MC 241
Box 1 Folder 26

Energy Losses Attending Thermionic Emission of Electrons from Metals, 1940

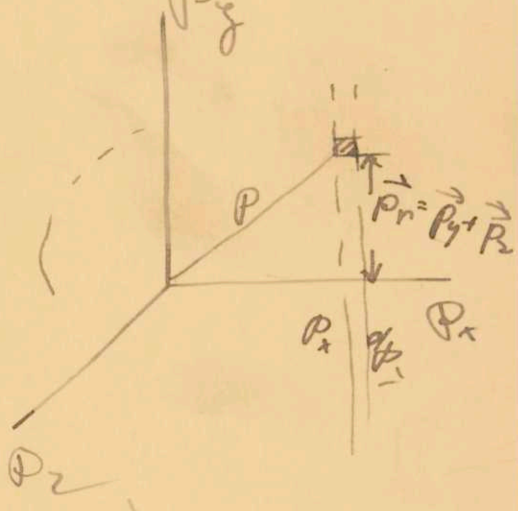
ⓐ

$$\frac{2}{h^3}\left[\frac{2\pi p_r\,dp_r\,dp_x}{e^{\frac{\frac{p^2}{2m}-W_i}{kT}}+1}\right]$$

F. + H. Eqn on p 892 wrong as shown below.
∴ All that follows is wrong in detail.

$$p_r\,dp_r = p\,dp - p_x\,dp_x$$

$$\frac{4\pi}{h^3}\,\frac{p\,dp\,dp_x - p_x(dp_x)^2}{e^{\frac{\frac{p}{2m}-W_i}{kT}}+1}$$




$$m v = p$$
$$m\,dv = dp$$
$$m\,dv_x = dp_x$$

$$\frac{4\pi}{h^3}\,\frac{m^3\left(v\,dv\,dv_x - v_x(dv_x)^2\right)}{e^{\frac{\frac{1}{2}mv^2-W_i}{kT}}+1}$$

(b)

$$\cancel{\frac{2m^3}{h^3}}$$

$$\frac{8\pi m^3}{h^3}\frac{v^2\,dv}{e^{-(\mu-E)/kT}+1}\times\frac{dv_x}{2v}$$

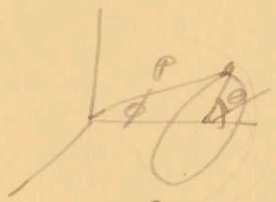
$$\frac{4\pi m^3}{h^3}\frac{v\,dv\,dv_x}{e^{-(\mu-E)/kT}+1}$$

Form used by Fleming and Henderson

Should be

$$\frac{4\pi m^3}{h^3}\frac{\left\{v\,dv\,dv_x-\underline{v_x(dv_x)^2}\right\}}{e^{-\frac{\mu-E}{kT}}+1}$$

Pages (a) and (b) above are not correct in that the slice which F + H are working with is not the shape assumed. This may be shown by computing their eqn (7) by polar coordinates.



current density for range $E$ to $E+dE$

$$= \frac{2}{h^3}\int_{\theta=0}^{\theta=2\pi}\int_{\phi=0}^{\phi=\frac{\pi}{2}} \frac{p^2\, d\phi\, p\sin\phi\, d\theta\, dp \times \frac{p\cos\phi}{m}}{e^{\frac{E-\mu}{kT}}+1}$$

$$= \frac{2\times\cancel{2}\pi\times\frac{1}{2}}{h^3}\,\frac{p^3\,dp\,\frac{1}{m}}{e^{\frac{E-\mu}{kT}}+1}$$

$$\frac{p^2}{2m} = E \qquad p^2 = 2mE$$

$$p\,dp = m\,dE$$

$$p^3\,dp = 2m^2 E\,dE$$

$$= \frac{4\pi m}{h^3}\,\frac{E\,dE}{e^{\frac{E-\mu}{kT}}+1}$$

at $T = 0°K$

~~current~~

$$I(E)_0\,dE = \frac{4\pi m}{h^3}\,E\,dE \quad \text{for } E<\mu$$

(d)

this checks eq. 7.

to get Eq 8.

In the extention in phase there are $\frac{2}{h^3} dp_x dp_y dp_z$ quantum states per unit vol. The probability that one is occupied is

$$\frac{1}{e^{\frac{E-\mu}{kT}}+1}$$

~~∴ The probability that it is **not** occupied is~~

$$\cancel{1 - \frac{1}{e^{\frac{E-\mu}{kT}}+1} = \frac{e^{\frac{E-\mu}{kT}}}{e^{\frac{E-\mu}{kT}}+1}}$$

The number occupied is

$$\frac{2}{h^3}\frac{dp_x dp_y dp_z}{e^{\frac{E-\mu}{kT}}+1}$$

(No page e) (f)

The number not occupied

is $\frac{2}{h^3} dp_x dp_y dp_z \left(1 - \frac{1}{e^{\frac{E-\mu}{kT}}+1}\right)$

The <u>fraction</u> not occupied is

$$f = 1 - \frac{1}{e^{\frac{E-\mu}{kT}}+1} = \frac{e^{\frac{E-\mu}{kT}}}{e^{\frac{E-\mu}{kT}}+1}$$

This is equation (8) and gives the fraction of energy states for a small range $dE$ at $E$ which are <u>not</u> <u>occupied</u>.

The current density for the energy range $dE$ at $E$ is

$$\frac{4\pi m}{h^3} E\,dE \text{ for } 0°K$$

This might be said to be proportional to the average velocity $\overline{v_x} \cdot \rho$ the density

This ~~th~~ average velocity $\overline{v_x}$ should depend on E but be independent of $\rho$

$$\overline{v_x} = \frac{1}{\rho_0} \times \frac{4\pi m}{h^3} E\,dE$$

where $\rho_0$ = density at 0°K and is of course a function of E.

At the temperature T the density is $\rho_0(1-f)$

$\therefore$ the current density is

$$\overline{v_x}\,\rho_0(1-f) = (1-f)\frac{4\pi m}{h^3} E\,dE$$

and the decrease in current density

is $f\frac{4\pi m}{h^3}E\,dE = \frac{4\pi m}{h^3} E \frac{e^{\frac{E-\mu}{kT}}}{e^{\frac{E-\mu}{kT}}+1} dE$

which is Eq. 9

Consider a metal in a temperature gradient with $T = f(x)$ only.

then the current density of electrons with energy $E$ to $E+dE$ <u>changes</u> with temperature from 0°K from

$$\frac{4\pi m}{h^3} E\,dE \xrightarrow{\text{by the amount}} \frac{4\pi m}{h^3} E \frac{e^{\frac{E-\mu}{kT}}}{e^{\frac{E-\mu}{kT}}+1} dE$$

as we progress along the metal in the direction of increasing $T$.

This is good only for $E < \mu$.

~~Another way of putting it is that the above measures the current of holes if we write~~

$$\cancel{\frac{4\pi m}{h^3} E \left( 1 - \frac{e^{\frac{E-\mu}{kT}}}{\phantom{x}} \right.}$$

$$I_h^{T}(E)\,dE = \frac{4\pi m}{h^3} E \frac{e^{\frac{E-\mu}{kT}}\,dE}{e^{\frac{E-\mu}{kT}}+1}$$ must be equal

to the "current density of holes".

This is "random" current in the sense that at any boundary there is an equal flow of "hole" current in the opposite direction to the +x direction except insofar as the temperature gradient $\frac{dT}{dx}$ is so great that there is a measureable change in $I_h(E,T)$ in the range $\Delta x$.

There would tend to be a net current flow of holes at a given energy E of

$$\int_{T_1}^{T_2} \frac{\partial I_h}{\partial T}\,dT.$$

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As we progress along the metal in the direction of increasing temp, there are more transitions out of a given energy band $dE$ at $E$ than there are into it. Following in the opposite direction the reverse is true.

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In order to calculate the contribution to the atomic heat of a metal resulting from the electron transitions it seems to me that at a given temperature the density of holes (not the current) could be calculated and the density of occupied energy levels above $\mu$ could be calculated. Then the average energy of the occupied states above $\mu$ might be equal to $\bar{\varepsilon}_-$ and that of the holes $-\bar{\varepsilon}_0$ relative to $\mu$. or on the basic scale of $E$ we would have $\bar{E}_- = \mu + \bar{\varepsilon}_-$ and $\bar{E}_0 = \mu - \bar{\varepsilon}_0$

average energy put in would be

$$\bar{E}_{-} - \bar{E}_{0} = \bar{\varepsilon}_{-} + \bar{\varepsilon}_{0} \equiv \bar{\varepsilon}_{-0}$$

If $n$ electrons made the transision ~~the~~ the total energy put in is $n\bar{\varepsilon}_{-0}$ and if there are $N$ elect. per mole of the metal then

$$\frac{n\bar{\varepsilon}_{-0}}{N} = \text{average energy change}$$

per electron

The contribution to the atomic heat at constant vol. would be

$$\bar{C}_v = \frac{d\left(\frac{n\bar{\varepsilon}_{-0}}{N}\right)}{dT} = \frac{1}{N}\left(\frac{dn}{dT}\varepsilon_{-0} + n\frac{d\bar{\varepsilon}_{-0}}{dT}\right)$$

F and H calculate what seems to me to be the average energy carried across a boundary by the "holes" when they write

$$\frac{\displaystyle\int_0^{\mu} \frac{E^2\, e^{\frac{E-\mu}{kT}}\, dE}{e^{E-\mu/kT}+1}}{\displaystyle\int_0^{\mu} \frac{E\, e^{\frac{E-\mu}{kT}}\, dE}{e^{\frac{E-\mu}{kT}}+1}}$$

11/29/40

carry out integration!

let $x = -\dfrac{E-\mu}{kT}$ $\qquad \mu - kTx = E = \mu\left(1-\dfrac{kTx}{\mu}\right)$

$dx = -\dfrac{dE}{kT}$ $\qquad E^2 = \mu^2(\ -\ )^2$

when $E = 0 \quad x = \dfrac{\mu}{kT}$

$E = \mu \quad x = 0$

$$\overline{E}_0 = \frac{\displaystyle \cancel{kT}\int_{x=0}^{x=\mu/kT} \frac{\mu^{\cancel{2}}\left(1-\frac{kTx}{\mu}\right)^2 e^{-x}\, dx}{e^{-x}+1}}{\displaystyle \cancel{\mu kT}\int_{x=0}^{x=\mu/kT} \frac{\left(1-\frac{kTx}{\mu}\right) e^{-x}\, dx}{e^{-x}+1}} = \frac{\mu\displaystyle\int_0^{\mu/kT} \frac{e^{-x}\,dx}{e^{-x}+1}}{\displaystyle\int_0^{\mu} \frac{e^{-x}\,dx}{e^{-x}+1}} - \frac{2kT\displaystyle\int_0^{\mu/kT} \frac{xe^{-x}\,dx}{e^{-x}+1}}{\displaystyle\int \frac{e^{-x}}{e^{-x}+1}\,dx} +$$

$$\longrightarrow + \left(\frac{kT}{\mu}\right)^2 \mu \frac{\displaystyle\int_0^{\mu} \frac{x^2 e^{-x}\,dx}{e^{-x}+1}}{\displaystyle\int_0^{\mu} \frac{e^{-x}\,dx}{e^{-x}+1}}$$

(m)

$$\overline{E_0} = \mu - 2kT \frac{\displaystyle\int_0^{\mu} \frac{x e^{-x}\, dx}{e^{-x}+1}}{\displaystyle\int_0^{\mu} \frac{e^{-x}\, dx}{e^{-x}+1}} + \frac{k^2T^2}{\mu} \frac{\displaystyle\int_0^{\mu} \frac{x^2 e^{-x}\, dx}{e^{-x}+1}}{\displaystyle\int_0^{\mu} \frac{e^{-x}\, dx}{e^{-x}+1}}$$

$$\int_0^{\mu/kT} \frac{x e^{-x}\, dx}{e^{-x}+1} = \int_0^{\mu/kT} \frac{x\, dx}{e^{x}+1}$$

See Peirce (409) on notes on photoeffect p 110

$$I_0 = \int_0^{\mu/kT} \frac{dx}{1+e^{x}} = -\ln(1+e^{-x})\Big|_0^{\mu/kT} = \ln 2 - \ln(1+e^{-\mu/kT})$$

$$\int_0^{\mu/kT} \frac{x\, dx}{e^{x}+1} = x I_0 - \int_0^{\mu/kT} I_0\, dx$$

$$I_1 = -x \ln(1+e^{-x})\Big|_0^{\mu/kT} + \int_0^{\mu/kT} \ln(1+e^{-x})\, dx = -\frac{\mu}{kT}\ln(1+e^{-\mu/kT}) + \int_0^{\mu/kT} \ln(1+e^{-x})\, dx$$

$$-\frac{\mu}{kT}\left(e^{-\mu/kT} - \frac{e^{-2\mu/kT}}{2} - - -\right)$$

$$-\left(e^{-x} - \frac{e^{-2x}}{2^2} + \frac{e^{-3x}}{3^2} - \right)\Big|_0^{\mu/kT}$$

$$= \frac{\pi^2}{12} - \left(\frac{\mu}{kT}+1\right)e^{-\mu/kT} + \frac{1}{2}e^{-2\mu/kT}\left(\frac{\mu}{kT}+\frac{1}{2}\right) - \frac{e^{-3\mu/kT}}{3}\left(\frac{\mu}{kT}+\frac{1}{3}\right) + \cdots$$

$$e^{-x} - \frac{e^{-2x}}{2} + \frac{e^{-3x}}{3}$$

$$\overline{E}_0 = \frac{\mu \int_0^{\mu/kT} \frac{\left(1 - \frac{kTx}{\mu}\right) e^{-x} dx}{e^{-x}+1} \,_{\wedge}^{(2-1)} kT \int_0^{\mu/kT} \frac{x e^{-x} dx}{e^{-x}+1} + \left(\frac{kT}{\mu}\right)^2 \int_0^{\mu/kT} \frac{x^2 e^{-x} dx}{e^{-x}+1}}{\int_0^{\mu/kT} \frac{\left(1 - \frac{kTx}{\mu}\right) e^{-x} dx}{e^{-x}+1}}$$

