Vacuum Tube Committee, Notes, 1929-30

W. B. Nottingham

X 389

Media 225

9/15/30

54245 Grayparts
910 Cherry

M W Deeki

9/6/29

350
4
1400

1.4×10^{-6}

$2.2 \times 4 \times 10^{-9}$

8.8×10^{-9}

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Vacuum Tube Committee Meeting September 15, 1930

Present: Messrs. Dike, Nottingham, Peters, Behr

CC - Messrs. Stein and Doyle

Williams described the 60 cycle amplifier at some length. The most serious difficulty was the coupling of the galvanometer to the output. The galvanometer stability was considered satisfactory when the electrical control did not exceed one-third the mechanical control. A satisfactory circuit for the galvanometer has a leading phase angle of 0.2 degree and this angle should be constant to 0.1 degree. The shunt inductance of a transformer would cause the phase angle to be about twenty degrees, and if this is to be reduced by means of a condenser, a rather precise balance is necessary, which is moreover not constant because of the effect of d.c. plate current on the transformer characteristics. A resistance capacity coupling has therefore been used, as this is not sensitive to comparatively wide variations (±10%) in the elements.

The work on the Ives optical pyrometer has been principally a study of the properties of the important elements, including the high resistances, vacuum tubes, photocell and voltage regulator.

High Resistance. The thin metal film resistances have a high temperature coefficient and questionable stability. Wire wound resistances, in units of 2-1/2 megohms, can be purchased for about $8.00. Their temperature coefficient is 0.0001 per degree C and the guaranteed accuracy is 1 %. A group of four is to be used to obtain ten megohms.

Vacuum Tubes. When it became necessary to replace the tubes in the amplifier it was impossible to find any in the radio shops with a sufficiently small grid current. Tubes have been obtained from Westinghouse, however, similar to a 250, but apparently with a better vacuum and these are satisfactory. Bell Telephone representatives have informed us that well evacuated tubes similar to 101D, 102D and 104D are being made. We have not yet received them, however.

Photocell. We are principally interested in the temperature coefficient and the change with time in the shape of the wave length-sensitivity characteristic. From what information can be gathered, the temperature coefficient is apparently small. No definite information is available in regard to the second point and it is proposed to check up on it here, by using the photocell as an optical pyrometer to measure black body temperatures over as wide a range of temperature as possible.

"copy" Mr. W. B. Nottingham

July 3, 1930.

TO: Messrs. J. V. Adams
L. Behr
A. J. Williams, Jr.

FROM: C. S. Redding

Re: Record of Invention No. 127 - "Two-Speed Drive System"

You will be interested in learning that we have instructed Mr. Ehret to proceed with a patent application covering Record of Invention No. 127 - "Two-Speed Drive System". In preparing this application, Mr. Ehret has been requested to have in view the following records of invention:-

No. 128 - "Armature for Quick Acting Electromagnetic Brake" - Adams

No. 129 - "Microphone Detector Amplifier" - Williams

No. 132 - "Elimination of Pickup in Low Range Potentiometer Circuits" - Nottingham

No. 134 - "Special Shielded Core Type Transformer" - Williams

No. 136 - "A Circuit for Tandem Power Tubes" - Behr and Williams

C. S. R.

P

cc Mr. W. B. Nottingham ✓

Vacuum Tube Committee Meeting September 29, 1930

Present: Messrs. Dike, Nottingham, Peters, Behr

CC- Messrs. Stein and Doyle

In regard to the Ives optical pyrometer the following figures were given for photo-cell current versus temperature of source:

Temperature of Source	Current - Amps. x 10^{-9} Gas Filled Cell	Vacuum Cell
1800 °F.	38	—
2200 °F.	190	5.+
2600 °F.	540	34
3000 °F.	1100	180

Some photo-cells have been ordered from General Electric and it is intended to put these and some Westinghouse and Western Electric cells through an aging test.

The "dc-ac" amplifier using a chopper has been set up. With a pointer type ac. galvanometer in the output circuit the sensitivity is 5 mv. per ~~minute~~ for input impedances not exceeding 10 megohms. The sensitivity goes down as the input impedance increases and with 50 megohms the sensitivity is 10 mv.

The next meeting of this Committee will be held on October 13th.

L.B.

R

October 20, 1930

X-389 - Vacuum Tube Committee Meeting of Oct. 20, 1930

Present: Messrs. Dike, Nottingham, Peters, Williams and Behr

CC- Messrs. Stein and Doyle

Some of the high vacuum amplifier tubes, referred to in the minutes of September 15, have been received and their characteristics measured. Three Western Electric D 86326 tubes, similar to the 101D, gave grid currents of 4.5, 5 and 14 x 10-9 amperes with 90 volts on the plate, approximately -4. volts on the grid and with 3 milliamperes in the plate circuit. Two Westinghouse 593428 tubes, similar to the 250, gave grid currents of 2 and 6 x 10^{-9} amperes with the plate at 90, the grid at -8. volts and with a plate current of 6 milliamperes. The mutual conductance of the Western Electric tubes is about 450 and of the Westinghouse tubes 600 micro-mhos.

The apparatus for the photocell tests is about set up and the tests should get under way at an early date. The voltage regulators on order with Ward Leonard have not yet been received.

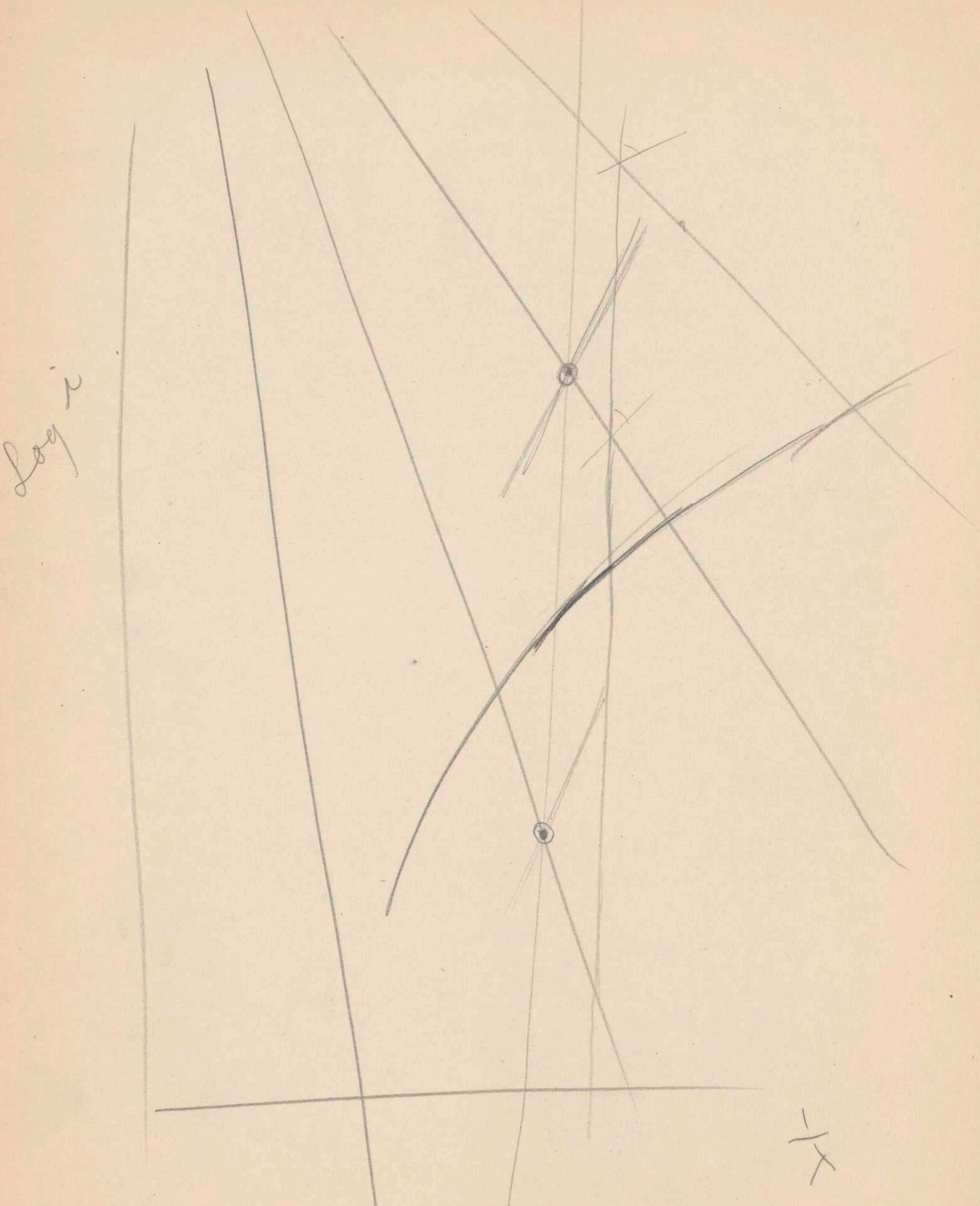
The chopper type dc-ac amplifier has been provided with an additional stage of amplification resulting in a sensitivity of 100 micro-volts per mm. With a 10 megohm resistance this means 10^{-11} amperes per mm. However, the stability is bad, the interrupter being under suspicion. The immediate plan of attack is to try various contact materials.

There was a general discussion of the use of the thyratron in conjunction with a photocell, particularly as regards the effect of the thyratron grid current.

