

LUFTPOST

TIL Dr. J. Charney

Institute for Advanced Study

Princeton, New Jersey

U. S. A.

BRETT HER

BRETT HER

BRETT HER TIL SLUTT

SENDER R. Fjørtoft

Meteorologisk Institutt

Oslo

Norge.

MERK! DET ER FORBUDT Å LEGGE NOE INN I AEROGRAMMET. LEGGES NOE INN I AEROGRAMMET GÅR SENDINGEN IKKE SOM LUFTPOST, MEN MED VANLIGE BEFORDRINGSMIDLER (SKIP, JERNBANE).

TO OPEN CUT HERE

Oslo 22-7-49

Dear Charney!

I and my family will leave from France with a ship going to Québeck where it will arrive around August 28 th. It is kind of you that you will try to get an apartment for us. – I have just seen the article of you and Arnt in "Tellus". It is remarkable that your simple model allows for such good numerical forecasts. – I am looking forward to cooperating with you and the other members of the staff at the meteorological project. I hope only that I shall be of some use for the work. –

Best greetings. Fjørtoft

Charney

Appendix D

Report on work at U.C.L.A.
Department of Meteorology
through the period January 29 to April 3, 1951
under contract (W28-099 ac-403)

By Ragnar Fjörtoft

In a series of eight seminars the subject of small waves in a baroclinic atmosphere was discussed. Particular attention was paid to their physical interpretation. The recent work in the Meteorology group in Princeton, New Jersey was also presented and discussed. In an additional seminar some aspects of barotropic fluids were discussed.

The subjects dealt with are rather well covered by parts of an article to appear in the Meteorological Compendium. A copy of the manuscript for this article has been added to this report.

Added is also a summarization paper on a problem related to the stability of atmospheric flow, which was taken up for investigation under the above mentioned contract.

On the influence from horizontal shear on baroclinically unstable waves.

1. In the Compendium article it has been demonstrated that for sufficiently long waves the horizontal motion in some upper horizontal level is essentially governed by the equations:

$$\left(\frac{\partial}{\partial t} - \mathbb{V}\cdot\nabla\right)\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{df}{dy}v + \mathbb{V}_T\cdot\nabla\left(\frac{\partial v_T}{\partial x} - \frac{\partial u_T}{\partial y}\right) = 0,$$

(1) $$\left(\frac{\partial}{\partial t} + \mathbb{V}\cdot\nabla\right)\psi_T = 0, \qquad \nabla\cdot\mathbb{V} = 0,$$

$$u_T = \frac{\partial \psi_T}{\partial y}, \qquad v_T = -\frac{\partial \psi_T}{\partial x}$$

Here $\mathbb{V}$ is the actual wind velocity and $\mathbb{V}_T$ is some thermal wind.

Considering the problem of small waves superimposed upon a basic zonal flow one finds easily on the basis of eqs. (1) the solutions when the basic flow is horizontally uniform. When this assumption of uniformity is no longer maintained the problem becomes a more complex one from a physical point of view. This is in virtue of the fact that the mean basic flow may now itself act as a source or sink of energy for the small disturbances. In addition the problem becomes much more difficult from a mathematical point of view. It is this latter point which will be considered in this note.

To study the problem as simply as possible it will be assumed that

$$\frac{df}{dy} = 0$$

U : linear function of y.

$$U_T = \text{const.}$$

Here U and U_T are the speeds in the basic flow of the wind and thermal wind, respectively. Assuming solutions at the form

$$V_T = a(y)\, e^{i(\mu x + \sigma t)}, \qquad \mu = \text{real}$$

and similarly for the remaining perturbation quantities, and neglecting second order terms in eqs. (1), one gets the following two equations:

$$(\sigma + \mu U)(V'' - \mu^2 V) + \mu U_T (V_T'' - \mu^2 V_T) = 0$$

(2)

$$V = \frac{\sigma + \mu U}{\mu U_T} V_T$$

By elimination of V this leads to the differential equation

(3) $$\frac{d^2 V_T}{dy^2} + \frac{2\mu \frac{dU}{dy}(\sigma + \mu U)}{(\sigma + \mu U)^2 + \mu^2 U_T^2} \frac{dV_T}{dy} - \mu^2 V_T = 0$$