(n)

$$= \mu - \frac{+kT\left(\frac{\pi^2}{12} - 10^{-20}\right)}{\ln 2 - \frac{kT}{\mu}\frac{\pi^2}{12}}$$

$$\doteq \mu - \frac{\frac{\pi^2}{12} kT}{\ln 2\left(1 - \frac{kT}{\mu}\frac{\pi^2}{12 \ln 2}\right)}$$

$$\frac{1}{20}\frac{\pi^2}{12 \cdot .69}$$

$$= \mu - \frac{\overset{.823}{\frac{\pi^2}{12}} kT \left(1 + \frac{kT}{\mu}\frac{\pi^2}{12 \ln 2}\right)}{\ln 2}$$

.04

.06

$$= \mu - \frac{.86}{\ln 2} kT \doteq \mu - 1.24\, kT$$

This equation

$$\overline{E_0} = \mu - 1.2\,kT$$

is Eq 10 and gives the average energy associated with the flow of "holes" across a boundary. Although this result is not exact it is near enough for all practical purposes when $\mu \sim 5$ ev. and $kT < .2$ ev.

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Eq. 11 of F+H cannot be right because it has the dimensions of ~~E and not~~

$$\frac{ME}{h^3} = \frac{MLM\frac{L}{T^2}}{\frac{M^3L^3}{T^3} \times L^3} = M^{-1}L^{-4}T$$

instead of

$$\frac{ME^2}{h^3} = M^{-1}L^{-4}T \times ML^2T^{-2}$$

$$= L^{-2}T^{-1}$$ number per cm² per sec.

P

Eq. ⑪ may be justified from

current density =

$$= \frac{2}{h^3} \frac{\int_{\theta=0}^{\theta=2\pi} \int_{\phi=0}^{\phi=\frac{\pi}{2}} \frac{p^3}{m} \sin\phi \cos\phi \; D(W) \, d\phi \, d\theta \, dp}{e^{\frac{E-\mu}{kT}}+1}$$

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$$W = \frac{p^2}{2m} \cos^2\phi$$

$$dW = -\frac{p^2}{m} \cos\phi \sin\phi \, d\phi$$

$$m \, dE = p \, dp$$

$$\phi = 0 \quad W = E$$
$$\phi = \frac{\pi}{2} \quad W = 0$$

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$$= \frac{4\pi m}{h^3} \frac{dE}{e^{\frac{E-\mu}{kT}}+1} \int_0^E D(W) \, dW$$

This is the electron-emission current in number per $cm^2$ per sec. with total energy within $dE$ at $E$

There is no direct limitation in the use of this formula for all values of $E$.

To obtain the energy lost one should multiply this current by $(E-\mu)$ since for all practical purposes the electrons flow <u>in</u> at $\mu$. F+H use instead of $\mu$, $(\mu - 1.2kT)$ which is the average energy <u>carried</u> <u>across a boundary</u> by the <u>holes</u>.

Further more they integrate the expression from $\mu$ to $\infty$ and it is hard to see how this measures the average energy of the field emission electrons since some of these (in fact most of them) have energy below $\mu$.

The calculation is further complicated by the choice of limits in Eq 13 since the range for E in the denominator is from E = 0 to $\infty$ as one might also expect above.

It seems to me that

$$\omega = \frac{\displaystyle\int_{E=0}^{E=\infty} \frac{(E-\mu)\,dE}{e^{\frac{E-\mu}{kT}}+1} \int_0^E D(W)\,dW}{\displaystyle\int_{E=0}^{E=\infty} \frac{dE}{e^{\frac{E-\mu}{kT}}+1} \int_0^E D(W)\,dW}$$

Let $y = \dfrac{E-\mu}{kT}$ $\qquad kTy + \mu = E$

$\qquad\qquad\qquad\qquad kT\,dy = dE$

$$\bar{\omega} = \frac{\displaystyle (kT)\int_{-\frac{\mu}{kT}}^{\infty} \frac{y\,dy}{e^{y}+1} \int_0^{kTy+\mu} D(W)\,dW}{\displaystyle \cancel{kT}\int_{-\frac{\mu}{kT}}^{\infty} \frac{dy}{e^{y}+1} \int_0^{kTy+\mu} D(W)\,dW}$$

For thermionic case

assume $D(W) = 0$ for $W < C$

$D(W) = 1$ " $W > C$ $\quad \phi = C - \mu$

$$\overline{\omega} = \frac{kT\int_{\phi/kT}^{\infty} y e^{-y} dy \int_{\phi+\mu}^{kTy+\mu} dW}{\int_{\phi/kT}^{\infty} e^{-y} dy \int_{\phi+\mu}^{kTy+\mu} dW} = \frac{kT\int_{\phi/kT}^{\infty}(kTy-\phi) y e^{-y} dy}{\int_{\phi/kT}^{\infty} e^{-y} dy (kTy-\phi)}$$

$$\int y^2 e^{-y} dy = -y(y+1)e^{-y} - (y+1)e^{-y} - e^{-y}$$

$$\int y e^{-y} dy = -ye^{-y} + \int e^{-y} = -ye^{-y} - e^{-y}$$

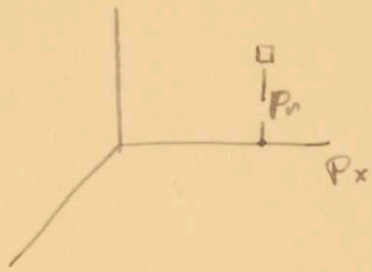
$$= -e^{-y}(y+1)\Big|_{\phi/kT}^{\infty} = \left(\frac{\phi}{kT}+1\right)e^{-\frac{\phi}{kT}}$$

$$\int y^2 e^{-y} dy = -\underline{y^2 e^{-y}} + 2\int y e^{-y} dy$$

$$= -y^2 e^{-y} - 2e^{-y}(y+1)\Big|_{\phi/kT}^{\infty} = \frac{\phi^2}{(kT)^2} e^{-\phi/kT} + 2e^{-\frac{\phi}{kT}}\left(\frac{\phi}{kT}+1\right)$$

$$\overline{\omega} = \frac{kT\left[\cancel{\frac{\phi^2 e^{-\phi/kT}}{kT}} + \cancel{2\phi e^{-\phi/kT}} + 2kTe^{-\phi/kT} - \cancel{\frac{\phi^2}{kT}} - \cancel{\phi}\right]}{\cancel{\phi} + kT - \cancel{\phi}} = \phi + 2kT$$

$$\rho = \frac{2}{h^3}\,\frac{1}{e^{\frac{\frac{p^2}{2m}-W_i}{kT}}+1}$$




$$f(p,p_x,p_r)\,d\ldots = \frac{2}{h^3}\,\frac{\frac{a\,p_x\,\Delta t}{m}\;2\pi\,p_r\,dp_r\,dp_x}{e^{\frac{\frac{p^2}{2m}-W_i}{kT}}+1}$$

= no. of el. crossing area a in time $\Delta t$
with total momentum $p$ to $p+dp$ and momentum along x of $p_x$ to $p_x+dp_x$.

with $p_r^2 \equiv p^2 - p_x^2$

$$p_r\,dp_r = p\,dp - p_x\,dp_x$$

$$= \frac{4\pi}{h^3}\,\frac{\frac{a\,\Delta t\,p_x}{m}\left(p\,dp - p_x\,dp_x\right)dp_x}{e^{\frac{p^2/2m-W_i}{kT}}+1}$$

$$E = \frac{p^2}{2m} \qquad W_x = \frac{p_x^{\,2}}{2m}$$

$$dE = \frac{p\,dp}{m} \qquad dW_x = \frac{p_x\,dp_x}{\cancel{2}m}$$

$$\cancel{\frac{2m\,dW_x}{p_x} = dp_x}$$

$$\cancel{\frac{2m\,dW_x}{(2mW_x)^{1/2}} = dp_x}$$

$$= \frac{4\pi}{h^3}\,\frac{a\,\Delta t\,m\,(dE - dW_x)\,dW_x}{e^{\frac{E-W_i}{kT}} + 1}$$

= no. of el. crossing a in $\Delta t$ with total energy $E$ to $E+dE$ and x energy $W_x$ to $W_x + dW_x$.

Energy carrying out $(E - W_i)$

$$\overline{w} = \frac{\displaystyle\int_{W_x=W_a}^{\infty}\int_{E=W_x}^{E=\infty} e^{-\left(\frac{E-W_i}{kT}\right)}(E-W_i)\,(dE - dW_x)\,dW_x}{\displaystyle\int_{W_a}^{\infty}\int_{W_x}^{\infty} e^{-\frac{E-W_i}{kT}}\,(dE - dW_x)\,dW_x}$$

$$X = E - W_x \qquad E = X + W_x$$

$$(E - w_i) = x + w_x - w_i$$

$$d\cancel{X} = dE - dW_x$$

$$\begin{cases} E = W_x \\ X = 0 \end{cases}$$

$$\frac{-kT \int_{w_a}^{\infty}\int_0^{\infty} e^{-\frac{x}{kT}} e^{-\frac{W_x - W_i}{kT}} (x + w_x - w_i) \frac{dx}{-kT} dW_x}{\int_{w_a}^{\infty}\int_0^{\infty} e^{-\frac{x}{kT}} e^{-\frac{W_x - W_i}{kT}} dx\, dW_x}$$

$$-kT \left\{ x e^{-\frac{x}{kT}} \Big|_0^{\infty} - \int_0^{\infty} e^{-\frac{x}{kT}} dx \right\}$$

$$+kT + \frac{kT \int_{w_a}^{\infty}\int_0^{\infty} e^{-\frac{x}{kT}} \frac{dx}{kT} (w_x - w_i) e^{-\frac{w_x - w_i}{kT}} dw_x}{\iint}$$

④

$$+kT - W_i + \frac{kT\iint_{x=0}^{\infty} e^{-\frac{x}{kT}} W_x \frac{dx}{kT} e^{-\frac{W_x}{kT}} dW_x}{\iint_{x=0}^{\infty} e^{-\frac{x}{kT}} dx\, e^{-\frac{W_x}{kT}} dW_x}$$

$$+ \frac{kT\int_{W_a}^{\infty} W_x e^{-\frac{W_x}{kT}} \frac{dW_x}{kT}}{\int_{W_a}^{\infty} e^{-\frac{W_x}{kT}} dW_x}$$

$$\frac{-kT\left\{ W_x e^{-\frac{W_x}{kT}} \Big|_{W_a}^{\infty} + \cancel{kT}\int_{W_a}^{\infty} e^{-\frac{W_x}{kT}} dW_x \right\}}{}$$

$$+2kT - W_i + \frac{kT W_a e^{-\frac{W_a}{kT}}}{-kT\int_{W_a}^{\infty} e^{-\frac{W_x}{kT}} \frac{dW_x}{-kT}}$$

$$e^{-\frac{W_x}{kT}} \Big|_{W_a}^{\infty}$$

$$W_a - W_i + 2kT$$

$$di' = \frac{4\pi}{h^3} \frac{m\,(dE - dW_x)\,dW_x}{e^{\frac{E - W_i}{kT}} + 1}$$

assume transmission $\left(1 - e^{-\frac{W_x - W_a}{R}}\right)$

$$di_R' = \frac{4\pi}{h^3}\, m\, e^{-\frac{E - W_i}{kT}} (dE - dW_x)\, dW_x \left(1 - e^{-\frac{W_x - W_a}{R}}\right)$$