At the meeting of June 30, it was pointed out that Bonn's capacity bridge was not suited for an industrial limit bridge because of poor stability and that further work on this problem should logically await the completion of the 60 cycle amplifier. The 60 cycle amplifier having been completed, it is intended to start on the limit bridge as soon as the other active jobs permit.

The next meeting of this Committee will be held on November 3.

L.B.


Log i

<u>Voltage Regulator</u>. The voltage regulator has been a source of trouble because of its short life. The difficulty is apparently due to poor design rather than wrong principle of operation. Ward-Leonard are to supply us with three regulators for test which are to meet the following specifications:

> Output voltage 114-116 volts, input 105-125 volts, frequency 59-61 cycles, load 50 watts unity power factor.

The next meeting of this committee will be on September 29.

HU L. B.

W. B. Nottingham

Media 225

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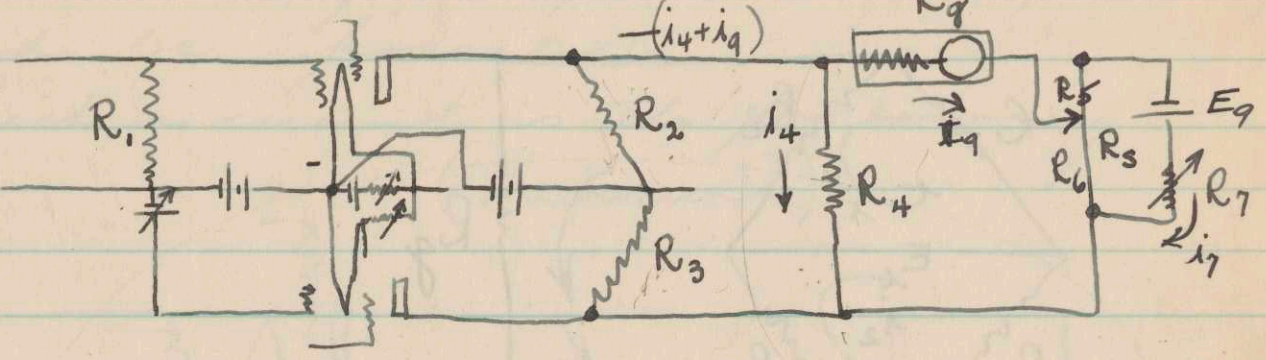
Calculations on D.C. amplifier of Balanced type.

fig 1



given

$R_g = 1400^{\omega}$ $\quad$ $i_g = 5\times10^{-7}$ per step

$R_4 = 3000^{\omega}$

$R_4 i_4 = i_g(R_g + R_5')$ $\quad$ where R_5' = combination of $R_5, R_6, \& R_7$

Per step

$R_4 i_{41} = 5\times10^{-7}(1400 + 100)$

$= 7.5\times10^{-4} = 7.5\times10^{-4}$

$i_{41} = \frac{7.5\times10^{-4}}{3.0\times10^{3}} = 2.5\times10^{-7}$

For 400 steps

$i_4 = 10^{-4}$ amp.

$R_4 i_4 = 3000\times10^{-4} = 0.3$ volt max IR

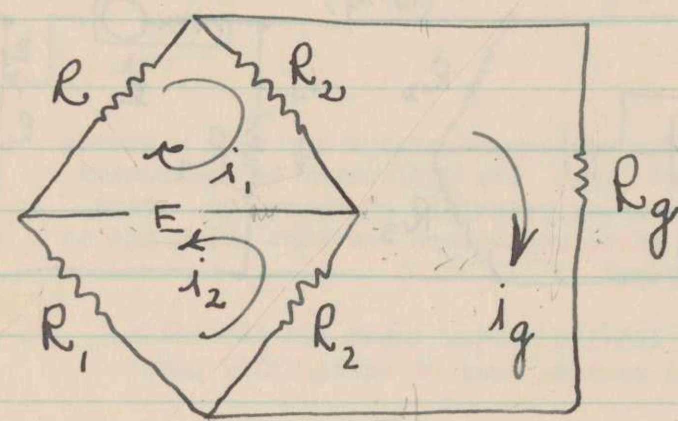
Take $E_g = 1.5$

$R_5 = 150^{\omega}$

$i_7 = \frac{0.3}{150} = 2\times10^{-3}$ amp.

$R_7 =$ ~~2×10~~ $\frac{1.2}{2\times10^{-3}} = 600^{\omega}$

Bridge problem to find relation between galvanometer current and circuit resistances



$$\begin{cases} i_1 R + (i_1 - i_g) R_2 = E \\ i_2 R_1 + (i_2 + i_g) R_2 = E \\ i_g R_g + (i_g + i_2) R_2 + (i_g - i_1) R_2 = 0 \end{cases}$$

Solution of these gives

$$i_g = \frac{E(R_1 - R) R_2}{R_2^2 (R_1 + R) + 2 R R_1 R_2 + (R_1 + R_2)(R + R_2) R_g}$$

This solution shows that the bridge circuit shown in fig 1 will not be compensated for changes in E because the i_4 of that circuit which is analogous to i_g of the simplified circuit is a linear function of E. If the recorder operated some compensating

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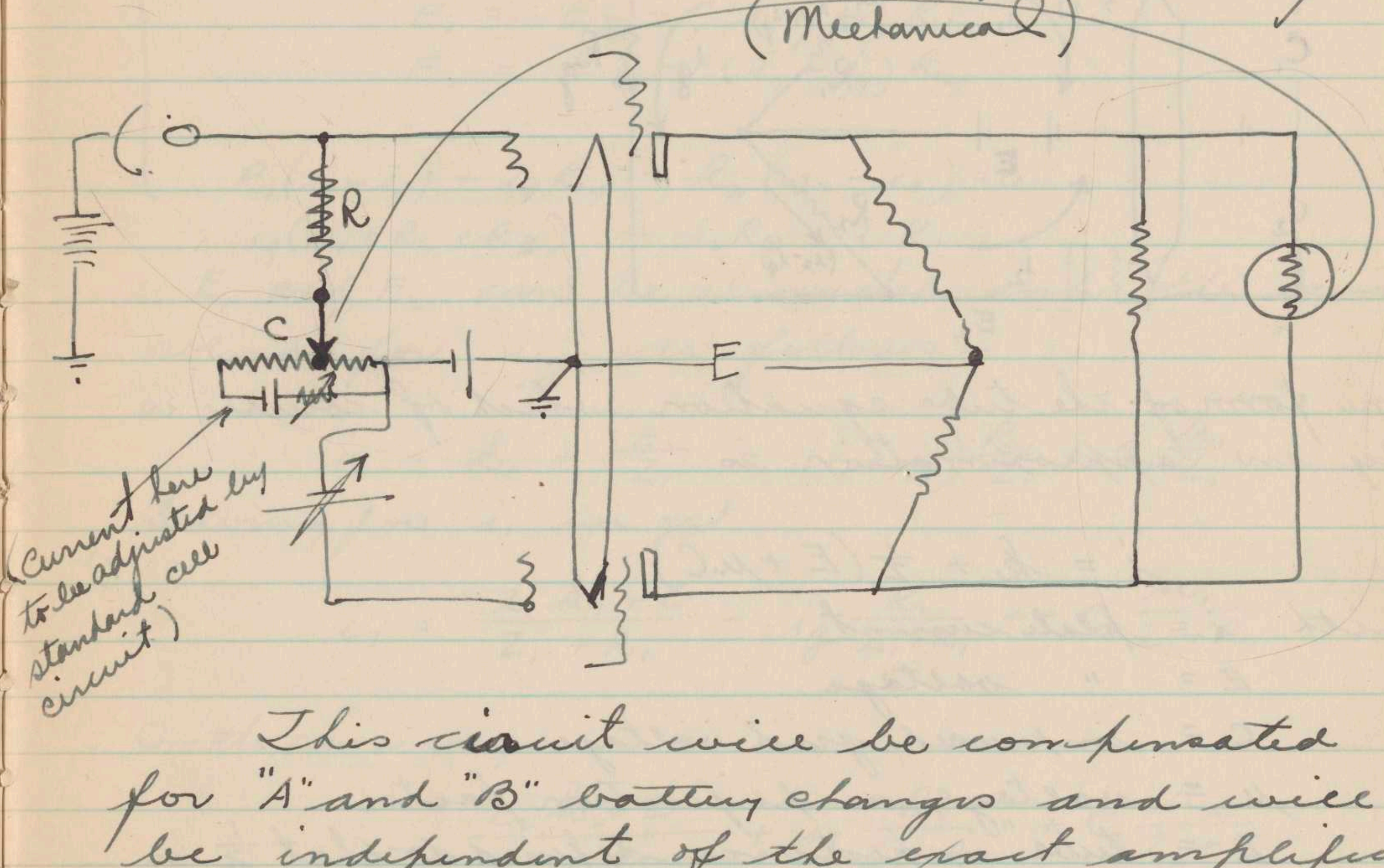
mechanism in the input circuit to bring about the balance then a compensation for plate battery changes would be possible.

Such a circuit is the following.