It is assumed that the boundary conditions are

$$V = 0 \quad \text{for} \quad y = 0, \quad y = B$$

From (2) it follows then also that

(4) $$V_T = 0, \quad \text{for} \quad y = 0, \quad y = B$$

To find the solutions of the system (3), (4) is of course a very difficult problem. The solutions will probably in the general case have singularities which make the corresponding waves impossible to realize. This can be demonstrated in the case where

$$\left(\frac{d^2}{dy^2} - \mu^2\right) V_T \approx \frac{d^2 V_T}{dy^2} \tag{5}$$

Since V_T must have a periodic variation along y, the largest possible wave length being 2B, (5) will obviously be a good approximation when $\frac{2\pi}{\mu} = L >> 2B$. In this case eq. (3) may be approximated with

$$\frac{d^2 V_T}{dy^2} + \frac{2\mu \frac{dU}{dy}(\sigma + \mu U)}{(\sigma+\mu U)^2 + \mu^2 U_T^2} \frac{dV_T}{dy} = 0$$

Applying the first of the boundary conditions in (4), the solution of the above equation is

$$V_T \sim \int_0^y \frac{dy}{(\sigma+\mu U)^2 + \mu^2 U_T^2} \tag{6}$$

In all cases when

$$(\sigma + \mu U)^2 + \mu^2 U_T^2 \neq 0, \qquad 0 \leq y \leq B \tag{7}$$

The integrand in (6) has no singularities. Using then the boundary conditions (4), equation (6) becomes

$$\int_0^B \frac{dy}{(\sigma+\mu U)^2 + \mu^2 U_T^2} = 0$$

Hence

$$\ln \frac{\sigma + \mu U + \mu U_T i}{\sigma + \mu U - \mu U_T i}\Bigg|_0^B = 0$$

which, with some closer inspection leads to the contradiction that

$$U(B) = U(0)$$

It can therefore be concluded that (7) cannot be true. Instead it follows that

$$(\sigma + \mu U)^2 + \mu^2 U_T^2 = 0, \quad \text{for some } y = y', \ 0 \leq y' \leq B \tag{8}$$

This relation can be used to a closer examination of the stability of the present waves. In the first place, real values of σ is inconsistent with (8). Accordingly, the solutions are certainly unstable. Substituting $\sigma = \sigma_R + i\sigma_i$ in (8) one gets the equivalent conditions:

(9) $$U(y) = c_R \qquad \text{for some} \quad y = y', \quad 0 \leq y \leq B$$

(10) $$\sigma_i = \pm \mu V_T$$

having introduced the notation $c_R = -\frac{\sigma_R}{\mu}$ for the real velocity of propagation. From the second of these relations it is seen that the amplification of the found waves will be independent of the existence of a horizontal windshear in a basic flow. So far as these waves are considered there is accordingly no source or sink of energy in the basic velocity field.

The question is, however, how much confidence one can have in this result. It is clearly understood that because of (8) the solution (6) for V_T will have a singularity at $y = y'$ where $c_R = U$, which makes these waves impossible to realize, at least not without coming in conflict with the conditions underlying the linearization of the equations of motion. To make these singular solutions useful, one must by means of them compose the general solution for an arbitrary and non-singular initial distribution of the perturbation quantities. (One may think of this being done by letting y continuously assume all values between 0 and B and adding in some integral fashion all the corresponding singular solutions). The question whether or not this is possible in the present case will not be investigated here. For these composed solutions, now, the physical influence from the existence of a horizontal shear will again have a chance to appear. This will be demonstrated in the following for the simple case of a constant two-dimensional shear, the so-called "Couette"-flow.