$\cancel{\overline{\omega} = W_a - W_i + kT -}$

$$\overline{\omega} = \frac{\displaystyle\int_{W_a}^{\infty}\!\!\int_{W_x}^{\infty} (E - W_i)\, e^{-\frac{E - W_i}{kT}} \left(1 - e^{-\frac{W_x - W_a}{R}}\right) dW_x\,(dE - dW_x)}{\displaystyle\int_{W_a}^{\infty}\!\!\int_{W_x}^{\infty} e^{-\frac{E - W_i}{kT}} \left(1 - e^{-\frac{W_x - W_a}{R}}\right) dW_x\,(dE - dW_x)}$$

$$X = E - W_x$$

$$dX = dE - dW_x$$

$$\overline{\omega} = -W_i + \frac{\displaystyle\int_{W_a}^{\infty}\!\!\int_{0}^{\infty} (W_x + X)\, e^{-\frac{X + W_x - W_i}{kT}} \left(1 - e^{-\frac{W_x - W_a}{R}}\right) dW_x\, dX}{\displaystyle\int_{W_a}^{\infty}\!\!\int_{0}^{\infty} e^{-\frac{X + W_x - W_i}{kT}} \left(1 - e^{-\frac{W_x - W_a}{R}}\right) dW_x\, dX}$$

$$\cancel{\int_{W_a}^{\infty}\left(1-e^{-\frac{W_x-W_a}{R}}\right)dW_x\left[\int x_x e^{-}\right.}$$

$$\bar{\omega} = -W_i + \frac{\displaystyle\int_{W_a}^{\infty}\left(1-e^{-\frac{W_x-W_a}{R}}\right)e^{-\frac{W_x-W_i}{kT}}dW_x\left[W_x\int_0^{\infty}e^{-\frac{x}{kT}}dx + \int_0^{\infty}xe^{-\frac{x}{kT}}dx\right]}{\displaystyle\int_{W_a}^{\infty}\left(1-e^{-\frac{W_x-W_a}{R}}\right)e^{-\frac{W_x-W_i}{kT}}dW_x\left[\int_0^{\infty}e^{-\frac{x}{kT}}dx\right]}$$

$$\int_0^{\infty}e^{-\frac{x}{kT}}dx = kT$$

$$=$$

$$\int_0^{\infty}xe^{-\frac{x}{kT}}dx = (kT)^2$$

$$\bar{\omega} = -W_i + \frac{\cancel{kT}\displaystyle\int_{W_a}^{\infty}(kT+W_x)\left(1-e^{-\frac{W_x-W_a}{R}}\right)e^{-\frac{W_x-W_i}{kT}}dW_x}{\cancel{kT}\displaystyle\int_{W_a}^{\infty}\left(1-e^{-\frac{W_x-W_a}{R}}\right)e^{-\frac{W_x-W_i}{kT}}dW_x}$$

$$\bar{w} = -W_i + \frac{\int_{W_a}^{\infty} kT e^{-\frac{W_x-W_i}{kT}} dW_x + \int_{W_a}^{\infty} W_x e^{-\frac{W_x-W_i}{kT}} dW_x - \int_{W_a}^{\infty} kT e^{-\frac{W_x-W_a}{R}} e^{-\frac{W_x-W_i}{kT}} dW_x - \int_{W_a}^{\infty} W_x e^{-\frac{W_x-W_a}{R}} e^{-\frac{W_x-W_i}{kT}} dW_x}{\int_{W_a}^{\infty} e^{-\frac{W_x-W_i}{kT}} dW_x - \int_{W_a}^{\infty} e^{-\frac{W_x-W_a}{R}} e^{-\frac{W_x-W_i}{kT}} dW_x}$$

$$-kT\int e^{-\frac{W_x-W_i}{kT}}\left(-\frac{dW_x}{kT}\right) = -kT e^{-\frac{W_x-W_i}{kT}}\Big|_{W_a}^{\infty} = kT e^{-\frac{W_a-W_i}{kT}}$$

$$+(kT)^2 e^{\frac{W_i}{kT}} \int_{W_a}^{\infty} \left(-\frac{W_x}{kT}\right) e^{-\frac{W_x}{kT}} \left(-\frac{dW_x}{kT}\right) = +kT e^{+\frac{W_i}{kT}} \left[ \frac{W_a}{kT} e^{-\frac{W_a}{kT}} - \int_{W_a}^{\infty} e \right.$$

$$\int_{-\frac{W_a}{kT}}^{-\infty} Y e^{Y} dY = Y e^{Y}\Big|_{-\frac{W_a}{kT}}^{-\infty} - \int_{-\frac{W_a}{kT}}^{\infty} e^{Y} dY = \frac{W_a}{kT} e^{-\frac{W_a}{kT}} + e^{-\frac{W_a}{kT}}$$

$$= \left(\frac{W_a}{kT} + 1\right) e^{-\frac{W_a}{kT}}$$

$$\overline{w} = -W_i + \frac{(kT)^2\left[e^{-\frac{W_a-W_i}{kT}} + \left(\frac{W_a}{kT}+1\right)e^{-\frac{W_a-W_i}{kT}}\right] - (kT)^2\int_{W_0}^{\infty} e^{-\frac{W_x-W_a}{R}}\, e^{-\frac{W_x-W_i}{kT}}\,\frac{dW_x}{kT} - (kT)^2\int_{W_0}^{\infty}\frac{W_x\, e\, e\, dW_x}{(kT)^2} \cdots}{kT\, e^{-\frac{W_a-W_i}{kT}} - kT\int_{W_0}^{\infty} e^{-\frac{W_x-W_a}{R}}\, e^{-\frac{W_x-W_i}{kT}}\,\frac{dW_x}{kT}}$$

$$= -W_i + \frac{kT\left(\frac{W_a}{kT}+2\right)e^{-\frac{W_a-W_i}{kT}} - kT\int_{W_a}^{\infty} e^{-\frac{W_x-W_a}{R}}\, e^{-\frac{W_x-W_i}{kT}}\,\frac{dW_x}{kT} - kT\int_{W_a}^{\infty}\frac{W_x}{kT}\, e^{-\frac{W_x-W_a}{R}}\, e^{-\frac{W_x-W_i}{kT}}\,\frac{dW_x}{kT}}{e^{-\frac{W_a-W_i}{kT}} - \int_{W_0}^{\infty} e^{-\frac{W_x-W_a}{R}}\, e^{-\frac{W_x-W_i}{kT}}\,\frac{dW_x}{kT}}$$

Let $\quad \dfrac{W_x - W_a}{kT} = y \qquad\qquad \dfrac{W_x}{kT} = y + \dfrac{W_a}{kT}$

$$\frac{W_x - W_i}{kT} = y + \frac{W_a - W_i}{kT}$$

$$\frac{W_x - W_a}{R} = y\left(\frac{kT}{R}\right)$$

$$\overline{w} = -W_i + \frac{kT\left(\frac{W_a}{kT}+2\right)e^{-\frac{W_a-W_i}{kT}} - kT\, e^{-\frac{W_a-W_i}{kT}}\left[\int_0^{\infty} e^{-y\left(\frac{kT}{R}\right)}\, e^{-y}\, dy + \int_0^{\infty}\left(y+\frac{W_a}{kT}\right)e^{-y\left\{\left(\frac{kT}{R}\right)+1\right\}}\, dy\right]}{e^{-\frac{W_a-W_i}{kT}} - e^{-\frac{W_a-W_i}{kT}}\int_0^{\infty} e^{-y\left(\frac{kT}{R}+1\right)}\, dy}$$

$$\overline{\omega} = -W_i + \frac{kT\left(\frac{W_a}{kT}+2\right) - kT\int_0^\infty e^{-y\left(\frac{kT}{R}+1\right)}\,dy - kT\int\left(y+\frac{W_a}{kT}\right)e^{-y\left(\frac{kT}{R}+1\right)}\,dy}{1 - \cancel{k}\int_0^\infty e^{-y\left(\frac{kT}{R}+1\right)}\,dy}$$

$$= -W_i + kT\,\frac{\left(\frac{W_a}{kT}+2\right) - \left(1+\frac{W_a}{kT}\right)\int_0^\infty e^{-\left(\frac{kT}{R}+1\right)y}\,dy - \int_0^\infty y\,e^{-y\left(\frac{kT}{R}+1\right)}\,dy}{1 - \frac{1}{\frac{kT}{R}+1}\int_0^\infty e^{-\alpha}\,(-d\alpha)}$$

$$= -W_i + kT\,\frac{\left(\frac{W_a}{kT}+2\right) - \frac{\left(1+\frac{W_a}{kT}\right)}{\left(1+kT/R\right)} - \frac{1}{\left(1+\frac{kT}{R}\right)^2}}{1 - \frac{1}{\frac{kT}{R}+1}}$$

$$= -W_i + \cancel{kT}\,\frac{\left(\frac{W_a}{kT}+2\right)\left(1+\frac{kT}{R}\right) - \left(1+\frac{W_a}{kT}\right) - \frac{1}{1+\frac{kT}{R}}}{\frac{\cancel{kT}}{R}}$$

$$= -W_i + \left(\frac{W_a}{kT}+2\right)\left(R+kT\right) - R\left(1+\frac{W_a}{kT}\right) - \frac{R^2}{R+kT}$$

$$= W_a - W_i + 2kT + \frac{R\cancel{W_a}}{\cancel{kT}} + \cancel{2}R - \cancel{R} - \frac{R\cancel{W_a}}{\cancel{kT}} - \frac{R^2}{R+kT}$$

$$= W_a - W_i + 2kT + R\left(1 - \frac{R}{R+kT}\right)$$

$$+ R\left(\frac{\cancel{R} + kT - \cancel{R}}{R+kT}\right)$$

$$= W_a - W_i + 2kT + \frac{R\,kT}{R+kT}$$

$$= W_a - W_i + 2kT + \frac{kT}{1 + \frac{kT}{R}}$$

$$= W_a - W_i + kT\left(2 + \frac{1}{1+\frac{kT}{R}}\right)$$

$$R \approx kT$$

$$\therefore \overline{\omega} = W_a - W_i + 2.5\,kT$$

$$\bar{\omega} = \frac{\displaystyle\int_0^\infty\int_{W_x}^\infty \frac{(E-W_i)\,D(W_x)\,(dE-dW_x)\,dW_x}{e^{\frac{E-W_i}{kT}}+1}}{\displaystyle\int_0^\infty\int_{W_x}^\infty \frac{D(W_x)\,(dE-dW_x)\,dW_x}{e^{\frac{E-W_i}{kT}}+1}}$$

$$X = E - W_x \qquad E - W_i = X + W_x - W_i$$

$$dX = dE - dW_x$$

$$E = X + W_x$$

$$\bar{\omega} = -W_i + \frac{\displaystyle\int_0^\infty\int_0^\infty \frac{(X+W_x)\,D(W_x)\,dX\,dW_x}{e^{\frac{X+W_x-W_i}{kT}}+1}}{\displaystyle\int_0^\infty\int_0^\infty \frac{D(W_x)\,dX\,dW_x}{e^{\frac{X+W_x-W_i}{kT}}+1}}$$

$$= -W_i + \frac{\displaystyle\int_0^\infty\int_0^\infty \frac{X\,D(W_x)\,dX\,dW_x}{e^{\frac{X+W_x-W_i}{kT}}+1} + \int_0^\infty W_x D(W_x)\,dW_x \int_0^\infty \frac{dX}{e^{\frac{X+W_x-W_i}{kT}}+1}}{\displaystyle\iint \frac{D(W_x)\,dX\,dW_x}{e^{\frac{X+W_x-W_i}{kT}}+1}}$$

*For field emission*

$$D(W_x) = \left(\frac{4}{W_a}\right) W_x^{1/2} (W_a - W_x)^{1/2} e^{-\frac{4K(C-W)^{3/2}}{3F}}$$

$$di = \frac{4\pi}{h^3} m \; e^{-\frac{x}{kT}} \; e^{-\frac{W_x - W_i}{kT}} \, dx \, D(W_x) \, dW_x$$

$$\overline{W} = -W_i + \frac{\int_0^\infty \int_0^\infty (\cancel{X} + W_x) \, e^{-\frac{x}{kT}} \, e^{-\frac{W_x - W_i}{kT}} \, dx \, D(W_x) \, dW_x}{\int_0^\infty \int_0^\infty e^{-\frac{x}{kT}} \, d\cancel{x} \, D(W_x) \, dW_x}$$

$$= -W_i + \frac{kT \int_0^\infty (kT + W_x) \, e^{-\frac{W_x - W_i}{kT}} \, D(W_x) \, dW_x}{kT \int_0^\infty e^{-\frac{W_x - W_i}{kT}} \, D(W_x)}$$

$$Z = e^{\frac{x}{kT} + \beta}$$

$$dZ = e^{\frac{x}{kT} + \beta} \frac{dx}{kT}$$

$$\frac{kT\,dZ}{Z} = \frac{dx}{\cancel{kT}}$$

$$\int_{e^\beta}^{\infty} \frac{kT\,dZ}{Z(Z+1)} = kT \int_{e^\beta}^{\infty} \frac{dZ}{Z} - \int_0^{\infty} \frac{dZ}{Z+1}$$

$$\ln Z - \ln Z+1 \Big| = -\ln\left(1 + \frac{1}{Z}\right)\Big|_{e^\beta}^{\infty}$$

$$= kT \ln\left(1 + e^{-\beta}\right)$$

$$\frac{W_x - W_a}{kT} = y$$

$$\frac{dW_x}{kT} = dy$$

when $W_x = W_a \quad y = 0$

$$e^{-\frac{W_a - W_i}{kT}} \int e^{-\frac{y}{k}} e^{-y}\, dy$$

$$-kT\int_0^\infty x e^{-\frac{x}{kT}} \left(-\frac{dx}{kT}\right) = -kT x e^{-\frac{x}{kT}} \Big|_0^\infty + kT \int_0^\infty e^{-\frac{x}{kT}}\, dx$$

$$= -(kT)^2 e^{-\frac{x}{kT}} \Big|_0^\infty = (kT)^2$$

$$\overline{\omega} = -W_i + \frac{(kT)^2 \int_{W_a}^\infty W_x e^{-\frac{W_x - W_i}{kT}} \left(1 - e^{-\frac{W_x - W_a}{R}}\right) dW_x}{kT \int_{W_a}^\infty e^{-\frac{W_x - W_i}{kT}} \left(1 - e^{-\frac{W_x - W_a}{R}}\right) dW_x}$$

(13)

$$\overline{W} = -W_i + \frac{\displaystyle\int_0^\infty\int_0^\infty \frac{x\,D(W_x)\,dx\,dW_x}{e^{\frac{x+W_x-W_i}{kT}}+1} + kT\int_0^\infty W_x D(W_x)\,dW_x \ln\left(1+e^{-\frac{W_x-W_i}{kT}}\right)}{\displaystyle kT\int_0^\infty D(W_x)\ln\left(1+e^{-\frac{W_x-W_i}{kT}}\right)dW_x}$$

---

$$\int_0^\infty \frac{x\,dx}{e^{\frac{x}{kT}+\beta}+1} = kT\int_{e^\beta}^\infty \frac{\ln z\,dz}{z(z+1)} - kT\beta\int_{e^\beta}^\infty \frac{dz}{z(z+1)}$$

$$\ln z = \frac{x}{kT}+\beta$$

$$kT\ln z - kT\beta = x$$

$$= kT\left[\int_{e^\beta}^\infty \frac{\ln z}{z}\,dz - \int_{e^\beta}^\infty \frac{\ln z\,dz}{(z+1)} - \beta\ln\left(1+e^{-\beta}\right)\right]$$

$$\left.\frac{(\ln z)^2}{2}\right|_{e^\beta}^\infty - \frac{(\ln(z+1))^2}{2} -$$

Remarks on Energy Losses Attending Thermionic Emission of Electrons from Metals.