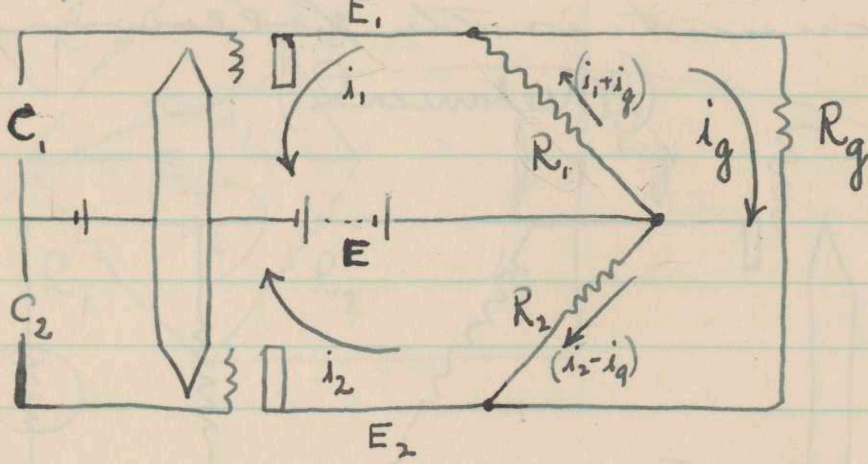
This circuit will be compensated for "A" and "B" battery changes and will be independent of the exact amplification factor of the vacuum tubes once they have been balanced for power changes etc.

The apparent limitations are:-

1.) R must be small enough to make the "IR" drop due to grid leakage current small.

2) A contact at "C" is apparently necessary but since this is a high resistance circuit the disadvantage should not be so great.

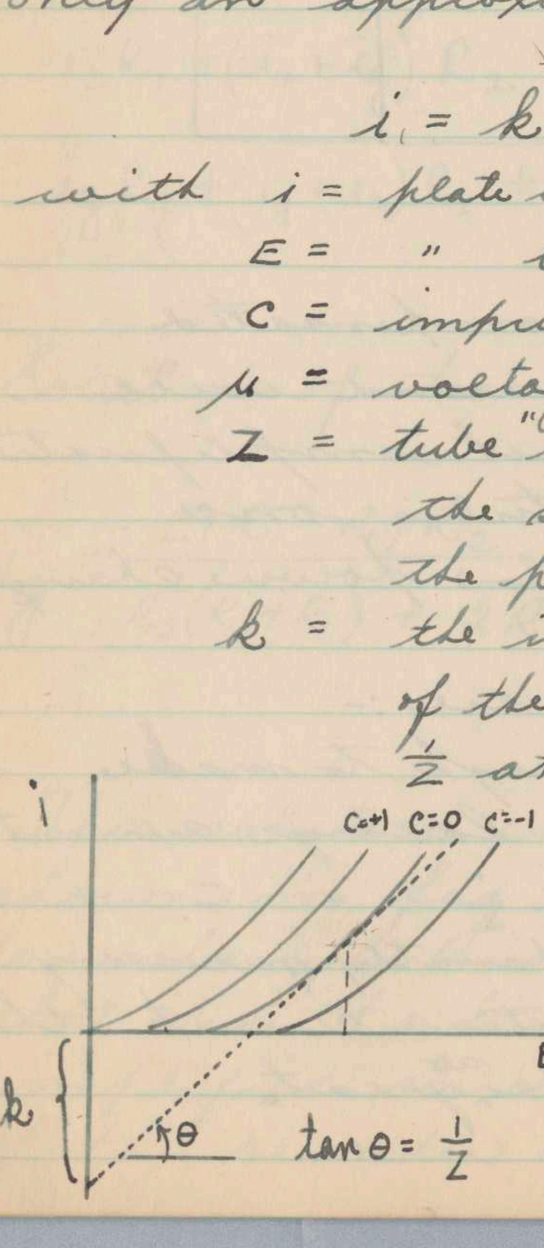
The following analysis of the balanced tube problem shows more definitely the possibilities and the limitations of this method of amplification.—



One form of the tube equation which of course is only an approximation is

$$i_1 = k_1 + \frac{1}{Z_1}(E_1 + \mu C) \qquad (1)$$

with i = plate current
E = " voltage
C = impressed grid voltage
μ = voltage amplification factor
Z = tube "impedance" defined so that $\frac{1}{Z}$ is the slope of the i vs E curve at the point of operation.
k = the intercept on the current axis of the tangent drawn with slope $\frac{1}{Z}$ at the point of operation when $C=0$



In case the $(i\text{--}E)_c$ curve with constant c is curved than $k = f(E)$ and $Z = F(E)$ and are not constants.

Using the notation in the figure we have

$$i_1 = k_1 + \frac{1}{Z_1}(E_1 + \mu_1 C_1) \qquad (2)$$
$$i_2 = k_2 + \frac{1}{Z_2}(E_2 + \mu_2 C_2) \qquad (3)$$

$$E_1 = E - (i_1 + i_g) R_1 \qquad (4)$$
$$E_2 = E - (i_2 - i_g) R_2 \qquad (5)$$

$$R_1(i_1 + i_g) + i_g R_g - R_2(i_2 - i_g) = 0 \qquad (6)$$
$$i_g(R_1 + R_2 + R_g) = i_2 R_2 - i_1 R_1$$

E_1 and E_2 can be eliminated and the equations solved for i_g as follows:-

$$i_1 = k_1 + \frac{E}{Z_1} - (i_1 + i_g)\frac{R_1}{Z_1} + C_1 \frac{\mu_1}{Z_1}$$

Solving for i_1 we get

$$i_1 = \frac{Z_1 k_1 + E}{Z_1 + R_1} - i_g \frac{R_1}{Z_1 + R_1} + C_1 \frac{\mu_1}{Z_1 + R_1}$$

In the same way

$$i_2 = \frac{Z_2 k_2 + E}{Z_2 + R_2} + i_g \frac{R_2}{Z_2 + R_2} + C_2 \frac{\mu_2}{Z_2 + R_2}$$

$$i_g(R_1 + R_2 + R_g) = \frac{Z_2 k_2 + E}{Z_2 + R_2} R_2 + C_2 \frac{\mu_2 R_2}{Z_2 + R_2} + i_g \frac{R_2^2}{Z_2 + R_2}$$
$$- \frac{Z_1 k_1 + E}{Z_1 + R_1} R_1 - C_1 \frac{\mu_1 R_1}{Z_1 + R_1} + i_g \frac{R_1^2}{Z_1 + R_1}$$

Collecting terms we get

$$\underbrace{i_g\left(R_1 + R_2 + R_g - \frac{R_1^2}{Z_1 + R_1} - \frac{R_2^2}{Z_2 + R_2}\right)}_{= X} = \underbrace{-\frac{R_1}{Z_1 + R_1}(Z_1 k_1 + \mu_1 R_1 + E) + \frac{R_2}{Z_2 + R_2}(Z_2 k_2 + \mu_2 R_2 + E)}_{= Y}$$

$$(Z_1+R_1)(Z_2+R_2)X = R_1Z_1Z_2 + R_2Z_1Z_2 + R_gZ_1Z_2 + R_1Z_1R_2 + R_2Z_1R_2 + R_gZ_1R_2$$
$$R_1R_1Z_2 + R_2R_1Z_2 + R_gR_1Z_2 + R_1R_1R_2 + R_2R_1R_2 + R_gR_1R_2$$
$$- R_1^2 Z_2 \qquad - R_1^2 R_2 - R_2^2 Z_1 - R_2^2 R_1$$

$$(Z_1+R_1)(Z_2+R_2)X = R_1Z_1Z_2 + R_2Z_1Z_2 + R_1R_2Z_1 + R_1R_2Z_2 + R_g(Z_1+R_1)(Z_2+R_2)$$

$$(Z_1+R_1)(Z_2+R_2)Y = (R_2Z_1+R_1R_2)(Z_2k_2+c_2\mu_2+E) - (R_1Z_2+R_1R_2)(Z_1k_1+c_1\mu_1+E)$$

$$i_g = \frac{(R_2Z_1+R_1R_2)(Z_2k_2+c_2\mu_2+E) - (R_1Z_2+R_1R_2)(Z_1k_1+c_1\mu_1+E)}{R_1Z_1Z_2+R_2Z_1Z_2+R_1R_2Z_1+R_1R_2Z_2+R_g(Z_1+R_1)(Z_2+R_2)} \quad (7)$$

$$i_g = \frac{(R_2Z_1+R_1R_2)(Z_2k_2+c_2\mu_2) - (R_1Z_2+R_1R_2)(Z_1k_1+c_1\mu_1) + (R_2Z_1-R_1Z_2)E}{R_1Z_1Z_2 + \ldots} \quad (8)$$

This equation shows that i_g is independent of the ~~plate~~ plate ~~+~~ battery supply when

$$R_2Z_1 - R_1Z_2 = 0 \quad (9)$$

This conclusion requires that over the range that E is permitted to change Z, k, and μ must be independent of E.

Under this condition $R_1Z_2 = R_2Z_1 = W^2$ (10)

$$i_g = \frac{\left(1+\frac{R_1R_2}{W^2}\right)(Z_2k_2+c_2\mu_2 - Z_1k_1 - c_1\mu_1)}{R_1+R_2+Z_1+Z_2+R_g\left(\frac{R_1R_2}{W^2}+\frac{Z_1Z_2}{W^2}+2\right)} \quad (11)$$

If it were possible to select tubes with
$Z_1 = Z_2$ and $k_1 = k_2$ and $\mu_1 = \mu_2$
and we adjusted $R_1 = Z_1 = R_2$
Then

$$\left(i_g\right)_{R=Z} = \frac{1}{2} \cdot \frac{\mu(C_2 - C_1)}{(Z + R_g)} \qquad (12)$$

This equation shows that if the sensitivity is defined as

$$\left(S\right)_{R=Z} = \frac{i_g}{C_2 - C_1} = \frac{1}{2} \frac{\mu}{(Z + R_g)} \qquad (13)$$

Since in general $Z \gg R_g$ we see that

$$S = \frac{1}{2} \frac{\mu}{Z} = \frac{1}{2} G_m \text{ where } G_m \text{ is the} \qquad (14)$$

"mutual conductance" of the tube use and we see that the sensitivity of the "balanced bridge" circuit is exactly $\frac{1}{2}$ that of the <u>straight</u> amplifier circuit.