2. Two-dimensional disturbances of a "Couette" flow.

Let the motion take place in the xz-plane. The governing equations are

(11) $$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla_2\right)\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = 0, \qquad \nabla_2 = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{k}\frac{\partial}{\partial z}$$

(12) $$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

Eq. (11) simply states the individual conservation of vorticity. Suppose now in the basic flow the velocity U to be a linear function of z, and the motion to take place in an infinite layer with the rigid boundaries at heights ~~y~~ $z=0$, ~~y~~ $z=H$. The boundary conditions then are

(13) $$w = 0 \quad \text{for} \quad z = 0, \quad z = H$$

Differentiation of (11) with respect to x, and application of (12) give

(14) $$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla_2\right)\nabla_2^2 w - \frac{\partial \mathbf{V}}{\partial x}\cdot\nabla_2\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = 0$$

By linearization of this equation one obtains in the case of "Couette"-flow:

(15) $$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\nabla_2^2 w = 0$$

For the "linearized" motion, therefore, also $\nabla_2^2 w$ is an individually conserved quantity. (This is of course no longer true when second order terms are taken into account as is seen from eq. (14).) To obtain a wave-solution of (15) one substitutes

$$w = w_a(z)\,e^{i(\mu x + \sigma t)}$$

~~Where~~ The amplitude w_a must satisfy the equation

$$(\sigma + \mu U)\left(\frac{d^2 w_a}{dz^2} - \mu^2 w_a\right) = 0$$

Since μU is real a complex value of σ would require that everywhere

$$\frac{d^2 w_a}{dz^2} - \mu^2 w_a = 0$$

which together with the conditions (13) gives $w_a = 0$ everywhere. Therefore σ has to be real corresponding to permanent waves with unchanged amplitude.

Further, $\sigma + \mu U$ must be equal to zero for some $z = z'$, $0 \leq z' \leq H$, so that $\frac{d^2 W_a}{dz^2} - \mu^2 W_a = 0$ for $z \neq z'$ and $\frac{d^2 W_a}{dz^2} - \mu^2 W_a \neq 0$ for $z = z'$. This obviously also gives a singular solution for W'. The method of composing the general solution from such singular solutions has been given by E. Höiland and R. Fjörtoft (as yet unpublished). However, this composition is not needed in the present case, because the general solution can be found very easily, as shown by Kelvin, directly as an initial value problem. In that connection suppose that

$$\nabla_2^2 W_{t=0} = f(x, z)$$

Eq. (15) now states that the values of f in any level Z is transported along the x-axis with the velocity U of that level. One therefore gets immediately

$$\nabla_2^2 W = f(x - Ut, z) \tag{16}$$

Suppose as an example

$$f(x, z) = e^{i(\mu_1 x + \mu_2 z)}$$

Then from (16)

$$\nabla_2^2 W = e^{i\mu_1 x} e^{i(\mu_2 z - \mu_1 U t)}$$

We substitute here

$$W = W_a(z, t) e^{i\mu_1 x}$$

and obtain

$$\frac{d^2 W_a}{dz^2} - \mu_1^2 W_a = e^{i(\mu_2 z - \mu_1 U t)}$$

The solution of this equation is

$$W_a = K_1 e^{\mu_1 z} + K_2 e^{-\mu_1 z} - \frac{e^{\mu_1 z}}{2\mu_1} \int e^{-\mu_1 z + i(\mu_2 z - \mu_1 U t)} dz + \frac{e^{-\mu_1 z}}{2\mu_1} \int e^{\mu_1 z + i(\mu_2 z - \mu_1 U t)} dz$$

or

$$W_a = K_1 e^{\mu_1 z_1} + K_2 e^{-\mu_1 z_1} + \left[\frac{1}{i(\mu_2 - \mu_1 \frac{dU}{dz} t) + \mu_1} - \frac{1}{i(\mu_2 - \mu_1 \frac{dU}{dz} t) - \mu_1} \right] \frac{e^{i(\mu_2 z - \mu_1 U t)}}{2\mu_1}$$

or

$$W_a = K_1 e^{\mu_1 z_1} + K e^{-\mu_1 z_1} + \frac{e^{i(\mu_2 z - \mu_1 U t)}}{(\mu_2 - \mu_1 \frac{dU}{dz} t)^2 + \mu_1^2}, \tag{17}$$

where K_1 and K_2 now are constants to be determined from the boundary conditions (13). It is immediately understood that K_1 and K_2 have to be proportional to the last term in (17). Therefore, the disturbances will be damped with time as $\left[\left(\mu_2 - \mu_1 \frac{dU}{dz} t\right)^2 + \mu_1^2\right]^{-1}$. This result is remarkable when it is compared with the result obtained for the wave solutions which have no damping of the amplitude. In the present case it is demonstrated that the mean basic flow acts as a sink of energy for the disturbances even if this is not apparent from the singular wave solutions.