The recent publication of a paper on "The Energy Losses Attending Field Current and Thermionic Emission of Electrons from Metals" by Dr. Gertrude M. Fleming and Professor Joseph E. Henderson[1] presents the results of a very valuable experimental study of this subject but, in the opinion of the writer, an error has been made in the assumed physical processes involved. If one considers that the free electrons in a metal can be described as having a Fermi distribution characterized by the thermodynamic potential $\mu$, then at any temperature T the "random" current flow in the positive x direction across any boundary can be calculated. As far as the flow of heat energy is concerned this may be calculated by determining how many electrons with energy greater than $\mu$ cross the boundary in a given time and multiplying this by the average energy carried by each electron. In addition to this one may consider that there is a "current" of "holes" in the Fermi band crossing the same boundary and that energy is carried by these and may be computed by multiplying the number of holes crossing in a given time by the average energy carried by each hole. Fleming and Henderson compute this average energy associated with the current flow of holes and find $\bar{\varepsilon}_0 = \mu - 1.2\ kT$. The determination of the 1.2 is necessarily an approximation but is good to better than four percent. They then proceed to compute the heat carried away by electrons emitted in an accelerating field as though the emission current were supplied at the cool part of the filament at the level $\bar{\varepsilon}_0$ instead of $\mu$. This leads to the result $\bar{w} = \phi + 3.2\ kT$ for the average energy carried away per electron when emitted thermionically. Here $\phi$ is the work function $(C - \mu)$ and C is essentially the potential energy of an electron just outside of the emitter relative to the bottom of the Fermi band as zero.

Consider the receiving plate of a tube for studying thermionic emission to be at $0^{\circ}K$. Then the electrons in the emission current cannot fall into quantum states lower than $\mu$ upon being received because all of those states are filled.

The electric current flows around the circuit at the level $\mu$ (except for batteries) and flows into the emitter at this level. Richardson[2] visualized a situation essentially no different from this and showed that the true heat loss per electron attending thermionic emission is $\overline{W} = \phi + 2\,kT$. A reflection effect, or a transmission coefficient $D(W)$ which is constant and therefore independent of $W$ for all values of $W > C$, does not alter $\overline{W}$ where $W$ is the energy associated with the motion normal to the surface. Thermionic studies[3] indicate that $D(W) = 1 - \exp(W - C)/R$ represents the experimentally determined energy distributions accurately where $R$ is an empirical constant equal to 0.191 electron volt. A paper is being prepared which shows that with this transmission coefficient, $\overline{W} = \phi + kT\left[2 + 1/(1 + kT/R)\right]$. For the temperature range 1500°K to 2200°K this coefficient of $kT$ varies from 2.6 to 2.5 as compared with 2 for non-selective transmission.

The possibility that there are missprints on page 893 of Fleming and Henderson makes a detailed checking of their results difficult since three of the integrations have limits $\mu$ to $\infty$ instead of those expected of 0 to $\infty$ and the brackets are not completed in front of the exponentials as it seems they should be. Although the writer has not yet been able to duplicate the final equation giving $\overline{W}$ as computed for the case of field emission using the indicated limits of $\mu$ to $\infty$, there can be no doubt concerning the final result that an inappreciable heat loss is to be expected for the electrons emitted even though the temperature of the emitter is fairly high. In fact it seems likely that one would find a detectable heating effect when a strong emission takes place from a very sharp point. If one uses integration limits 0 to $\infty$ and assumes that the electrons enter the emitter at the $\mu$ level, the calculation of such heating, if it exists, is straight forward.

W. B. Nottingham

George Eastman Laboratory of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts
November 30, 1940

1. G.M.Fleming and J.E.Henderson, Phys.Rev. 58,887(1940)
2. O.W.Richardson, Phil. Trans. A 201, 497(1903)
3. W.B.Nottingham, Phys. Rev. 49,78(1936)

November 29, 1940

Dr. Gertrude M. Fleming
Russell Sage College
Troy, New York

Dear Dr. Fleming:

Your paper on "The Energy Losses Attending Field Current and Thermionic Emission of Electrons from Metals" is very interesting, and I think important from the point of view of establishing with still greater certainty the view that the field emission electrons come more or less directly from the Fermi band, and therefore their emission entails practically no change in the temperature of the metal from which they are emitted.

Although it is relatively unimportant from the standpoint of the validity of your results and conclusions, the claim that the energy <u>w</u> carried away per electron is $\phi$ + 3.2 kT is in my opinion not correct. In view of the fact that this energy was calculated by Richardson as long ago as 1903, and the fact that it can be calculated directly from the Fermi-Sommerfeld model to be $\phi$ + 2 kT, it is surprising that you have calculated it as $\phi$ + 3.2 kT without an extensive justification of your method.

Of course what you have done is to calculate, as indicated by the last equation on page 892, the average energy "per hole" carried across the boundary by the current of "holes" below the thermodynamic potential $\mu$ and taken this average energy as the reference point for calculating the energy carried away on the average by the thermionically emitted electrons. This is in my mind a very unusual point of view, since I would consider that the proper reference point should be $\mu$ instead of $\mu$ - 1.2 kT.

In view of the fact that I feel that it is important not to let an error of this type become perpetuated in the literature, I am submitting the enclosed letter to the editor with the hope that it may possibly arrive in time to be accepted by the third of December, which is the closing date for letters to the editor which will appear in Vol. 58.

Of course, if I am wrong and you can justify the procedure which you have followed, I think that the publication of my letter is still worth while in that it is surely a point which most other workers in this field would support, and in case you are in a position to defend your belief in this matter, we should all be enlightened.

I am sending a copy of this letter to Professor Tate along with my letter to the editor, and am also sending a copy to Professor Henderson.

I hope that it will not be rushing you too much to submit to Professor Tate your reply in time for publication in this volume, although I realize full well that may be impossible to do. It might facilitate matters if at the same time you sent your reply to Professor Tate, you also sent me a copy so that I can assure Professor Tate that the matter can stand as "closed" as far as letters to the editor are concerned.

Very truly yours,

Wayne B. Nottingham
Associate Professor of Physics

WBN:W
CC to Professor Tate
Professor Henderson

Encs.

November 29. 1940

Professor Joseph E. Henderson
University of Washington
Seattle, Washington

Dear Professor Henderson:

I am enclosing a copy of a letter which I have written to Dr. Fleming, and also a copy of a letter to the editor which I hope can be published in Vol. 58 of the PHYSICAL REVIEW, although I realize that it will be difficult to manage it.

I was not sure whether or not you or Dr. Fleming would prefer to defend your point of view. Although the matter is practically of no importance as far as the principal experimental results which you have presented are concerned, I think that it is important not to leave this result uncorrected in case my point of view is the correct one.

Very truly yours,

Wayne B. Nottingham
Associate Professor of Physics

WBN:W
CC to Professor Tate
Dr. Fleming

Encs.

November 29, 1940

Professor John T. Tate, Editor
PHYSICAL REVIEW
University of Minnesota
Minneapolis, Minnesota

Dear Professor Tate:

I am enclosing a "letter to the editor" which I hope you will find suitable for publication in the December 15 issue. I realize that the time will be very short and that it may not be possible to do this.

I am also enclosing copies of my letter to Dr. Fleming and Professor Henderson; I think these are all self-explanatory.

Very truly yours,

Wayne B. Nottingham
Associate Professor of Physics

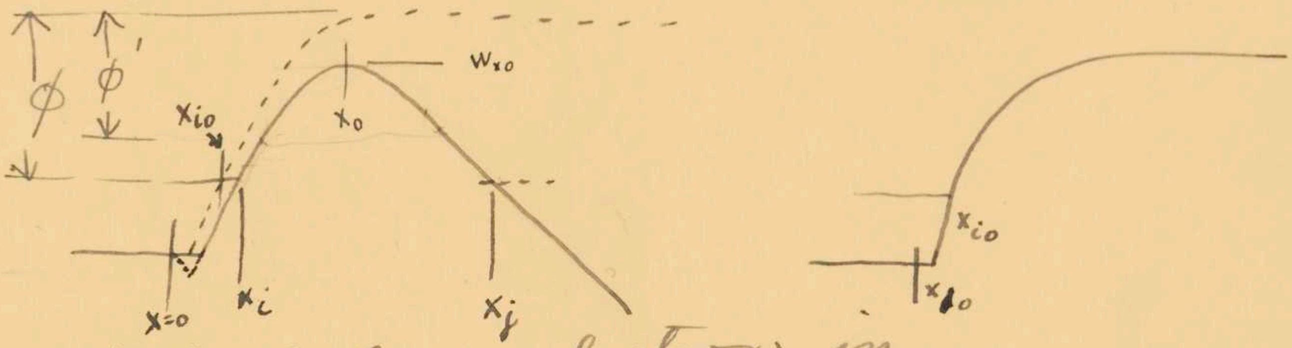
WBN:W
Encs.

Would it be possible to compute (1)

field emission by substituting

a parabola for the mirror

image barrier.




potential of an electron in

image field is $-\frac{e^2}{4X}$

and potential in a uniform

elec field $E$ is $eEx$

at $x_1$ | $W_a = \frac{e^2}{4x_1}$

at $x_{i0}$ | $(W_a - W_i) = \frac{e^2}{4x_{i0}}$

In distance $x_j - x_i$ work done

by field is $eE(x_j - x_i)$

~~Work done~~

Potential $W_a - \frac{e^2}{4X} - eEx = W$

$W = W_i$ at $x_i$ and $x_j$

$$\phi = (W_a - W_i) = \frac{e^2}{4x_{ij}} + eEx_{ij}$$

$$x_{ij}^2 - \frac{2\phi}{2eE}x_{ij} + \left(\frac{\phi}{2eE}\right)^2 = \left(-\frac{e}{4E} + \left(\frac{\phi}{2eE}\right)^2\right)$$

$$x_{ij} - \frac{\phi}{2eE} = \pm\sqrt{-\frac{e}{4E} + \left(\frac{\phi}{2eE}\right)^2}$$

$$x_{ij} = \frac{\phi}{2eE} \pm \sqrt{\frac{e}{4E} \mp \left(\frac{\phi}{2eE}\right)^2}$$

$$= \frac{\phi}{2eE}\left(1 \pm \sqrt{1 - \frac{e^3E}{\phi^2}}\right)$$

$$x_i = \frac{\phi}{2eE}\left(1 - \sqrt{1 - \frac{e^3E}{\phi^2}}\right) \doteq \frac{e^2}{4\phi}$$

$$x_j = \frac{\phi}{2eE}\left(1 + \sqrt{1 - \frac{e^3E}{\phi^2}}\right) \doteq \frac{\phi}{eE}$$

$$\frac{4\pi}{h}(2m)^{1/2}\int$$

See also p 6

$$D(W) = \frac{1}{1+e^{\frac{4\pi}{h}(2m)^{1/2}\int_{x_1}^{x_2}(W-W_x)^{1/2}\,dx}}$$

Since for the important range of $W_x$, $D(W_x)$ is always very small compared with 1.