General considerations of <u>sensitivity</u> and <u>stability</u> when the bridge type of circuit is <u>not used</u>.

Assume the tube equation (1)

$$i = k + \frac{1}{Z}(E + \mu C) \qquad (1)$$

$S = \frac{\partial i}{\partial C} = \frac{\mu}{Z} = G_m$ showing that sensitivity is proportional to the mutual conductance under the usual conditions.

$\frac{\partial i}{\partial E} = \frac{1}{Z}$ which is thus a measure of the <u>instability</u>

of a ~~gu~~ tube. In an application we should use the maximum G_m ~~consistents~~ consistent with the stability required.

Since the stability might be taken as given by $\frac{1}{\text{instability}} = \bar{S} = Z$ (15)

This gives for the product

$$S \cdot \bar{S} = \frac{\mu}{Z} \cdot Z = \mu \quad (16)$$

Showing that considering both the sensitivity and the stability the value of "μ" can be taken as a measure of the value of a tube for direct-current amplification circuits.

The following table of tube characteristics shows that the UX 112 tube should have the maximum sensitivity but would be instable with power.

The UX 222 is only $\frac{1}{5}$ as sensitive but should be about 200 times as stable.

TYPE	USE	"A" BAT V.	TERM "A" V.	FIL CURRENT	DET "B" V.	AMP "B" V.	-C V.	PLATE CURRENT V	PLATE RES.	MUT. COND. Gm MICROMHOS	VOLTAGE AMP μ
UX-199	Amp Det	3.0 to 4.5	3.3	.063	22.5 to 45.0	45.0 67.5 90.0	.5-1.5 1.5-3.0 4.5	1.0 1.7 2.5	19,500 16,500 15,000	320 380 415	6.25
UX-120	P AMP	4.5	3.3	.132		90 135	16.5 22.5	3.2 7.0	7,700 6,600	428 500	3.3
UX-201-A		6.0	5.0	.25	45	45 67.5 90. 135.	.5-1.5 1.5-3.0 4.5 9.0	0.9 1.7 2.0 2.5	18,500 14,000 10,000 10,500	430 570 725 760	8.0
UX-240		6.0	5.0	.25	90	90 135 180	1.5 3.0 4.5	0.2 0.2 0.2	150,000 " "	200 " "	30.0
UX112		6.0	5.0	.25	45	90 135 157 180	4.5 9.0 10.5 13.5	4.8 5.8 7.9 7.8	5,300 5,000 4700 4700	1500 1600 1700 1700	8.0
UX226		A.C	1.5	1.05		90 135 180	6.0 12.0 16.5	3.7 3.0 3.8	9,400 10 000 9400	870 820 870	8.2
UX222			3.3	.123		135			1,100,000	300	330

Taken from

Cunningham Tube Data Book

October 1927 Price $2.50

When a bridge circuit is used then equation 8 shows that the circuit is perfectly stable with plate voltage when

$$R_2 Z_1 = R_1 Z_2$$

This result depends on the ~~constankey~~ constancy of "Z" for ~~st~~ small changes in E and C.

Dependence of sensitivity S on the ~~plate~~ resistance in the plate circuit R.

From equation (8) we have

$$i_g = \frac{\left(1+\frac{R_1R_2}{W}\right)\left(Z_2k_2 + c_2\mu_2 - Z_1k_1 - c_1\mu_1\right)}{R_1+R_2+Z_1+Z_2 + R_g\left(\frac{R_1R_2}{W} + \frac{Z_1Z_2}{W} + 2\right)} \qquad (8)$$

assume $Z_1 = Z_2 = Z$; $k_1 = k_2$; $\mu_1 = \mu_2$; and $R_1 = R_2 = R$

$$i_g = \frac{\left(1+\frac{R}{Z}\right)\left(c_2 - c_1\right)\mu}{2R + 2Z + R_g\left(\frac{R}{Z} + \frac{Z}{R} + 2\right)} \qquad (17)$$

$$S = \frac{\mu\left(1+\frac{R}{Z}\right)}{2R + 2Z + R_g\left(\frac{R}{Z} + \frac{Z}{R} + 2\right)} \qquad (18)$$

If there is value of R for which S is a max. we find it by $\frac{\partial S}{\partial R} = 0$

$$\left[2R+2Z+R_g\left(\frac{R}{Z}+\frac{Z}{R}+2\right)\right]\frac{\mu}{Z} - \mu\left(1+\frac{R}{Z}\right)\left(2+\frac{R_g}{Z} - \frac{Z}{R^2}\right) = 0 \qquad (19)$$

Solving this equation for R we get

$$R = \pm\sqrt{-Z} \qquad (20)$$

which shows that for positive values of Z and R there is no max.

For small values of R compared with Z we have

$$S = \frac{\mu R}{Z(2R+R_g)} \qquad (21)$$

For large values of R we see that S approaches a value given by (22) which is independent of R.

Let $X = (1+\frac{R}{Z})\mu$

$Y = 2R + 2Z + R_g(\frac{R}{Z} + \frac{Z}{R} + 2)$

$$\left(\frac{dX}{dR}\right)_{R=\infty} = \frac{1}{Z}$$

$$\left(\frac{dY}{dR}\right)_{R=\infty} = 2 + \frac{R_g}{Z}$$

$$\therefore \quad (S)_{R=\infty} = \frac{\mu \frac{1}{Z}}{2 + \frac{R_g}{Z}} = \frac{\mu}{2Z + R_g} \qquad (22)$$

It is of interest to compare this with that of $(S)_{R=Z}$ when $R=Z$ given in equation (13)

$$(S)_{R=Z} = \frac{\mu}{2Z + 2R_g}$$

Now that we see the general way in which S depends on R we can get a more useful expression by letting $R = mZ$ where in general applications $m < 1$. Equation (18) then reduces to the following

$$S = \frac{\mu}{2Z + R_g\left(\frac{m+1}{m}\right)} \qquad (23)$$

Let us define a quantity "F" as $\frac{(S)_{R=mZ}}{(S)_{R=\infty}} = F$

This "F" is then the fraction of the greatest possible sensitivity which will be obtained with a given value of $m = \frac{R}{Z}$

Using equations (22) and (23)

$$\frac{2Z + R_g}{2Z + R_g\left(\frac{m+1}{m}\right)} = F \qquad (24)$$

From which we can calculate F from given values of m.