Returning now to the baroclinic waves considered earlier it is now clear that it is likely that some physical influence from the horizontal shear will be apparent (when passing) from the singular wave solutions to the general solution. The above result also suggests that it may be advantageous to find the general solution directly as an initial-value problem.

3. Three-dimensional perturbations of a "Couette"-flow.

The following will show that in the general solution for three-dimensional perturbations of a "Couette"-flow one may expect to find a linear amplification of the disturbances with time. For three-dimensional motion there is of course in general no conservation of the vorticity components, and the governing equations have to be written*

$$\left.\begin{aligned} \left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)u &= -\frac{\partial \alpha p}{\partial x} \\ \left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)v &= -\frac{\partial \alpha p}{\partial y} \\ \left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right)w &= -\frac{\partial \alpha p}{\partial z} \end{aligned}\right\}, \qquad \nabla\cdot\mathbf{V} = 0$$

where α is the specific volume, and p is the pressure

* The fluid having supposed to be homogeneous and possessing rigid boundaries there can of course be no influence from gravity. Gravity may therefore be neglected at the outset.

As boundary conditions it is assumed:

$$W = 0 \qquad \text{for} \qquad z = 0\,, \quad z = H \tag{18}$$

$$v = 0 \qquad \text{for} \qquad y = 0\,, \quad y = B \tag{19}$$

The corresponding linearized equations are

$$\left\{\begin{aligned} \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) u &= -\frac{\partial \alpha p}{\partial x} - w\frac{dU}{dz} \\ \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) v &= -\frac{\partial \alpha p}{\partial y}\,, \\ \left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right) w &= -\frac{\partial \alpha p}{\partial z}\,, \\ \nabla\cdot\mathbf{v} &= 0\,, \end{aligned}\right. \tag{20}$$

$\supset: \ \frac{\partial \mathbf{v}}{\partial t} + U\frac{\partial \mathbf{v}}{\partial x} = -\nabla\alpha p - w\frac{dU}{dz}\mathbf{i}$

Hence by elimination of $\frac{\partial \mathbf{v}}{\partial t}$:

$$\nabla^2 \alpha p = -2\frac{dU}{dz}\frac{\partial w}{\partial x}\,,$$

Then also

$$\nabla^2 \frac{\partial \alpha p}{\partial z} = -2\frac{dU}{dz}\frac{\partial^2 w}{\partial x \partial z}\,,$$

or, by substitution from the third of the equations (20)

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\nabla^2 w = 0 \tag{21}$$

For the "linearized" motion (15) is therefore valid also in the case of three-dimensional perturbations if only ∇_2^2 is replaced by the three-dimensional Laplacian operator ∇^2. If now again

$$\nabla^2_{t=0} w = f(x, y, z)$$

one finds immediately from (21) that

$$\nabla^2 w = f(x - Ut, y, z) \tag{22}$$

To solve for w one needs the boundary condition in terms of w at the boundaries $y = 0\,,\ y = B$. From (19) and the second eqs of (20) one obtains:

$$\frac{\partial \alpha p}{\partial y} = 0 \qquad \text{for} \qquad y = 0\,, \quad y = B$$

Hence also

$$\frac{\partial^2 \alpha p}{\partial y \partial z} = 0 \qquad \text{for} \qquad y = 0\,, \quad y = B$$

By substitution from the third of eqs. (20) one then finds:

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\frac{\partial w}{\partial y} = 0\,, \quad \text{for } y = 0\,,\ y = B$$

Therefore, if

$$\frac{\partial w}{\partial y}_{t=0,\,y=0} = g_1(x,z)\,, \quad \text{and} \quad \frac{\partial w}{\partial y}_{t=0,\,y=B} = g_2(x,z)$$

one gets

(23) $$\frac{\partial w}{\partial y}_{y=0} = g_1(x-Ut,z)\,, \qquad \frac{\partial w}{\partial y}_{y=B} = g_2(x-Ut,z)$$