then $D(W_x) \doteq e^{-\frac{4\pi(2m)^{1/2}}{h}\int_{x_1}^{x_2}(W-W_x)^{1/2}\,dx}$

$$= e^{-\frac{4\pi(2m)^{1/2}}{h}A}$$

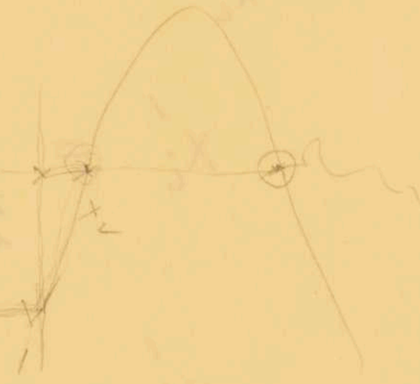
To calculate the transmission at level $W_a - W_x = \phi'$ we need to know the integral $\int (W - W_x)^{1/2}\, dx$

$$A \equiv \int_{x_i'}^{x_j'} \left(\phi' - \frac{e^2}{4x} - eEx\right)^{1/2} dx$$

$$x_i' = \frac{\phi'}{2eE}\left(1 - \sqrt{1 - \frac{\varepsilon^3 E}{\phi'^2}}\right)$$

$$x_j' = \frac{\phi'}{2eE}\left(1 + \sqrt{1 - \frac{\varepsilon^3 E}{\phi'^2}}\right)$$

$$\frac{x_j' - x_i'}{2} = \frac{\phi' \sqrt{1 - \frac{\varepsilon^3 E}{\phi'^2}}}{2eE}$$


$x_2$

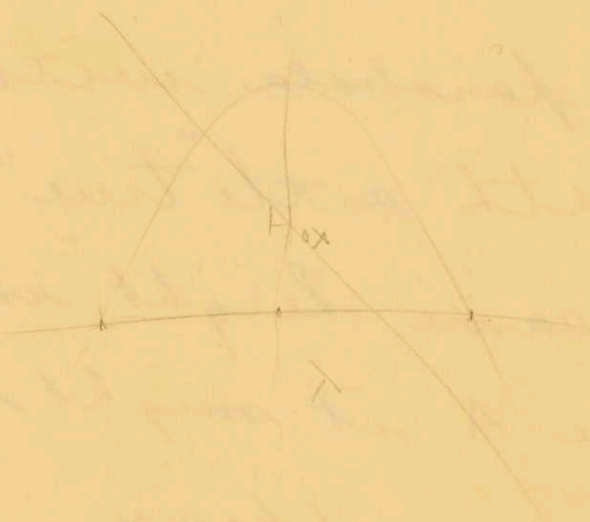
The above forms show that it is not an easy matter to evaluate A and thus determine D(W) but if an approximation is made by which a parabola with the same width as the "true" barrier and the same height instead of the same A it might make a workable approx.

~~Although $\phi$ above is thought of as $W_a - W_i$ it may be generalized to mean any level below $W_a$.~~

at $x_0$ $\quad \frac{dW}{dx} = 0$

$$+\frac{e^2}{4x_0^2} - Ee = 0$$

$$x_0 = \frac{1}{2}\sqrt{\frac{e}{E}}$$



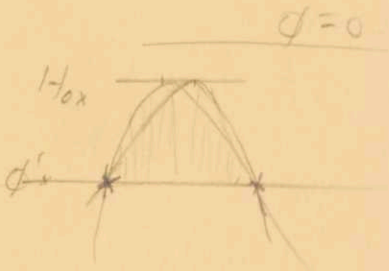
and $W_0 = W_a - \frac{2e^2\sqrt{E}}{4e^{1/2}} - \frac{eEe^{1/2}}{2E^{1/2}}$

$$= W_a - \cancel{\#}\, e^{3/2}E^{1/2}$$

$\cancel{W_0 = W}$ - $W_0 - W_x = H_{ox}$

$$W_a - W_x - e^{3/2}E^{1/2} = H_{ox}$$

$$\phi' - e^{3/2}E^{1/2} = H_{ox}$$




let $H = H_{ox} - \alpha(\lambda)^2 \cancel{- x_0)^3}$

$$H = 0 \text{ at } \lambda = \pm\left(\frac{x_j' - x_i'}{2}\right)$$

$$\alpha = \frac{4H_{ox}}{(x_j' - x_i')^2} = \frac{4\left(\phi' - e^{3/2}E^{1/2}\right)}{\frac{\phi'^2}{\cancel{4}e^2E^2}\left(\cancel{4}\left(1 - \frac{e^3E}{\phi'^2}\right)\right)}$$

$$= \frac{4e^2E^2\left(\phi' - e^{3/2}E^{1/2}\right)}{\phi'^2 - \cancel{4}e^3E}$$

$$\boxed{H = \left(\phi' - e^{3/2}E^{1/2}\right)\left(1 - \frac{4e^2E^2\lambda^2}{\phi'^2 - e^3E}\right)}$$

wanted

$$2\left(\phi' - e^{3/2} E^{1/2}\right)^{1/2} \int_0^{\frac{(x_j - x_i)}{2}} \left(1 - \frac{1}{\beta^2}\lambda^2\right)^{1/2} d\lambda$$

Let $\beta^2 = \dfrac{\phi'^2 - e^3 E}{4e^2 E^2}$

$$\frac{2\left(\phi' - e^{3/2} E^{1/2}\right)^{1/2}}{\beta} \int_0^{\frac{(x_j - x_i)}{2}} \left(\beta^2 - \lambda^2\right)^{1/2} d\lambda$$

$$\frac{2\left(\phi' - e^{3/2} E^{1/2}\right)^{1/2}}{\beta} \; \frac{1}{2}\left(\lambda\sqrt{\beta^2 - x^2} + \beta^2 \sin^{-1}\frac{\lambda}{\beta}\right)\Bigg|_0^{\frac{x_j - x_i}{2}}$$

zero at both limits

$$\frac{\lambda}{\beta} = \frac{\frac{\phi'}{2eE}\left(\cancel{2}\left(1 - \frac{\epsilon^3 E}{\phi'^2}\right)^{1/2}\right)\cancel{2eE}}{\left(\phi'^2 - e^3 E\right)^{1/2}} = 1$$

$$\sin^{-1} 1 = \frac{\pi}{2}$$

$$\frac{2\pi}{2 \times 2}\frac{\left(\phi' - e^{3/2} E^{1/2}\right)^{1/2}\sqrt{\phi'^2 - e^3 E}}{2e^4 E} = \frac{\pi}{2}\;\frac{\left(\phi' - e^{3/2} E^{1/2}\right)^{1/2}\left(\phi'^2 - e^3 E\right)^{1/2}}{2eE}$$

$$A = \frac{\pi \phi'^{3/2} \left(1 - \frac{e^{3/2} E^{1/2}}{\phi'}\right)^{1/2} \left(1 - \frac{e^{3} E}{\phi'^{2}}\right)^{1/2}}{2 \times 2 e E}$$

(See back of page 2)

$$D(\phi, E) = e^{-\frac{2\pi^2 (2m)^{1/2}}{\nu h\, eE} \phi'^{3/2} \left(1 - \frac{e^{3/2} E^{1/2}}{\phi'}\right)^{1/2} \left(1 - \frac{e^{3} E}{\phi'^{2}}\right)^{1/2}}$$

$$= e^{-\frac{2\pi^2 (2m)^{1/2}}{\nu h e} \times \frac{\phi'^{3/2}}{E} \times \left(1 - \frac{e^{3/2} E^{1/2}}{\phi'}\right)^{1/2} \left(1 - \frac{e^{3} E}{\phi'^{2}}\right)^{1/2}}$$

$$D = e^{-\frac{2\pi^2 (2m)^{1/2}}{\nu h \epsilon} \frac{\phi'^{3/2}}{E} \left(1 - \frac{\Delta\phi}{\phi'}\right)^{1/2} \left(1 - \frac{(\Delta\phi)^2}{\phi'^{2}}\right)^{1/2}}$$

---

See. E.C. Kemble – McGraw Hill Book Co.
1st Edition (1937) p 111.

$$D = \frac{1}{1 + e^{2K}}$$

where $K = \frac{2\pi}{h} \int_{x_1}^{x_2} |p| \, d\xi$

$p^2 = 2m(E - V) = 2m\, H$ of page 4b above.

$$K = \frac{2\pi}{h} (2m)^{1/2} \int H \, dx$$

$$\frac{1}{\gamma} = \frac{2\pi^2 (2m)^{1/2}}{\nu h \varepsilon \quad E}$$

$$N(W_x)\,dW_x = \frac{4\pi m kT}{h^3}\ \ln\left(1+e^{-\frac{W_x-W_i}{kT}}\right)dW_x$$

$$\mu = \frac{W_x-W_i}{kT}$$

$$\cancel{k}\ dW_x = kT\,d\mu$$

$$N(\mu)\,d\mu = \frac{4\pi m (kT)^2}{h^3}\ln(1+e^{-\mu})\,d\mu$$

$$\phi' = W_a - W_x$$

$$W_x = W_a - \phi' =$$

$$W_x - W_i = W_a - W_i - \phi' = \phi - \phi' = \mu kT$$

$$\phi' = \phi - \mu kT$$

$$= \phi\left(1-\frac{\mu kT}{\phi}\right)$$

$$i = \frac{4\pi m (kT)^2}{h^3}\int_{-\frac{W_i}{kT}}^{\frac{\phi-\Delta\phi}{kT}} \ln(1+e^{-\mu})\, e^{-\frac{\phi^{3/2}\left(1-\frac{\mu kT}{\phi}\right)^{3/2}}{\gamma}\left(1-\frac{e^{3/2}E^{1/2}}{\phi-\mu kT}\right)^{1/2}\left(1-\frac{e^3E}{(\phi-\mu kT)^2}\right)^{1/2}}\,d\mu$$

$$\int f(\mu)\,d\mu = \int_{-\frac{W_i}{kT}}^{0} \left\{\ln(1+e^{\mu}) \cancel{*} - \mu\right\} e^{-\frac{\phi^{3/2}\left(1-\frac{\mu kT}{\phi}\right)^{3/2}(\;)(\;)}{\gamma}}\,d\mu \quad +$$

$$+ \int_{0}^{\infty} \ln(1+e^{-\mu})\, e^{-\overline{\qquad\qquad}}\,d\mu$$

The chief range of interest is for $\mu = \cancel{\pm}-3$ to $+3$ at most

$\frac{kT}{\phi}$ range is 0 to .03

$\therefore$ range of $\mu\frac{kT}{\phi}$ is 0 to $\pm .1$

use $\left(1-\frac{\mu kT}{\phi}\right)^{3/2} \doteq 1 - \frac{3}{2}\frac{\mu kT}{\phi}$

---

$$\left\{1 - \frac{e^{3/2}E^{1/2}}{\cancel{\phi}\left(1-\frac{\mu kT}{\phi}\right)}\right\}^{1/2} \doteq \left\{1 - \frac{e^{3/2}E^{1/2}}{\phi}\left(1+\frac{\mu kT}{\phi}\right)\right\}^{1/2}$$

$$\doteq 1 - \frac{\varepsilon^{3/2}E^{1/2}}{2\phi} - \frac{\varepsilon^{3/2}E^{1/2}}{2\phi^2}\mu kT$$

$$\left\{1 - \frac{e^3 E}{\phi^2\left(1-\frac{\mu kT}{\phi}\right)^2}\right\}^{1/2} \doteq \left\{1 - \frac{e^3 E}{\phi^2}\left(1+\frac{2\mu kT}{\phi}\right)\right\}^{1/2}$$

$$\doteq 1 - \frac{\varepsilon^3 E}{2\phi^2} - \frac{\varepsilon^3 E\,\mu kT}{\phi^3}$$

---

$$\ln(1+e^{-\mu}) \doteq e^{-\mu} - \frac{e^{-2\mu}}{2} + \frac{e^{-3\mu}}{3} - \cdots$$

$$e^{-\text{---}(\ )(\ )} = e^{-\frac{\phi^{3/2}}{8}\left(1-\frac{3}{2}\frac{\mu kT}{\phi}\right)\left(1-\frac{\varepsilon^{3/2}E^{1/2}}{2\phi}-\frac{\varepsilon^{3/2}E^{1/2}}{2\phi^2}\mu kT\right)\left(1-\frac{\varepsilon^3 E}{2\phi^2}-\frac{\varepsilon^3 E\mu kT}{\phi^3}\right)}$$

$$= e^{-\frac{\phi^{3/2}}{8}} \times e^{-\frac{\phi^{3/2}}{8}\left(\frac{\varepsilon^{3/2}E^{1/2}}{2\phi}\right)\left(\frac{\varepsilon^3 E}{2\phi^2}\right)}\; e^{-\frac{\phi^{3/2}}{8}\mu\left(\frac{3}{2}\frac{kT}{\phi}\right)}$$

$$a = \frac{\mu kT}{2} \qquad b = \frac{\varepsilon^{3/2}E^{1/2}}{2\phi} = \frac{\Delta\phi}{2\phi}$$

$$(1-3a)(1-b-2ab)(1-2b^2-8b^2a)$$

$$(\qquad)(1-b-2ab-2b^2+2b^3+4ab^3-8b^2a+8b^3a)$$

$$(\qquad)(1-b-2b^2+2b^3-(2b+8b^2-12b^3)a)$$

$$1-b-2b^2+2b^3-(2b+8b^2-12b^3+3-3b-6b^2+6b^3)a$$

$$1-b-2b^2+2b^3-(3-b+2b^2-6b^3)a$$

$$\cancel{\frac{\phi^{3/2}}{\gamma}\left(1-\frac{\varepsilon^{3/2}E^{1/2}}{2\phi}-\right.}$$

$$\frac{\phi^{3/2}}{\gamma}\underbrace{\left\{1-\frac{\Delta\phi}{2\phi}-2\left(\frac{\Delta\phi}{2\phi}\right)^2+2\left(\frac{\Delta\phi}{2\phi}\right)^3\right\}}_{A}-\frac{\phi^{3/2}}{\gamma}\underbrace{\left\{3-\frac{\Delta\phi}{2\phi}+2\left(\frac{\Delta\phi}{2\phi}\right)^2-6\left(\frac{\Delta\phi}{2\phi}\right)^3\right\}}_{B}\frac{kT}{2}\mu$$

$$i=\frac{4\pi m(kT)^2}{h^3}e^{-\frac{\phi^{3/2}}{\gamma}A}\left[\int_{0}^{\cancel{\frac{\phi-\Delta\phi}{kT}}}\left\{e^{-\mu\left(1-\frac{\phi^{3/2}}{\gamma}B\frac{kT}{2}\right)}-\frac{e^{-2\mu\left(1-\frac{1}{2}\frac{\phi^{3/2}}{\gamma}B\frac{kT}{2}\right)}}{2}+e^{-3\mu\left(1-\frac{1}{3}\frac{\phi^{3/2}}{\gamma}B\frac{kT}{2}\right)}\right\}d\mu\right.$$

$$\left.+\int_{-\frac{W_i}{kT}}^{0}\left\{-\mu+e^{\mu\left(1-\underbrace{\frac{\phi^{3/2}}{\gamma}B\frac{kT}{2}}_{\beta}\right)}-\frac{e^{2\mu\left(1-\frac{1}{2}\frac{\phi^{3/2}}{\gamma}B\frac{kT}{2}\right)}}{2}+\right\}d\mu\right.$$