Solving this equation for "m" we get

$$m = \frac{R_g}{2Z + R_g} \times \frac{F}{1-F} \qquad (25)$$

If we introduce $M = \frac{Z}{R_g}$

$$\text{we get } m = \frac{1}{2M+1} \times \frac{F}{1-F} \qquad (26)$$

Taking $M = \frac{Z}{R_g}$ and $L = \frac{R}{R_g}$

we find from (24) that

$$F = \frac{1}{1 + \frac{1}{L\left(2 + \frac{1}{M}\right)}}$$

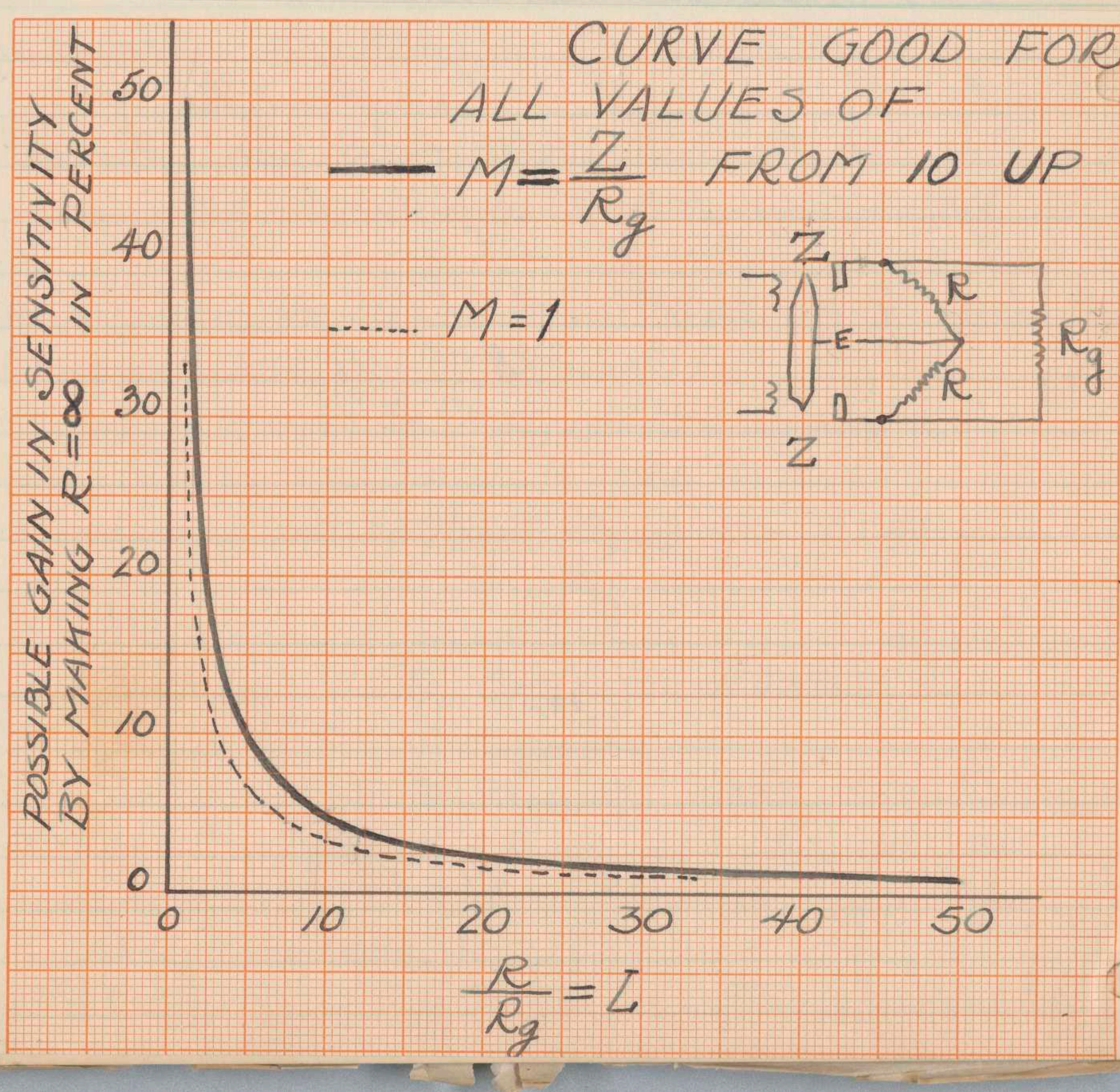
$\frac{L}{M} = m$

Since $100\left(\frac{1}{F}-1\right)$ is the ~~loss~~ gain in sensitivity in percent which we ~~have~~ could realize by having $R = \infty$

Possible gain in sensitivity $= 100\left(\frac{1}{F}-1\right) = 100\left(\frac{S_\infty - S_R}{S_R}\right)$

$$\text{Percent gain} = \frac{100}{L\left(2+\frac{1}{M}\right)} \qquad (27)$$

which is shown in the attached graph.



One interpretation which can be made on the basis of this calculation is that the value of R which it is practical to use is determined primarily by the value of R_g. Therefore if $R_g = 1000^\omega$ then R need not exceed $10,000^\omega$ to $20,000^\omega$ so long as $Z > 10,000^\omega$.

Numerical example:-

Given (1) the circuit shown in figure on p. 3.
(2) galvanometer resistance with proper shunt $R_g = 1000^\omega$ (i.e. about 1500^ω coil with 3000^ω shunt)
(3) Min. useful deflection produced by 5×10^{-7} amp.

Take $R = 20,000^\omega$ which will give ~~within~~ about 98% the max possible sensitivity

Using equation (23) in terms of L + M since $m = \frac{L}{M}$

$$S = \frac{\mu}{2Z + R_g\left(\frac{L+M}{L}\right)}$$

Case I - UX112

$L = 20$ $\quad M_{112} = 5$
$Z_{112} = 5000^\omega$
$\mu_{112} = 8$

approx
$E_B = 300$ volts
$I_B = 16$ mils
$E_c = -9$ volts
$I_A = 0.5$ amp.

$$S_{112} = \frac{.8}{10,000 + 1000\,\frac{25}{20}} = 7.1 \times 10^{-4} \text{ amp. per volt}$$

Min. useful voltage $\frac{5 \times 10^{-7}}{7.1 \times 10^{-4}} = 7 \times 10^{-4}$ volt

Thus the voltage "amplification" of this circuit is

$$\frac{(V_g)_{min}}{(V_c)_{min}} = \alpha_V = \frac{1000 \times 5 \times 10^{-7}}{7 \times 10^{-4}} = 0.71$$

which is of course an actual loss.

The current amplification on the other hand is given by

$(i_c)_{min} R_c = 7 \times 10^{-4}$ = min useful voltage

$(i_g)_{min} = 5 \times 10^{-7}$ = min galv. current

~~$\alpha_c = \frac{i_g}{i_c} = \frac{5 \times 10^{-7}}{7 \times 10^{-4}} R_c = 7.1 \times 10^{-4} R_c$~~

~~Since R_c can be as high as 100 megs in many applications with photo-cells, etc.~~

~~$\alpha_c = 7.1 \times 10^{4} = 71,000$~~

$$(i_c)_{min} = \frac{7 \times 10^{-4}}{R_c} = \frac{7 \times 10^{-10}}{R_c'}$$

where R_c' is expressed in "megs."

$$\alpha_c = \frac{i_g}{i_c} = \frac{5 \times 10^{-7}}{7 \times 10^{-10}} = 710 \times R_c'$$

UX-112

R_c'	α_c	$(i_c)_{min}$	V per step	400 steps
1	710	7×10^{-10}	7×10^{-4}	.28 volt
10	7,100	7×10^{-11}	"	"
100	71,000	7×10^{-12}	"	"
1000	710,000	7×10^{-13}	"	"

Case II - UX 222

Approx.

$L = 20 \qquad \mu = 300 \qquad \left\{ E_B = +160 \text{ volts} \right.$

$M = 1000 \qquad I_B = 3 \text{ mils}$

$Z = 10^6 \qquad E_c = -1.5 \text{ volts}$

$I_A = .3 \text{ amp.}$

$$S_{222} = \frac{300}{2\times10^6 + 1000\left(\frac{1020}{20}\right)} = 1.46\times10^{-4} \text{ amp/volt}$$

Min. useful voltage $= 34.2\times10^{-4}$

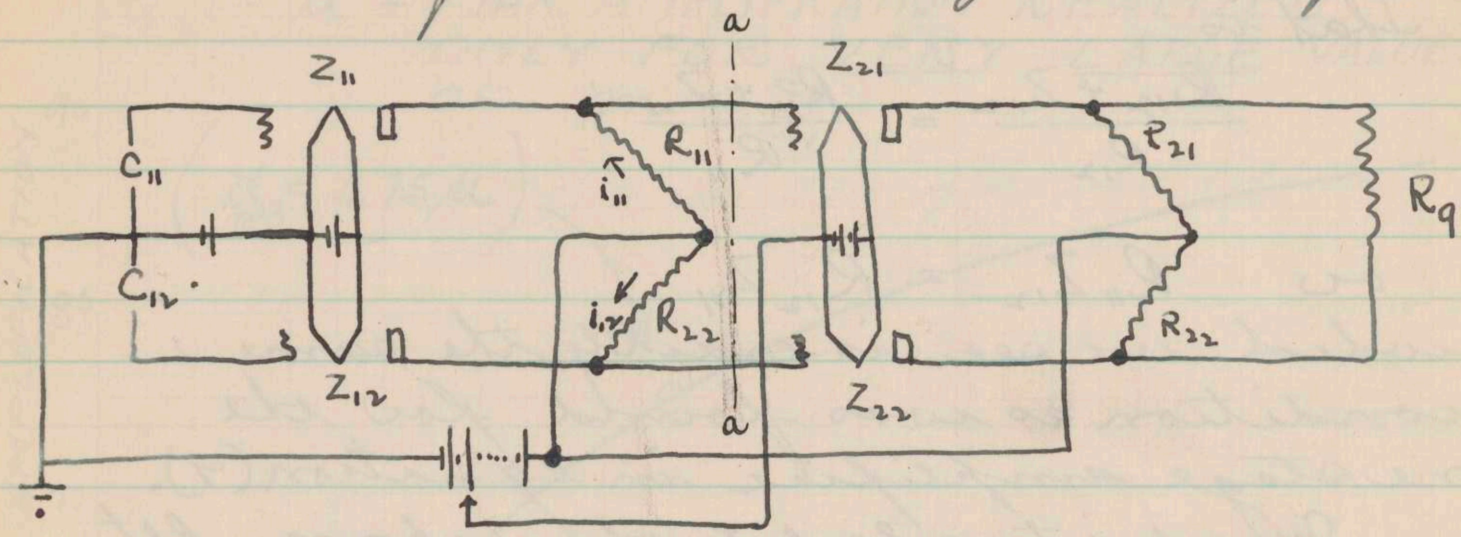
Voltage for 400 steps $= 1.368$ volts.