We shall not in this note be concerned with the solution of (22), (23) in general, since the existence of disturbance amplification can be shown in a much less laborious way. Let ζ denote the z-component of vorticity

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

From the first and second of equations (20) one gets by elimination of αp

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\zeta = \frac{dU}{dz}\frac{\partial w}{\partial y}$$

Applying this equation ~~of~~ to either of the two boundaries $y=0$ or $y=B$, and denoting by g either of the two functions in (23), one gets

(24) $$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\zeta = \frac{dU}{dz}\,g(x-Ut,z)$$

Let it be supposed that

(25) $$g(x,z) = F(z)e^{i\mu x}$$
$$\zeta_{t=0} = A(z)e^{i\mu x}$$

The solution for ζ will then necessarily have to be of the form

(26) $$\zeta = H(z,t)e^{i\mu x}$$

By substitution from (25) and (26) into (24) one therefore obtains

$$\frac{\partial \zeta}{\partial t} + i\mu U\zeta = \frac{dU}{dz}F(z)\,e^{i\mu(x-Ut)}$$

the solution of which is

$$\zeta = \zeta_{t=0}\,e^{-iU\mu t} + e^{-iU\mu t}\int_0^t e^{iU\mu t}\frac{dU}{dz}F(z)\,e^{i\mu(x-Ut)}\,dt$$

or

$$\zeta = \zeta_{t=0}\,e^{-iU\mu t} + F(z)\frac{dU}{dz}\left\{t e^{-iU\mu t}\right\}e^{i\mu x}$$

There is therefore an amplification of ζ at the vertical boundaries provided $F(z)$ is chosen $\neq 0$.

This result is of considerable interest in the problem of creation of turbulence. It indicates how careful one should be in drawing too far-reaching conclusions from wave solutions when these have singularities, and further that it may be as profitable in such cases to try to get the general solution directly, and not via the singular wave solutions.

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ON A SIMPLIFIED THREE-DIMENSIONAL MODEL

IN NUMERICAL WEATHER FORECASTING

The following constitutes an attempt to simplify further the Charney-Eliassen equations with the aim of making them more fitted for numerical computations.

Notations:

p = pressure

ρ = density

T = temperature

g = acceleration of gravity

$\Gamma = \frac{c_v}{c_p RT}$

k = vertical unit vector

h = subindex used for "horizontal"

$\varkappa = \frac{c_v}{c_p}$

Equation of motion:

(1)
$$\rho\left[\frac{Dv}{dt} + f k \times v_h - g\right] = -\nabla p$$

Eliminating ∇p, we obtain:

$$\nabla\rho \times \left[\frac{Dv}{dt} + f k \times v_h - g\right] + \rho\nabla \times \left[\frac{Dv}{dt} + f k \times v_h\right] = 0$$

Noting that

$$\frac{\nabla\rho - \Gamma\nabla p}{\rho} = -\nabla \ln \vartheta$$

and

$$\nabla p \times \left[\frac{Dv}{dt} + f k \times v_h - g\right] = 0 ,$$

Let H be the top of the atmosphere and "—" a symbol for mean values defined from

$$\overline{\alpha} = \frac{\int_0^H I \alpha \, dz}{\int_0^H I \, dz}$$

Then, by integrating (13) between ground and the top level, one obtains:

(14) $$K = - \frac{g}{f} \overline{\nabla_h^2 \int_0^z \frac{\partial \ln \vartheta}{\partial t} dz} + \frac{\overline{J}}{f} + \overline{\frac{\Gamma(\zeta + f)\left(\frac{\partial}{\partial t} + v_h \cdot \nabla_h\right) p}{\varrho}} + \frac{I w \Big|_0^H}{\int_0^H I dz}$$

Having as boundary conditions:

For $z = 0 \begin{cases} w = 0 \\ f + \zeta > 0 \end{cases}$ (according to experience)