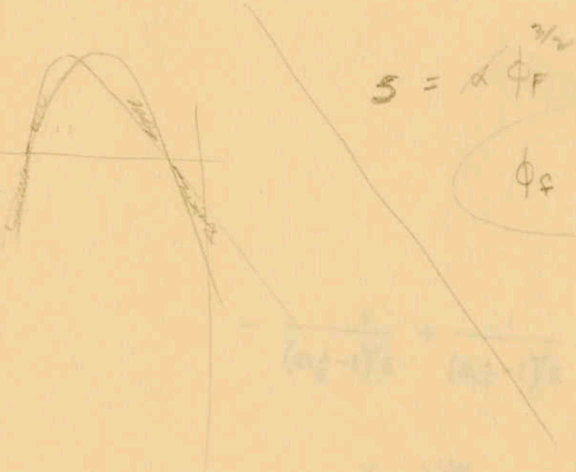
$$=\frac{4\pi m(kT)^2}{h^3}e^{-\frac{\phi^{3/2}}{\gamma}A}\left[\frac{W_i^2}{2(kT)^2}+\frac{1}{1-\beta}-\frac{1}{2^2(1-\frac{1}{2}\beta)}+\frac{1}{3^2(1-\frac{1}{3}\beta)}-\right.$$

$$-\frac{e^{-\frac{W_i}{kT}(1-\beta)}}{1-\beta}+\frac{e^{-2\frac{W_i}{kT}(1-\beta)}}{2(1-\frac{1}{2}\beta)}-\cdots$$

$$\left.+\frac{1}{1-\beta}-\frac{1}{2^2(1-\frac{1}{2}\beta)}+\frac{1}{3^2(1-\frac{1}{3}\beta)}\right]$$

$$\frac{4K}{3} = \frac{4\alpha(8)^{1/2}\pi\, m^{1/2}}{3h}$$

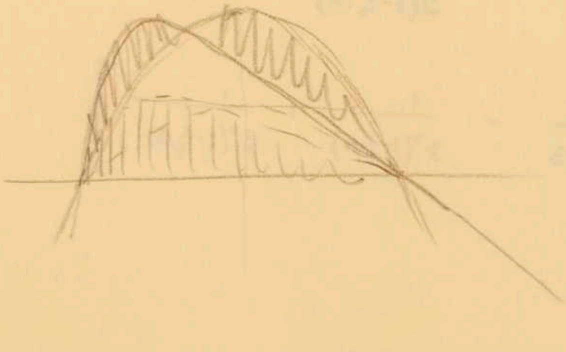
$$\frac{4\frac{8}{3}\,\frac{\sqrt{2}\,m^{1/2}\pi}{h}}{2\,\pi^{2}\,\frac{(2m)^{1/2}}{h}} = \frac{4}{3\pi} = \frac{1}{2.3}$$



$$S = \alpha\,\phi_F^{3/2} = 2\alpha\,\phi_n^{3/2}$$

$$\phi_f = 2.3^{2/3}\,\phi_n = 1.7\,\phi_n$$

$$\phi_n = \frac{\phi_F}{1.7}$$



$$i = \frac{\cancel{2}\pi m W_i^2}{\cancel{4}h^3} e^{-\frac{\phi^{3/2}}{\gamma}A} + \frac{\cancel{8}\pi m (kT)^2}{h^3} e^{-\frac{\phi^{3/2}}{\gamma}A} \left\{ \frac{1}{1-\beta} - \frac{1}{2\cancel{4}(1-\frac{1}{2}\beta)} + \frac{1}{9(1-\frac{1}{3}\beta)} - \right\}$$

since the second term of this equation is smaller than the first by the order of $\left(\frac{\cancel{4}2kT}{W_i}\right)^2$

$$i = \frac{2\pi m W_i^2}{h^3} e^{-\frac{2\pi^2(2m)^{1/2}}{\cancel{3}h\epsilon}\frac{\phi^{3/2}}{E}\left\{1 - \frac{\Delta\phi}{2\phi} - 2\left(\frac{\Delta\phi}{2\phi}\right)^2 + 2\left(\frac{\Delta\phi}{2\phi}\right)^3\right\}} \left[1 + \left(\frac{\cancel{4}2kT}{W_i}\right)^2 \left\{\frac{1}{1-\beta} - \frac{1}{4(1-\frac{1}{2}\beta)} + \frac{1}{9(1-\frac{1}{3}\beta)}\right\}\right]$$

The second term is known to be small since it contributes at most

$(.06)^2 = 3.6 \times 10^{-3}$ or $.4\%$

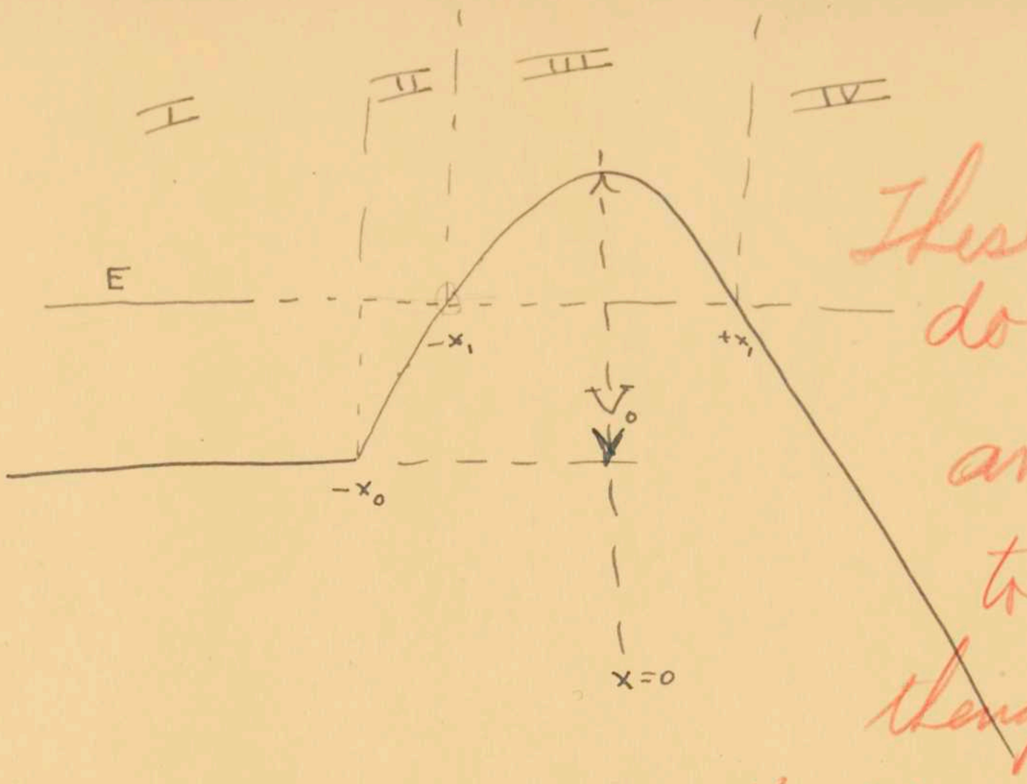
---

$$i = \frac{2\pi m W_i^2}{h^3} e^{-\underbrace{\frac{\cancel{4}2\pi^2(2m)^{1/2}}{\cancel{3}h\epsilon}}\frac{\phi^{3/2}}{E}\left\{1 - \frac{\Delta\phi}{2\phi} - 2\left(\frac{\Delta\phi}{2\phi}\right)^2\right\}}$$

Fowler uses $\frac{4}{3}K$ here where $K^2 = \frac{8\pi^2 m}{h^2}$

$K = \frac{2\pi(2m)^{1/2}}{h}$ my value is $\frac{\pi K}{2}$

$\therefore$ ratio is $N : F = \pi : \frac{4}{3}$ $\quad \frac{N}{F} = \frac{3\pi}{4 \times 2} = \cancel{\ } \; 1.18$




These do not amount to anything - See Kemble reference on p 6

$V = 0$

$x = -\infty$ to $-x_0$

$V = V_0 - a^2 x^2$

$x = -x_0$ to $x = +\infty$

---

Wave equation:-

$$\frac{d^2\phi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\,\phi = 0$$

Solution in I

$$\phi_1 = Ae^{i\alpha x} + Be^{-i\alpha x}$$

---

assume for test solutions in II, III, & IV

$$\phi = Ce^{\frac{2\pi i}{h}\int (2m)^{1/2}(E-V)^{1/2}dx} + De^{-\frac{2\pi i}{h}\int (2m)^{1/2}(E-V)^{1/2}dx}$$

or

$$\phi = C\,(E-V)^{-\frac{1}{4}} e^{i\beta\int (E-V)^{\frac{1}{2}} dx} + D(E-V)^{\frac{1}{4}} e^{-i\beta\int (E-V)^{\frac{1}{2}} dx}$$

---

$$E - V = E - V_0 + a^2 x^2$$

$$E - V = 0 \quad \text{at } x_1$$

$$\frac{V_0 - E}{a^2} = x_1^{\,2}$$

$$x_1 = \pm \frac{(V_0 - E)^{\frac{1}{2}}}{a}$$

---

$$x_0 = -\frac{V_0^{\frac{1}{2}}}{a}$$

---

let $y = \dfrac{x}{x_1}$ $\qquad dy = \dfrac{dx}{x_1}$

when $x = x_1$
$y = 1$

$$E - V = (E - V_0)\left(1 - \frac{a^2}{V_0 - E} x^2\right)$$

$$= (E - V_0)\left(1 - \frac{x^2}{x_1^{\,2}}\right)$$

$$= (E - V_0)(1 - y^2)$$

$$\int (E-V)^{1/2} dx = x_1 (\cancel{E} - V_0)^{1/2} \int (1-y^2)^{1/2} dy$$

For region II

$$X = X_0$$

$$y_0 = \frac{X_0}{X_1} = \frac{V_0^{1/2}}{a} \times \frac{a}{(V_0-E)^{1/2}} = \frac{V_0^{1/2}}{(V_0-E)^{1/2}}$$

$$\int (1-y^2)^{1/2} dy = \frac{1}{2}\left(y\sqrt{1-y^2} + \sin^{-1} y\right)$$

$$\int_{-\frac{V_0^{1/2}}{(V_0-E)^{1/2}}}^{1} (\quad) dy = \frac{1}{2}\left(0 + \frac{\pi}{2} + \frac{V_0^{1/2}}{(V_0-E)^{1/2}}\sqrt{1 - \frac{V_0}{V_0-E}} + \sin^{-1}\frac{V_0^{1/2}}{(V_0-E)^{1/2}}\right)$$

$$= \frac{1}{2}\left(\frac{\pi}{2} + y_0\sqrt{1-y_0^2} + \sin^{-1} y_0\right)$$

$$y_0 = \frac{V_0^{1/2}}{(V_0-E)^{1/2}}$$

$$(V_0-E)^{1/2} = \frac{V_0^{1/2}}{y_0}$$

$$\int (E-V)^{1/2} dx = V_0^{1/2} \frac{x_1}{2y_0}\left(\frac{\pi}{2} + y_0\sqrt{1-y_0^2} + \sin^{-1} y_0\right)$$

Region III

$$\int_{-x_1}^{+x_1} (E-V)^{1/2}\,dx = 2x_1 (E-V_0)^{1/2} \int_0^{*} (1-y^2)^{1/2}\,dy$$

$$=$$

$$\frac{N}{V} = \int N(p_x)\,dp = \frac{8\pi mkT}{h^3}\int_{p_x=0}^{p_x=\infty} \ln\left(1+e^{-\frac{p_x^2-2mW_i}{2mkT}}\right) dp.$$

at $T \rightarrow 0°K$.