$\alpha_c = 146\, R_c'$

UX-222

R_c'	α_c	$(i_c)_{min}$
1	146	3.4×10^{-9}
10	1,460	3.4×10^{-10}
100	14,600	3.4×10^{-11}
1000	146,000	3.4×10^{-12}

Problem of the two stage amplifier



Assuming the tube equation and that grid current in the tubes of the second stage can be neglected.

$$i_{11} = k_{11} + \frac{1}{Z_{11}}(E_{11} + \mu_{11} C_{11})$$

$$i_{12} = k_{12} + \frac{1}{Z_{12}}(E_{12} + \mu_{12} C_{12})$$

$$E_{11} = E - i_{11} R_{11}$$
$$E_{12} = E - i_{12} R_{12}$$

$$Z_{11} i_{11} = Z_{11} k_{11} + E - i_{11} R_{11} + \mu_{11} C_{11}$$

$$i_{11} = \frac{E + Z_{11} k_{11} + \mu_{11} C_{11}}{R_{11} + Z_{11}}$$

$$i_{12} = \frac{E + Z_{12} k_{12} + \mu_{12} C_{12}}{R_{12} + Z_{12}}$$

$$V_{C2} = i_{12} R_{12} - i_{11} R_{11} = R_{12} \frac{E + Z_{12} k_{12} + \mu_{12} C_{12}}{R_{12} + Z_{12}} - R_{11} \frac{E + Z_{11} k_{11} + \mu_{11} C_{11}}{R_{11} + Z_{11}}$$

In order that V_{C2} be independent of E we must have

$$E\left(\frac{R_{12}}{R_{12} + Z_{12}} - \frac{R_{11}}{R_{11} + Z_{11}}\right) = 0$$

That is

$$\frac{R_{12} + Z_{12}}{R_{12}} = \frac{R_{11} + Z_{11}}{R_{11}}$$

or $\quad R_{11} Z_{12} = R_{12} Z_{11}$

which we see is exactly the same condition as was found for the one stage amplifier in equation (9).

In order to calculate the response, let

$Z_{11} = Z_{12} = Z_1$; $k_{11} = k_{12} = k_1$; $\mu_{11} = \mu_{12} = \mu_1$; $R_{11} = R_{12} = R_1$

$$V_{c2} = \frac{R_1 \mu_1}{Z_1 + R_1} (c_{12} - c_{11})$$

$$S_1 = \frac{R_1 \mu_1}{R_1 + Z_1}$$

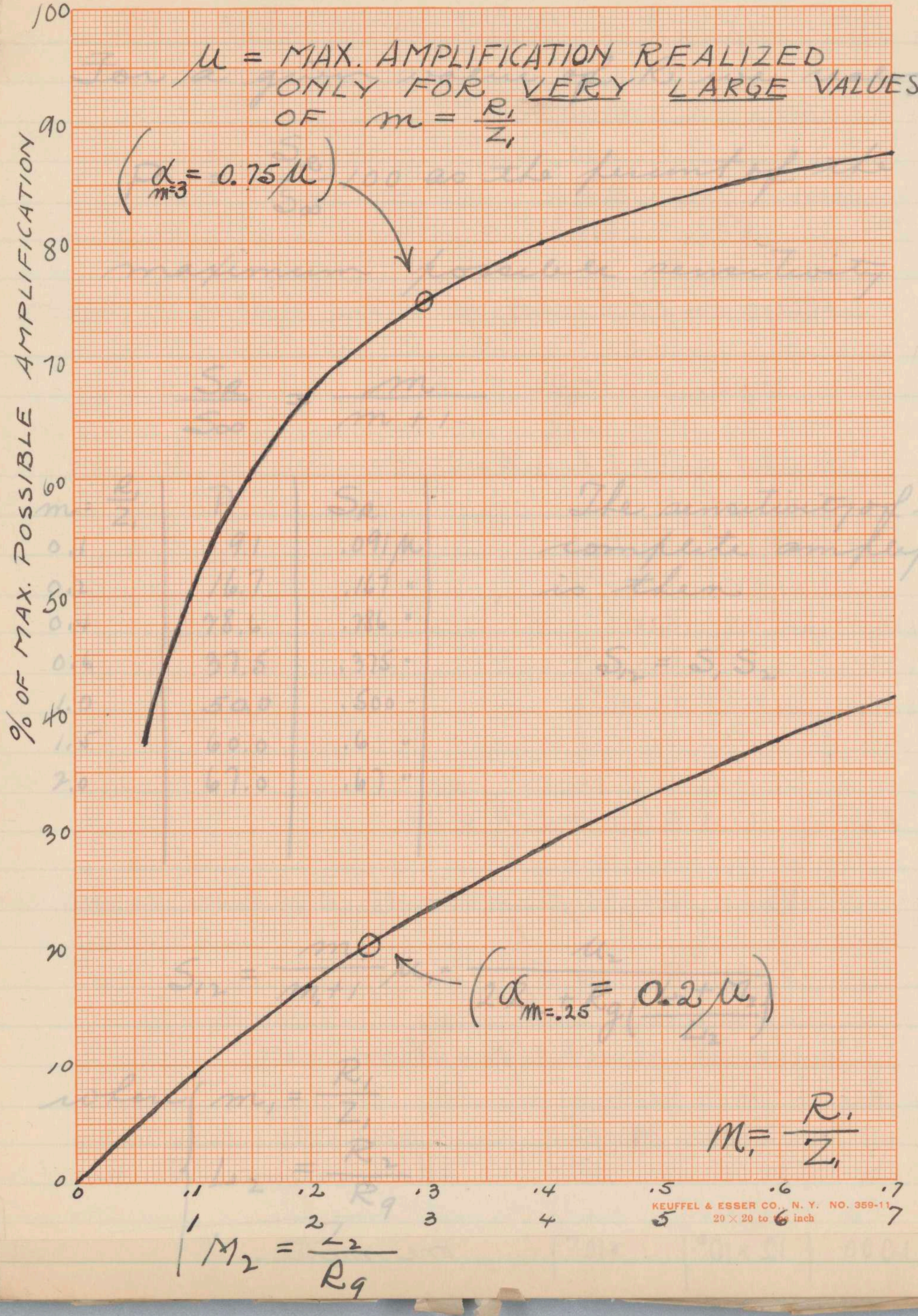
We can conclude from this that if $R_1 \gg Z_1$ then the amplification is $\underline{\mu_1}$ but if $Z_1 \gg R_1$ we have $\underline{R_1 G_m}$ as the amplification factor.

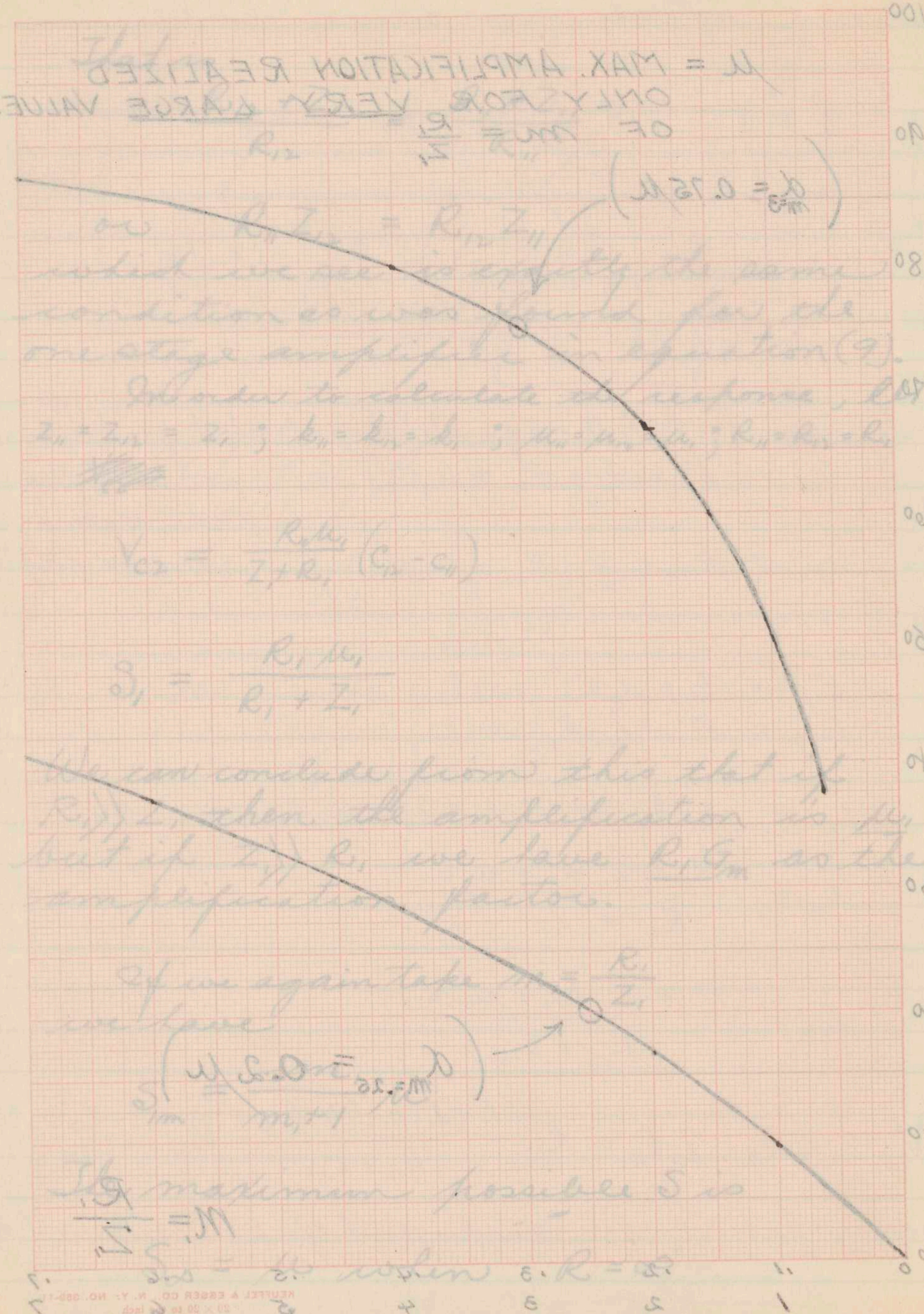
If we again take $m_1 = \frac{R_1}{Z_1}$ we have

$$S_{1m} = \frac{-m_1}{m_1 + 1} \mu_i$$

The maximum possible S is

$$S_\infty = \mu \text{ when } R = \infty$$


μ = MAX. AMPLIFICATION REALIZED ONLY FOR VERY LARGE VALUES OF $m = \frac{R_1}{Z_1}$
$\left(\alpha_{m=3} = 0.75\mu\right)$
$\left(\alpha_{m=.25} = 0.2\mu\right)$
% OF MAX. POSSIBLE AMPLIFICATION
100
90
80
70
60
50
40
30
20
10
0
0
.1
.2
.3
.4
.5
.6
.7
1
2
3
4
5
6
7
$M_1 = \frac{R_1}{Z_1}$
$M_2 = \frac{Z_2}{R_g}$
KEUFFEL & ESSER CO., N. Y. NO. 359-11
20 × 20 to the inch



that

so $R_{11} Z_{12} = R_{12} Z_{11}$

which we see is exactly the same condition as was found for the one stage amplifier in equation (9).