For $z = H \begin{cases} w = \text{finite} \\ p = 0 \\ f + \zeta > 0 \end{cases}$ (hypothesis) ,

one gets

(15) $$I w \Big|_0^H = 0$$

Combining equations (15), (14) and (7), one obtains:

(16) $$\nabla_h^2 \left(\frac{\partial \gamma}{\partial t} - g \int_0^z \frac{\partial \ln \vartheta}{\partial t} dz \right) = \overline{J} - g \overline{\nabla_h^2 \int_0^z \frac{\partial \ln \vartheta}{\partial t} dz} + \overline{\frac{f}{\varrho} (f + \zeta) \Gamma \left(\frac{\partial}{\partial t} + v_h \cdot \nabla_h \right) p}$$

An analysis of orders of magnitude reveals that the last term in this equation is $\sim 5 \, 10^{-3} \, \overline{J}$. It may, therefore, be neglected without introducing serious errors.*

The equation expressing conservation of entropy is:

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* In an appendix it is shown that the mentioned term is negligible to the same extent as c_i^2 (c_i = velocity of inertia waves) is negligible in comparison with c_s^2 (c_s = velocity of sound).

(6) $$\frac{\partial \zeta}{\partial t} = \frac{1}{f}\nabla_h^2 \frac{\partial \gamma}{\partial t} = \frac{1}{f} J\left(\frac{1}{f}\nabla_h^2 \gamma + f, \gamma\right) - w\frac{\partial(\zeta + f)}{\partial z} - (\zeta + f)\nabla_h \cdot v_h$$

One gets from the hydrostatic equation, equation (3):

$$\frac{1}{f}\nabla_h^2 \frac{\partial^2 \gamma}{\partial t \partial z} = \frac{g}{f}\nabla_h^2 \frac{\partial \ln \vartheta}{\partial t}$$

or

(7) $$\frac{1}{f}\nabla_h^2 \frac{\partial \gamma}{\partial t} = \frac{g}{f}\nabla_h^2 \int_0^z \frac{\partial \ln \vartheta}{\partial t} dz + K(x, y, t)$$

Elimination of $\frac{1}{f}\nabla_h^2 \frac{\partial \gamma}{\partial t}$ between equations (6), (7) gives:

(8) $$K = -\frac{g}{f}\nabla_h^2 \int_0^z \frac{\partial \ln \vartheta}{\partial t} dz + \frac{J}{f} - w\frac{\partial(\zeta + f)}{\partial z} - (\zeta + f)\nabla_h \cdot v_h$$

If K can be determined by quantities not involving the two last terms, equation (7) will represent the original vorticity equation, equation (6), free from the terms $-w\frac{\partial(\zeta + f)}{\partial z} - (f + \zeta)\nabla_h \cdot v_h$. From the equation of continuity

(9) $$\frac{D\rho}{dt} = -\rho \nabla \cdot v$$

and the equation of pieozotropy

(10) $$\frac{D\rho}{dt} = \Gamma \frac{Dp}{dt} = \Gamma \frac{\partial p}{\partial t} + \Gamma v_h \cdot \nabla_h p + \Gamma w \frac{\partial p}{\partial z}$$

one gets by combination

(11) $$-p^{\varkappa}\nabla_h \cdot v_h = \frac{\partial (p^{\varkappa} w)}{\partial z} + \left(\frac{\partial}{\partial t} + v_h \cdot \nabla_h\right) p^{\varkappa}$$

Multiply equation (8) by

(12) $$I = \frac{p^{\varkappa}}{(f + \zeta)^2}$$

and using equation (11), one gets:

(13) $$I K = -\frac{gI}{f}\nabla_h^2 \int_0^z \frac{\partial \ln \vartheta}{\partial t} dz + \frac{IJ}{f} + \frac{1}{(f + \zeta)}\left(\frac{\partial}{\partial t} + v_h \cdot \nabla_h\right) p^{\varkappa} + \frac{\partial I w}{\partial z}$$

by equation (1), one arrives at

$$-\frac{\nabla \vartheta}{\vartheta} \times \left[\frac{D\mathbf{v}}{dt} + f\mathbf{k}\times\mathbf{v}_h\right] + \nabla\times\left[\frac{D\mathbf{v}}{dt} + f\mathbf{k}\times\mathbf{v}_h\right] = -\nabla\times \ln\vartheta\, \mathbf{g}$$