$$\frac{N}{V} = \frac{8\pi mkT}{h^3}\int_{p_x=0}^{p_x=(2mW_i)^{1/2}} \cancel{\ln(1+e}^{-\frac{2mW_i-p_x^2}{2mkT}}) + \frac{2mW_i-p_x^2}{2mkT}\,dp_x$$

$$= \frac{\cancel{8}4\pi\,\cancel{kT}}{h^3}\left[\int_{p_x=0}^{p_x=(2mW_i)^{1/2}} +(2mW_i)\,dp_x - \int_{p_x=0}^{p_x=(2mW_i)^{1/2}} p_x^2\,dp_x\right]$$

$$\frac{N}{V} = \frac{4\pi}{h^3}\left[+(2m)^{3/2}W_i^{3/2} - \frac{1}{3}(2mW_i)^{3/2}\right]$$

$$\frac{N}{V} = \frac{4\pi}{h^3}\times\frac{2}{3}(2m)^{3/2}W_i^{3/2}$$

$$W_i^{3/2} = \frac{h^3\,3n}{8\pi(2m)^{3/2}}$$

(2)

$$W_i = \frac{h^2}{\cancel{2}m}\left(\frac{3n}{\cancel{2}\pi}\right)^{2/3} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3}$$

$$W_{i0} = \frac{h^2}{8m}\left(\frac{3n}{\pi}\right)^{2/3} \text{ for } \underline{0^\circ K}$$

$$\frac{N}{V} = \frac{8\pi m kT}{h^3}\left\{\int_0^{p_x=(2mW_i)^{1/2}} \ln\left(1+e^{-\frac{2mW_i-p_x^2}{2mkT}}\right) dp_x + \int_0^{p_x=(\ )^{1/2}} \frac{2mW_i-p_x^2}{2mkT}\, dp_x\right.$$

$$\left. + \int_{p_x=(2mW_i)^{1/2}}^{\infty} \ln\left(1+e^{-\frac{(p_x^2-2mW_i)}{2mkT}}\right) dp_x \right\}$$

$$\text{let } u = \frac{p_x^2-2mW_i}{2mkT}$$

$$2\cancel{kk}m kT + 2mW_i = p_x^2$$

$$p_x = (2m)^{1/2}\left(\frac{ukT}{\cancel{m}} + W_i\right)^{1/2}$$

$$du = \frac{p_x}{mkT}\, dp_x$$

$$\frac{mkT\, du}{p_x} = dp_x$$

$$p_x = (2mW_i)^{1/2} \longrightarrow u = 0$$

$$p_x = 0 \qquad u = -\frac{W_i}{kT}$$

$$\frac{(m)^{1/2}(kT)^{1/2}\, du}{(2\cancel{kk})^{1/2}\left(\frac{u\cancel{kT}}{\cancel{kk}} + \frac{W_i}{kT}\right)^{1/2}} = dp_x$$

$$\frac{m(kT)^{1/2}}{2^{1/2}\left(u + \frac{W_i}{kT}\right)^{1/2}} = dp_x$$

$$dp_x = \frac{(2m)^{1/2}(kT)^{1/2}}{2\left(u + \frac{W_i}{kT}\right)^{1/2}}$$

$$n = \frac{\cancel{8}^{\,4}\pi (m)^{3/2}(2)^{1/2}(kT)^{3/2}}{\cancel{p}\,h^3}\left[\int_{-\frac{W_i}{kT}}^{0}\frac{\ln(1+e^{+\mu})\,d\mu}{\left(\mu+\frac{W_i}{kT}\right)^{1/2}} + \int_{-\frac{W_i}{kT}}^{0}\frac{-\mu\,d\mu}{\left(\mu+\frac{W_i}{kT}\right)^{1/2}}\right.$$

$$\left. + \int_{0}^{\infty}\frac{\ln(1+e^{-\mu})\,d\mu}{\left(\mu+\frac{W_i}{kT}\right)^{1/2}}\right]$$

$$n = \frac{4\pi}{h^3}(2m)^{3/2}\,\frac{2}{3}\,W_{i0}^{3/2}\left[\frac{3}{4}\left(\frac{kT}{W_{i0}}\right)^{3/2}\int_{-\frac{W_i}{kT}}^{0}\frac{\ln(1+e^{\mu})\,d\mu}{\left(\mu+\frac{W_i}{kT}\right)^{1/2}} - \int_{-\frac{W_i}{kT}}^{0}\frac{\mu\,d\mu}{\left(\mu+\frac{W_i}{kT}\right)^{1/2}}\right.$$

$$\left. + \int_{0}^{\infty}\frac{\ln(1+e^{-\mu})\,d\mu}{\left(\mu+\frac{W_i}{kT}\right)^{1/2}}\right]$$

[ ] should be very near 1.

Instead of sub. $\mu$ as above.

take $x^2 = W_i - \frac{p_x^2}{2m}$

$$W_i - x^2 = \frac{p_x^2}{2m}$$

$$p_x = \sqrt{2m(W_i - x^2)}$$

$$dp_x = \frac{(2m)^{1/2}(-2x\,dx)}{2\sqrt{W_i - x^2}} = -\frac{(2m)^{1/2}\,x\,dx}{\sqrt{W_i - x^2}}$$

$$y^2 = \frac{x^2}{W_i}$$

$$y\,dy = \frac{x\,dx}{W_i}$$

$$dp_x = -\frac{W_i^{1/2}(2m)^{1/2}\,y\,dy}{\sqrt{1-y^2}}$$

$$y^2 = 1 - \frac{\frac{p_x^2}{2m}}{W_i}$$

when $p_x = 0$
$y = 1$

when $p^2 = 2mW_i$
$y = 0$

$$W_i(1-y^2) = \frac{p_x^2}{2m}$$

$$p_x = \sqrt{2mW_i(1-y^2)}$$

$$n = \frac{8\pi m kT}{h^3}\left[+\int_0^1 \frac{W_i^{3/2}(2m)^{1/2}\ln\left(1+e^{-\frac{W_i y^2}{kT}}\right)y\,dy}{W_i\sqrt{1-y^2}}\right.$$

$$- \frac{W_i^{3/2}(2m)^{1/2}}{kT}\int_0^1 \frac{\cancel{W_i}\, y^3\, dy}{\cancel{kT}\,\sqrt{1-y^2}} + \int_0^\infty \frac{\ln\left(1+e^{\frac{W_i y^2}{kT}}\right) W_i^{1/2}(2m)^{1/2}\, y\, dy}{\sqrt{1-y^2}} \Bigg]$$

$$\Vert$$

$$W_i^{3/2}(2m)^{1/2}\int_0^\infty \frac{\ln\left(1+e^{-\frac{W_i y^2}{kT}}\right) y\, dy}{W_i\, \sqrt{1-y^2}} + \frac{W_i^{3/2}(2m)^{1/2}}{kT}\int_0^\infty \frac{y^3\, dy}{\sqrt{1-y^2}}$$

$$\frac{4\pi(2m)^{3/2}W_i^{3/2}}{h^3}\,\frac{2}{3}\Bigg[\frac{3kT}{2W_i}\int_0^1\frac{\ln\left(1+e^{-\frac{W_i y^2}{kT}}\right) y\, dy}{\sqrt{1-y^2}} - \frac{3}{2}\int_0^1\frac{y^3\, dy}{\sqrt{1-y^2}}$$

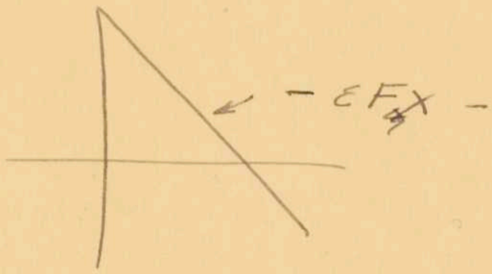
$$+ \frac{3}{2}\int_0^\infty \frac{y^3\, dy}{\sqrt{1-y^2}} + \frac{3}{2}\frac{kT}{W_i}\int_0^\infty\frac{\ln\left(1+e^{-\frac{W_i y^2}{kT}}\right) y\, dy}{\sqrt{1-y^2}}\Bigg]$$

at $T = 0°K$

$$n = \frac{4\pi(2m)^{3/2}W_i^{3/2}}{h^3}\times\frac{2}{3}\Bigg[0 - \cancel{\frac{3}{2}\int_0^1\frac{y^3\, dy}{\sqrt{1-y^2}}} + \cancel{\frac{3}{2}\int_0^1 \frac{y^3\, dy}{\sqrt{1-y^2}}} + \frac{3}{2}\int_1^\infty\frac{y^3\, dy}{\sqrt{1-y^2}} + 0\Bigg]$$

Something wrong here.

$$A_f = \frac{4\pi(2m)^{1/2}}{h}\int_{x=0}^{x=\frac{E}{\varepsilon F}} (\varepsilon F x - E)^{1/2}\,dx$$



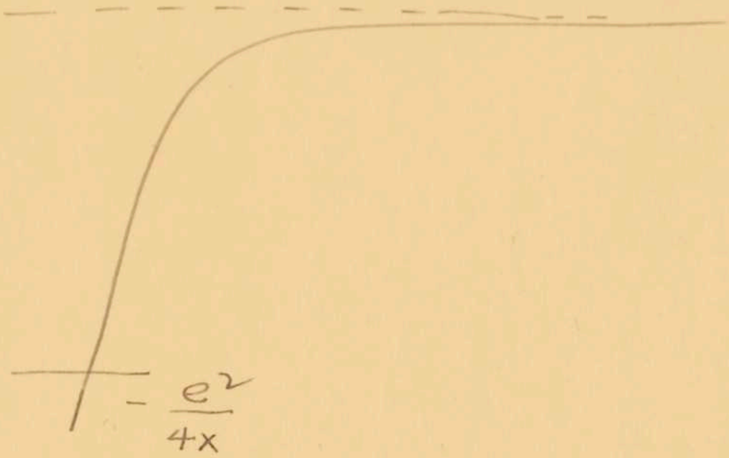

$$= \frac{4\pi(2m)^{1/2}}{h}\;\frac{2}{3}\frac{1}{\varepsilon F}(\varepsilon F x - E)^{3/2}\Big|_0^{\frac{E}{\varepsilon F}}$$

$$= \frac{4\pi(2m)^{1/2}}{h}\;\frac{2}{3}\;\frac{E^{3/2}}{\varepsilon F}$$

---

$$F = \frac{E^2}{r^2\varepsilon^3}$$

$$A = \frac{4\pi(2m)^{1/2}}{h}\;\frac{2}{3}\;\frac{E^{3/2}}{\varepsilon}\frac{r^2\varepsilon^3}{E^2} = \frac{2}{3}\;\frac{r^2\varepsilon^2}{E^{1/2}}$$

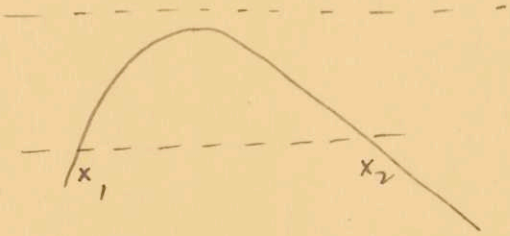


$$-\frac{e^2}{4x}$$

$$-W_a = -\frac{e^2}{4x_a^{*}}$$

$$x_a = \frac{e^2}{4W_a}$$

$$V = -eFx - \frac{e^2}{4x}$$




at $V_{max} \neq \frac{dV}{dx} = 0$

$$-\cancel{e}F + \frac{e^{\cancel{2}}}{4x_m^2} = 0$$

$$\frac{e}{4x_m^2} = F$$

$$x_m = \frac{1}{2}\left(\frac{e}{F}\right)^{1/2}$$

$$V_{max} = -\frac{1}{2}\varepsilon(eF)^{1/2} - \frac{2}{4}\varepsilon(\varepsilon F)^{1/2} = -\varepsilon^{3/2}F^{1/2}$$

| X | | | | H |
|---|---|---|---|---|
| ~~.324~~ | | | | 0 |
| .81 | 4.12 | .081 | | |
| 1 | 3.375 | .1 | 3.475 | 1.025 |
| 2 | 1.688 | .2 | 1.888 | 2.612 |
| 4 | .844 | .4 | 1.244 | 3.256 |
| 6 | .563 | .6 | 1.163 | 3.337 |
| 9 | .375 | .9 | 1. | |

$$\frac{4.5\,\cancel{\varepsilon}}{\cancel{300}} - \frac{\cancel{\varepsilon}\,10^{7}\,\lambda\cdot 10^{-8}}{\cancel{300}} - \frac{\varepsilon^{2}\times \cancel{300}}{4\lambda\cdot 10^{-8}} = H_{ev}$$

$$4.5 - \lambda\cdot 10^{-1} - \frac{\overset{1.725}{\cancel{4.8}}\times \overset{3.375}{\cancel{300}}\times \cancel{10^{8}}\times\cancel{10^{-16}}}{\cancel{4}\lambda}$$

$$4.5 - \lambda\cdot 10^{-1} - \frac{3.6}{\lambda} = H_{ev}$$

$$\cancel{H} = E - V = eFx - \frac{e^2}{4x} + E = \frac{p^2}{2m}$$

$$p = (2m)^{1/2}\sqrt{\varepsilon F x - \frac{\varepsilon^2}{4x} + E}$$

$$\xi = \frac{x\, 4W_a}{\varepsilon^2} \qquad x = \frac{\xi \varepsilon^2}{4W_a} \qquad dx = \frac{\varepsilon^2}{4w_a} d\xi$$

$$p = (2m)^{1/2}\left(\frac{E^2 \varepsilon^3 F}{E^2 4W_a}\xi - \frac{\cancel{\varepsilon^2} 4W_a}{4\cancel{\varepsilon^2}\xi} + E\right)^{1/2}$$

$$= (2m)^{1/2}\left(\frac{E^2}{4W_a r^2}\xi - \frac{E\,W_a}{E\,\xi} + E\right)^{1/2}$$

$$= (2m)^{1/2}(E)^{1/2}\left(1 - \frac{W_a}{E\xi} + \frac{E\xi}{4W_a r^2}\right)^{1/2}$$

$$A = \frac{4\pi(2m)^{1/2}}{h}\frac{(E^{1/2})\varepsilon^2}{4W_a}\int_{\xi_1}^{\xi_2}\left(1 - \frac{W_a}{E\xi} + \frac{E\xi}{4W_a r^2}\right)^{1/2} d\xi$$

Compute

$\cancel{EM}\ F = \frac{10^7}{300}$ esu.