In order to calculate the response, let $Z_{11} + Z_{12} = Z_1$; $k_{11} = k_{12} = k_1$; $\mu_{11} = \mu_{12} = \mu_1$; $R_{11} = R_{12} = R_1$

$$V_{c2} = \frac{R_1 \mu_1}{Z_1 + R_1}(c_{12} - c_{11})$$

$$S_1 = \frac{R_1 \mu_1}{R_1 + Z_1}$$

We can conclude from this that if $R_1 \gg Z_1$ then the amplification is μ_1 but if $Z_1 \gg R_1$ we take $R_1 G_m$ as the amplification factor.

If we again take $m_1 = \frac{R_1}{Z_1}$ we have

$$S_{1m} = \frac{m_1 \mu_1}{m_1 + 1}$$

The maximum possible S is

$S_1 = \mu_1$ when $R = \infty$

For a given value of R we realize

$P = \frac{S_R}{S_\infty} 100$ as the percent of the maximum possible sensitivity.

$$\frac{S_R}{S_\infty} = \frac{m}{m+1}$$

$m = \frac{R_1}{Z_1}$	P	S_R
0.1	9.1	.091 μ
0.2	16.7	.167 "
0.4	28.6	.286 "
0.6	37.5	.375 "
1.0	50.0	.500 "
1.5	60.0	.6 "
2.0	67.0	.67 "

The sensitivity of the complete amplifier is then

$$S_{12} = S_1 S_2$$

$$S_{12} = \frac{m_1}{m_1+1} \mu_1 \times \frac{\mu_2}{2Z_2 + R_g\left(\frac{L_2+M_2}{L_2}\right)}$$

when $\begin{cases} m_1 = \frac{R_1}{Z_1} \\ L_2 = \frac{R_2}{R_g} \\ M_2 = \frac{Z_2}{R_g} \end{cases}$

Case III - UX222 + UX112

Take $R_1 = 400,000^{\omega}$

$Z_1 = 1,000,000^{\omega}$

Then $m_1 = 0.4$

$S_1 = .286\mu = 86$

Voltage amplification of entire system is now

$\alpha_v S_1 = 0.71 \times 86 = 61$

Current amplification is

R'_{1C}	$\alpha_{c\,overall}$	$(i_c)_{min}$
1	61,000	8.1×10^{-12}
10	610,000	8.1×10^{-13}
100	6.1×10^{6}	8.1×10^{-14}
1000	61×10^{6}	8.1×10^{-15}

Case IV - UX 222 + UX 222

Voltage amplification of system is now

$\alpha_v S_1 = 12.5$

Current amplification is

R_{1C}	α_c	$(i_c)_{min}$
1	12,500	3.96×10^{-11}
10	125,000	$\times 10^{-12}$
100	1.2×10^{6}	$\times 10^{-13}$
1000	12×10^{6}	$\times 10^{-14}$

Using $R'_{1C} = 3000$ and a galvanometer sens. of 2000 meg 10^{-16} amp should give 1 cm at one meter.

Photoelectric current as a function of temperature

Current = i
Light energy from source of wave length between λ and $\lambda + d\lambda$ = $L\,d\lambda$
This is given by Planks equation

$$L\,d\lambda = \frac{C_1}{\lambda^5} \cdot \frac{1}{e^{\frac{ch}{k\lambda T}} - 1}\,d\lambda$$

c = velocity of light
k = Boltzmann's gas const.
h = Plank's constant
λ = wave length in meters
T = Temperature Kelvin.

Let $C_2 = \frac{ch}{k}$
In all cases of interest in photoelectric applications

$$\frac{ch}{k\lambda T} > 1 \text{ and } e^{\frac{ch}{k\lambda T}} \gg 1$$

∴ we can write

$$L\,d\lambda) = \frac{C_1}{\lambda^5}\, e^{-\frac{C_2}{\lambda T}}\,d\lambda \quad \text{when } C_2 = \frac{ch}{k}$$

Since our interest is in the variation of $L\,d\lambda$ when λ is fixed and T is varied we can write

$$i = m\,L\,d\lambda = C\,e^{-\frac{C_2}{\lambda T}}$$

when m is a proportionality constant and C = constant as long as the wave length is not changed.

$5 \overline{)273}$
546
9
491.5
32

~~$C_2 = 14,330$~~ when

$C_2 = 143.3 \times 10^6$ when λ is given in Å and T in °K (See Int. Crit. Tables vol V p 239.)

In order to use common log instead of natural we must take $.4343 \times C_2$ which is 62.235×10^6

$$\therefore \quad i = C\,10^{-\frac{62.235 \times 10^6}{\lambda T}}$$

If T is given in absolute °F instead of °K, we take

$$i = C\,10^{-\frac{112 \times 10^6}{\lambda F}}$$

$$F^\circ = f^\circ + 460^\circ$$

Take case of photocurrent at 1600°f compared with 1400°f

$$\frac{i_{1600}}{i_{1400}} = 10^{\frac{112 \times 10^6}{\lambda}\left(\frac{1}{1860} - \frac{1}{2060}\right)}$$

($\left(\frac{1}{1860} - \frac{1}{2060}\right) \leftarrow 52.2 \times 10^{-6}$)

assume $\lambda = 9000$ Å

$$\frac{i_{1600}}{i_{1400}} = 10^{.65} = 4.47$$

$$\rho = \frac{80}{4.47 - 1} = 23^{\omega}$$

If we adjust the slide wire current and the end coil resistance to give 1400°F for 0 and 1600°F at 80 on the slide we can calculate the resistance of the end coil since

$$\frac{i_{1600}}{i_{1400}} = \frac{r+80}{r}$$

$$r = \frac{80}{\frac{i_{1600}}{i_{1400}} - 1} = \frac{80}{3.47} = 23^{\omega}$$

$$\frac{i_{1800}}{i_{1600}} = 10^{1.244 \times 10^4 \left(\frac{1}{2060} - \frac{1}{2260}\right)} \quad \leftarrow .43 \times 10^{-4}$$

$$= 10^{.535} = 3.428$$

$$r = \frac{80}{2.428} = 32.9^{\omega}$$

In order to compute the pen position for a given temperature t ~~into~~ in terms of the current for temperature t_0 we have

$$\frac{i_t}{i_0} = \frac{R+r}{r}$$

$$R = r\left(\frac{i_t}{i_0} - 1\right)$$

for example take $f = 1500°$ or $F = 1960°$
$x = 23°$ to attend $f_0 = 1400°$ giving
$f_{80} = 1600°$