For the motions we are interested in, the order of magnitude of the first term in this equation is 1/100 of that of the second one. Simplifying according to this the following equations of motion result:

(2) $$\frac{D\mathbf{v}_h}{dt} + f\mathbf{k}\times\mathbf{v}_h = -\nabla_h \gamma$$

(3) $$0 = -\frac{\partial \gamma}{\partial z} + g\ln\vartheta \quad ,$$

having introduced the hydrostatic assumption in the vertical component of the equation of motion.

Let

$$\zeta = \nabla_h \times \mathbf{v}_h \cdot \mathbf{k}$$

Eliminating $\nabla_h \gamma$ from equation (2) one gets,

(4) $$\frac{\partial \zeta}{\partial t} = -\mathbf{v}_h\cdot\nabla_h(\zeta+f) - w\frac{\partial(\zeta+f)}{\partial z} - (\zeta+f)\nabla_h\cdot\mathbf{v}_h$$

when the term $\nabla_h w \times \frac{\partial \mathbf{v}_h}{\partial z}$ has been considered of secondary importance.

When $\frac{D\mathbf{v}_h}{dt}$ is neglected in equation (2) and the resulting equation is solved with respect to $\mathbf{v}_h = \mathbf{v}_g$ one gets:

(5) $$\mathbf{v}_g = -\frac{\nabla_h \gamma \times \mathbf{k}}{f}$$

For an approximate evaluation of ζ , and of $\mathbf{v}_h$ in the first right-hand - side term in equation (4) we substitute $\mathbf{v}_h = \mathbf{v}_g$. f being virtually constant when the curl of $\mathbf{v}_g$ is taken, we get:

(17)
$$\frac{\partial \ln \vartheta}{\partial t} = -\mathbf{v}_h \cdot \nabla_h \ln \vartheta - w \frac{\partial \ln \vartheta}{\partial z}$$

We substitute here $\mathbf{v}_h = \mathbf{v}_g$. Then

(17')
$$\frac{\partial \ln \vartheta}{\partial t} = -\mathbf{v}_g \cdot \nabla_h \ln \vartheta - w \frac{\partial \ln \vartheta}{\partial z}$$

Suppose now that it is allowed as a first approximation to assume that the advection of entropy is a horizontal one:

(18)
$$\frac{\partial \ln \vartheta}{\partial t} = -\mathbf{v}_g \cdot \nabla_h \ln \vartheta - w \frac{\partial \ln \vartheta}{\partial z} \rightarrow -\mathbf{v}_g \cdot \nabla_h \ln \vartheta$$

From equations (5) and (3) we get:

(19)
$$\mathbf{v}_g \cdot \nabla_h \ln \vartheta = \frac{f}{g} \mathbf{v}_g \times \frac{\partial \mathbf{v}_g}{\partial z} \cdot \mathbf{k}$$

Then from equations (18) and (19):

$$\frac{\partial \ln \vartheta}{\partial t} = -\frac{f}{g} \mathbf{v}_g \times \frac{\partial \mathbf{v}_g}{\partial z} \cdot \mathbf{k} = -\frac{1}{gf} \nabla_h \gamma \times \nabla_h \frac{\partial \gamma}{\partial z} \cdot \mathbf{k} = -\frac{1}{gf} \cdot J\left(\gamma, \frac{\partial \gamma}{\partial z}\right)$$

Substituting this in equation (16) we get, ignoring the term mentioned as negligible:

(20)
$$\nabla_h^2 \left[\frac{\partial \gamma}{\partial t} + \frac{1}{f} \int_0^z J\left(\gamma, \frac{\partial \gamma}{\partial z}\right) dz\right] = \overline{J\left(\frac{1}{f} \nabla_h^2 \gamma + f, \gamma\right) + \frac{1}{f} \nabla_h^2 \int_0^z J\left(\gamma, \frac{\partial \gamma}{\partial z}\right) dz}$$