$W_i = 6$ e.v. $= \frac{6 \times \varepsilon}{300}$ erg

$W_a - W_i = 4.5$ ev $= \frac{4.5 \times \varepsilon}{300}$

$$x_1 = \frac{\frac{4.5 \times \cancel{\varepsilon}}{\cancel{300}}}{\frac{2 \cancel{\varepsilon} \times 10^7 \times \cancel{\varepsilon}}{\cancel{300}}} \left(1 - \sqrt{1 - \frac{\varepsilon^{\cancel{2}} 10^7}{\cancel{300} \times \frac{(4.5)^2 \times \cancel{\varepsilon^2}}{\cancel{300}}}}\right)$$

$$x_1 = \frac{4.5}{2 \times 10^7}\left(1 - \sqrt{1 - \frac{4.8 \times 10^{-3} \times \cancel{300}}{\cancel{4.5} \times 4.5}}\right)$$

$$= \overset{22.5}{\cancel{9}} \times 10^{-8}\left(1 - \left(1 - .07\overset{15}{1}1\right)^{1/2}\right)$$

$$\underbrace{1 - .036}_{.964}$$

$= 22\cancel{2}5 \times 10^{-8} \times .036 = .\overset{.81}{\cancel{324}} \times 10^{-8}$ cm.

$x_2 = 9 \times 10^{-8} \times \cancel{1}.964 = \overset{44.2}{\cancel{17.6}}6 \times 10^{-8}$ cm.

~~Max at $9 \times 10^{-8}$ cm.~~

$$x = \frac{1}{2}\left(\frac{4.8 \times 10^{-10} \times \cancel{300}}{\cancel{10^7}}\right)^{1/2} = \underline{6 \times 10^{-8}}$$

$10^{-16}$

at level $-E$

$\ne V = -E$

$E - V = 0$ at $E - eFx - \frac{e^2}{4x} = 0$

$$x_1 = \frac{-E}{2\varepsilon F}\left(1 - \sqrt{1 - \frac{\varepsilon^3 F}{E^2}}\right)$$

$$x_2 = \frac{-E}{2\varepsilon F}\left(1 + \sqrt{1 - \frac{\varepsilon^3 F}{E^2}}\right)$$

if $\begin{cases} E = -\varepsilon\sqrt{\varepsilon F} = -\varepsilon^{3/2} F \\ x_1 = x_2 \end{cases}$

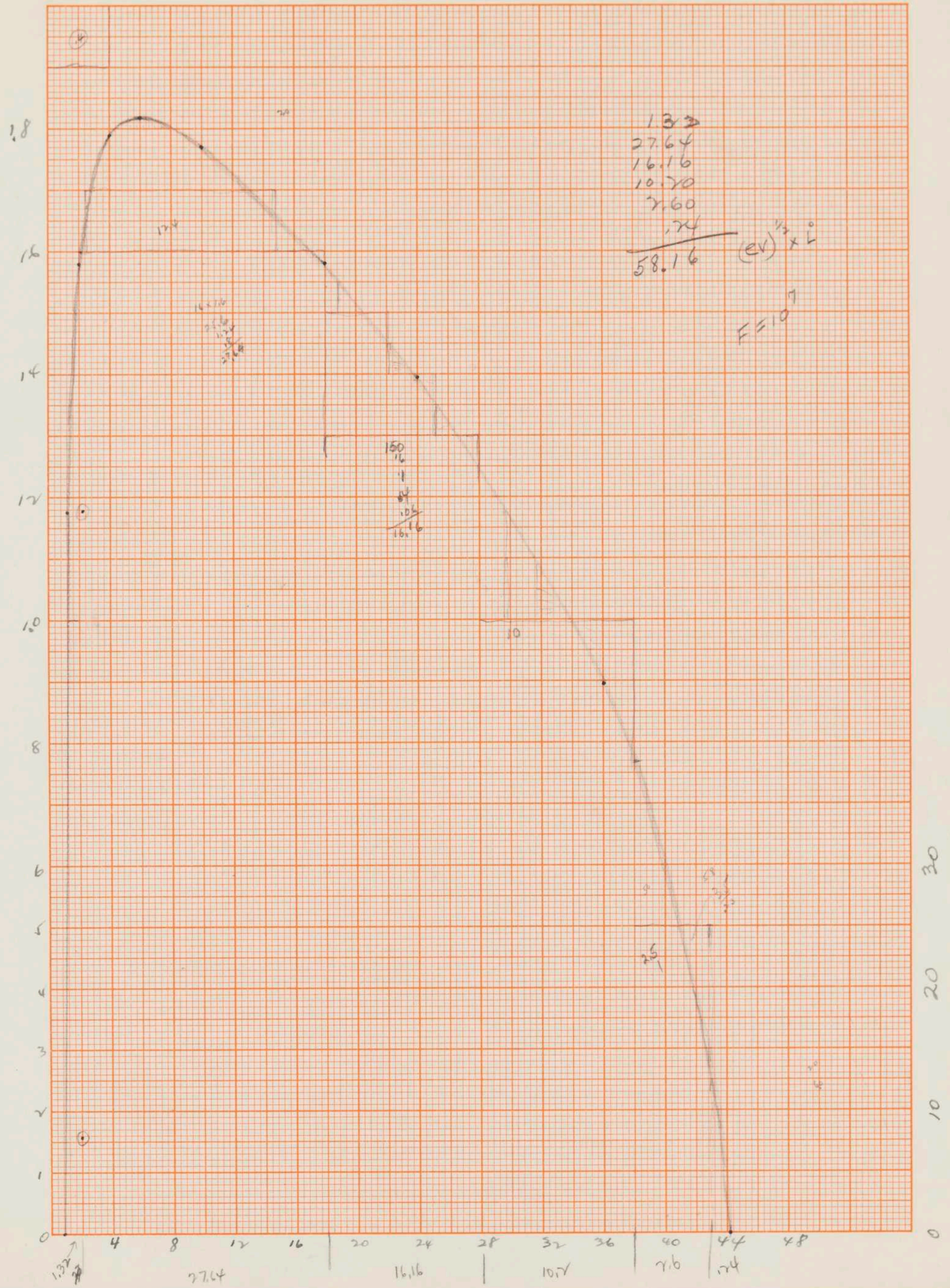
---

$$\xi_1 \equiv \frac{x_1}{x_a} = \frac{-E \; \cancel{4}^{2} W_a}{\cancel{2}\varepsilon F \; \varepsilon^2}\left(1 - \sqrt{1 - \frac{\varepsilon^3 F}{E^2}}\right)$$

$$r^2 = \frac{\cancel{4} E^2}{\varepsilon^3 F}$$

$$\xi_1 = -\frac{W_a}{E} r^2 \left(1 - \sqrt{1 - r^2}\right)$$

$$\xi_2 = -\frac{W_a}{E} r^2 \left(1 + \sqrt{1 - r^2}\right)$$


1.32
27.64
16.16
10.20
2.60
.24
58.16 (ev)^{1/2} × i
F = 10^7
1.8
16
14
12
10
8
6
5
4
3
2
1
0
4
8
12
16
20
24
28
32
36
40
44
48
30
20
10
1.32
27.64
16.16
10.2
2.6
.24

| $\lambda$ | | | | $H$ | $H^{1/2}$ |
|---|---|---|---|---|---|
| .815 | 4.42 | .08 | 4.5 | 0 | 0 |
| 2 | 1.8 | .2 | 2 | 2.5 | 1.58 |
| 4 | .9 | .4 | 1.3 | 3.2 | 1.79 |
| 6 | .6 | .6 | 1.2 | 3.3 | 1.817 |
| 10 | .36 | 1.0 | 1.36 | 3.14 | 1.77 |
| 18 | .2 | 1.8 | 2. | 2.5 | 1.58 |
| 24 | .15 | 2.4 | 2.55 | 1.95 | 1.395 |
| 36 | .1 | 3.6 | 3.7 | .8 | .895 |
| 44.2 | .0815 | 4.42 | 4.5 | 0 | .0 |
| 1.2 | 3 | .12 | 3.12 | 1.38 | 1.176 |

$$\text{"A"}^{1/2} = 58.16\ (ev)^{1/2}\ \mathring{A}$$

Fowler

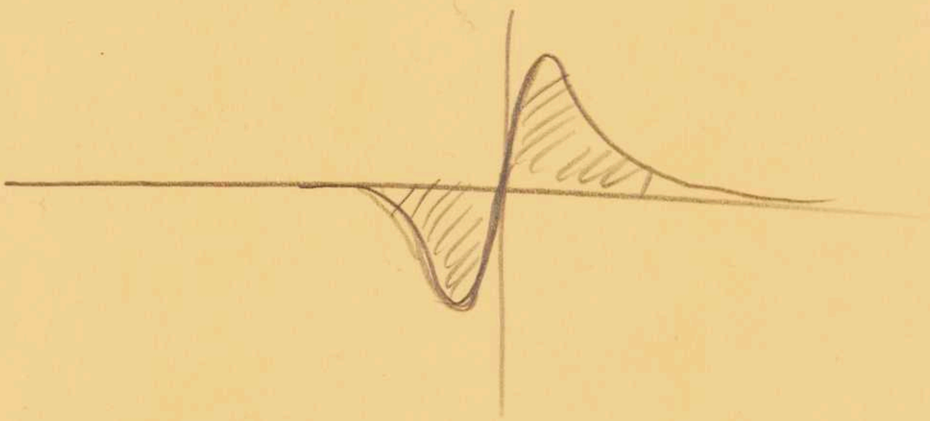
$$\cancel{\frac{2}{3}} \times \frac{\cancel{(4.5)^{3/2}}\ 4.5 \times \left(\frac{4.5}{300}\right)^{1/2}}{\underset{2.4}{\cancel{4.8}}\times 10^{-16} \times \frac{10^{7}}{\cancel{300}}}\ 1.225\times10^{-1}\times10^{5} = 2.3\times10^{4}$$

$$\cancel{\frac{2}{3}} \times \frac{(4.5)^{1/2}\ \frac{4.5}{300}\ 1.5 \times 4.8\times10^{-10} \times 10^{3}}{\underset{2.4}{\cancel{4.8}} \times \frac{10^{7}}{\cancel{300}}\ \cancel{\times 10^{-10}}} = \cancel{1.325\times10^{3}} = 63.6\times10^{-8}$$

for the first two terms of $\overline{E}_0$

$$\overline{E}_0 = \mu - \frac{2kT\left(\frac{\pi^2}{12} - e^{-\mu}(\mu+1) + \frac{1}{2}e^{-2\mu}(\mu+\frac{1}{2}) - \frac{e^{-3\mu}}{3}(\mu+\frac{1}{3}) + \cdots\right)}{\ln 2 - \ln(1+e^{-\mu})}$$

$$\overline{E}_0 \doteq \mu - \frac{\pi^2 kT}{6 \ln 2} = \mu - \frac{1.64\, kT}{\ln 2} = \mu - 2.37\, kT$$



$$\frac{4\pi^{\#}(2m)^{\frac{1}{2}}}{h} \quad \frac{\pi}{4} \quad \frac{(4.5)^{\frac{1}{2}}\ \frac{4.5 \times 4.8 \times 10^{-10}}{300}}{4.8 \times 10^{-16} \times \frac{10^{7}}{300}} = 75 \times 10^{-8}$$

$$\left(1 - \frac{1.2}{4.5}\right)^{\frac{1}{2}} = .856$$

$64.1 \times 10^{-8}$

$.856 \times .963 = .823$

$$(1 - .071)^{\frac{1}{2}} = .963$$

$$\begin{array}{r} 61.6 \times 10^{-8} \\ 58.16 \\ \hline 3.4 \end{array} \qquad \begin{array}{r} 63.6 \\ 58.2 \\ \hline 5.4 \end{array}$$

5.8% high

9.3% high