APPENDIX

A :

In the relation

$$\nabla_h^2 \frac{\partial \gamma}{\partial t} = f \, \nabla_h \times \frac{\partial \mathbf{v}_g}{\partial t} \cdot \mathbf{k}$$

we substitute the standard expression for $\mathbf{v}_g$:

$$\mathbf{v}_g = - \frac{\nabla_h p \times \mathbf{k}}{f \rho}$$

ρ may be considered as a virtual constant under the time differentiation $\frac{\partial}{\partial t}$ and the curl operator $\nabla_h \times$. Considering in addition, as before, f as virtual constant when the curl is taken, we get:

(1) $$\nabla_h^2 \frac{\partial \gamma}{\partial t} = \frac{\nabla_h^2 \frac{\partial p}{\partial t}}{\rho}$$

We have, as regards orders of magnitude:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_h \cdot \nabla_h\right) p \sim \frac{\partial p}{\partial t}$$

$$f + \zeta \sim f$$

$$\overline{\frac{f(f+\zeta)\Gamma\left(\frac{\partial}{\partial t} + \mathbf{v}_h \cdot \nabla_h\right) p}{\rho}} \sim \frac{f^2 \Gamma}{\rho} \frac{\partial p}{\partial t}$$

On the other side, from equation (1) we get

$$\nabla_h^2 \frac{\partial \gamma}{\partial t} \sim \frac{4\pi^2}{L_x^2 + L_y^2} \cdot \frac{\frac{\partial p}{\partial t}}{\rho} \sim \frac{2\pi^2}{L^2} \frac{\frac{\partial p}{\partial t}}{\rho}$$

when a periodic dependency upon x and y has been assumed with wavelengths L_x and $L_y \sim L_x$, respectively. One has

$$\nabla_h^2 \frac{\partial \gamma}{\partial t} \sim \overline{J}$$

Therefore, if C_s denotes the velocity of sound, and C_i that of inertia waves, one gets

$$\overline{\frac{1}{\rho} f(f+\zeta)\,\Gamma\left(\frac{\partial}{\partial t} + v_h \cdot \nabla_h\right) p} \sim \frac{1}{2}\,\frac{c_i^2}{c_s^2}\,\overline{J} \sim 5 \times 10^{-3}\,\overline{J}$$

B :

The mean values occurring in the previous equations were defined from:

(2)

$$\overline{\alpha} = \frac{\int_0^H I \alpha \, dz}{\int_0^H I \, dz}$$

$$I = \frac{p^{\varkappa}}{(\zeta + f)^2}$$

Using the hydrostatic equation one gets when

$$\frac{\partial \vartheta}{\partial z} = 0$$

$$\rho \sim p^{\varkappa}$$

In this case one may, therefore, as well define the mean values from:

(3)

$$\overline{\alpha} = \frac{\int_0^H I \alpha \, dz}{\int_0^H I \, dz}$$

$$I = \frac{\rho}{(\zeta + f)^2}$$

In the atmosphere generally

$$\frac{\partial \vartheta}{\partial z} \sim +5\,^{\circ}C / km$$

To see what we would then get if $p^{\varkappa}$ is replaced by ϱ , we return to equation (11):

$$p^{\varkappa} \nabla_h \cdot v_h = - \frac{\partial p^{\varkappa} w}{\partial z} - p^{\varkappa} \Gamma \frac{\left(\frac{\partial}{\partial t} + v_h \cdot \nabla\right) p}{\varrho}$$

We may also write:

$$\varrho \nabla_h \cdot v_h = - \frac{\partial \varrho w}{\partial z} - \varrho \Gamma \frac{\left(\frac{\partial}{\partial t} + v_h \cdot \nabla_h\right) p}{\varrho} + \varrho w \frac{\partial \ln \vartheta}{\partial z}$$

Therefore, by using ϱ instead of $p^{\varkappa}$ we will get an additional term

$$\overline{f(f+5)\, w \frac{\partial \ln \vartheta}{\partial z}}$$

on the right-hand side of equation (16). This is in general not a negligible term. Therefore, using the definition (2) for the mean values instead of (3) will result in more simple equations.