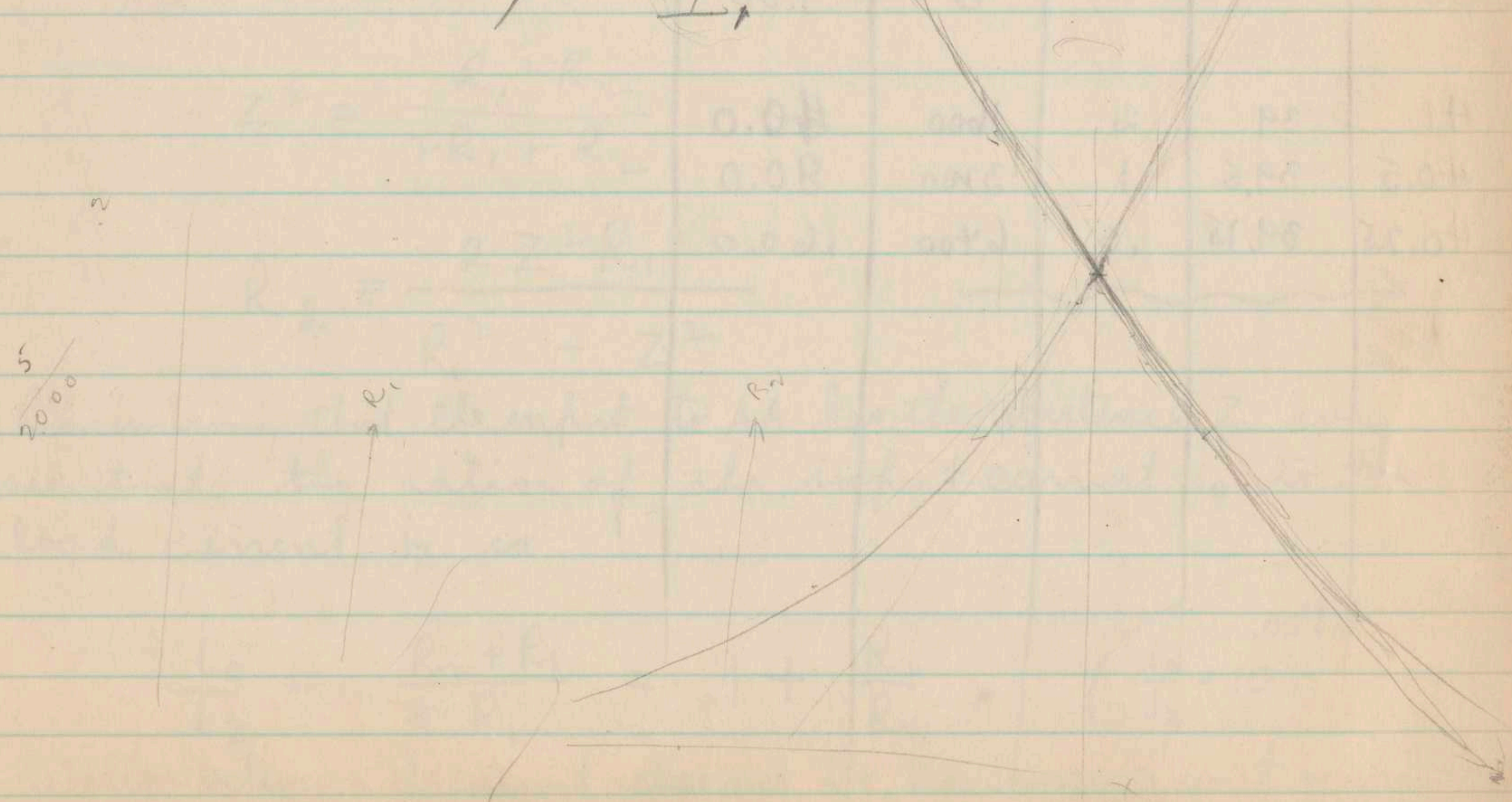
5376344
5102041
0274303

$$\frac{i_{1500}}{i_{1400}} = 10^{1.244 \times 10^4 \left(\frac{1}{1860} - \frac{1}{1960}\right)} \quad \swarrow .3415$$

$$= 2.198$$

$$R = 23(2.198 - 1) = 27.5$$

$$\lambda = \frac{112 \times 10^6 \left(\frac{1}{F_1} - \frac{1}{F_2}\right)}{\log_{10} \frac{I_2}{I_1}}$$

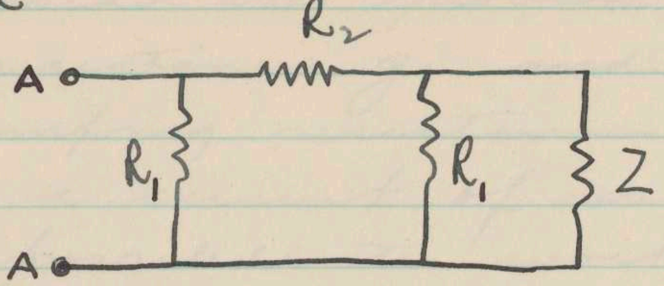


Given $Z = 40$ $\quad R_2 = \dfrac{3200}{R_1 - \dfrac{1600}{R_1}}$

R_1	$\frac{1600}{R_1}$	$R_1 - \frac{1600}{R_1}$	R_2	$\frac{R_2}{R_1} + 1$
40	40	0	∞	∞
42	38.1	3.9	82.0.	20.5
44	36.4	7.6	42.1.	10.55
48	33.3	14.7	21.8.	5.54
60	26.7	33.3	9.61.	2.60
80	20.0	60.0	5.34	1.667
100	16.0	84.0	3.81	1.381
160	10.0	150.0	2.13	1.133
240	6.67	233.3	1.322	1.057
300	5.33	294.7	1.09	1.0363
∞			0	1.00
41	39	2	1600	40.0
40.5	39.5	1	3200	80.0
40.25	39.75	.5	6400	160.0

Computation of an "H" type artifical line.

Symetric



Condition :- Input resistance to AA shall be equal to Z.

$$\frac{1}{Z} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{R_1 Z}{R_1 + Z}}$$

$$= \frac{R_1 R_2 + R_2 Z + R_1 Z + R_1^2 + R_1 Z}{R_1^2 R_2 + R_1 R_2 Z + R_1^2 Z}$$

$$R_1^2 R_2 + (R_1 R_2 + R_1^2) Z = Z(R_1 R_2 + R_1^2) + Z^2 (R_2 + 2R_1)$$

$$Z^2 = \frac{R_1^2 R_2}{2R_1 + R_2}$$

$$R_2 = \frac{2Z^2 R_1}{R_1^2 - Z^2} \longrightarrow$$

Remembering that the input to AA has the resistance Z we see that the ration of the input current i_0 to the load current i_Z is

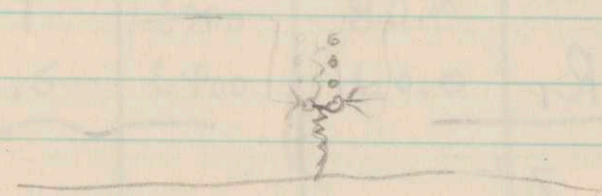
$$\frac{i_0}{i_Z} = \frac{R_2 + R_1}{R_1} = 1 + \frac{R_2}{R_1} \qquad \left(\frac{i_0}{i_2} = 10^{.05TU} \right.$$

The relation between the current ratio and the transmission unit is given by

$$a = \frac{mg - R}{m + 1}$$

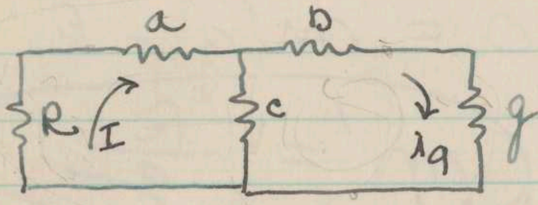
$$b = \frac{mR - g}{m + 1}$$

$$c = \frac{m}{m^2 - 1}(R + g)$$



Note on the nonsymetrical "T" type resistance network

Consider the problem of a "galvanometer of resistance g and external critical damping resistance R connected in a circuit of resistance R. Is it possible to work out a network which will work for this circuit to change the galvanometer current without changing the circuit current or the damping.



$$\begin{cases} g = a + \dfrac{c(g+b)}{c+g+b} \\ R = b + \dfrac{c(a+R)}{c+a+R} \\ Ic = i_g(b+c+g) \\ \dfrac{I}{i_g} = m = \dfrac{b+c+g}{c} \end{cases}$$

These equations can be solved for a, b, c to get.

$$a = g\left(\frac{m^2-m+1}{m(m+1)}\right) - \frac{R}{m+1} = \frac{(m^2+1)g - m(R+g)}{m(m+1)}$$

$(m-1)$

$$\frac{mg-R}{m+1}$$

$$b = \frac{Rm-g}{m+1}$$

$$c = \frac{m}{m^2-1}(R+g)$$

When $m=1$ $a=-b$ ~~but~~ if $R>g$ resistance "a" is <u>negative</u>

" $R<g$ " "b" " "

since in the usual case $R > g$ there is a certain limiting value of ~~...~~ m for which this network will function. This can be found by setting $a = 0$ and solving for m.

~~$(m^2+1)g = m(R+g)$~~

~~$m^2+1 = m\left(\frac{R}{g}+1\right)$~~

~~$m^2 - m\left(\frac{R}{g}+1\right) = -1$~~

~~$m = \frac{R+g}{2g} + \sqrt{\left(\frac{R+g}{2g}\right)^2 - 1}$~~

~~"+" sign because~~ $m > 1$

$$a = \frac{mg - R}{m+1}$$

$$0 = m'g - R$$

$$m' = \frac{R}{g}$$

$\frac{R}{g}$	Limiting m'
1	1
2	~~2.62~~
5	~~5.83~~
10	~~10.91~~
15	~~15.94~~

~~For $m > m'$ it is possible to design a network to control the circuit as desired.~~

1400 20 40 60 80 1500 20 40 60 80 1600 20 40 60 80 1700

λ_8	(R)	R_9				λ_9	λ_8	λ_9	λ_8
0	1400	0	1860	537634					1.0
3.65	20	4.1	1880	531915	005719	.0711	.08	1.178	1.2
9.5	40	8.77	1900	526316	011318	.141	.1582	1.381	1.52
13.1	60		20	520833	016801	.	.2350		1.72
19.0	80	20.4	40	515464	022170	.216	.31	1.885	2.04
25.9	1500		60	510204	027430		.384		2.42
33.8	20	35.4	80	505051	032583	.405	.456	2.54	2.85
43.1	40		2000	500000	037634		.526		3.36
53.6	60	55.0	20	495050	042584	.53	.596	3.385	3.94
65.9	80		40	490196	047438		.664		4.61
80.0	1600	80.0	60	485437	052197	.65	.731	4.47	5.38
96.0	20	94.5	80	480769	056865	.708	.796	5.10	6.25

$$r_8 = \frac{80}{4.38} = 18.25\ \omega$$

$$r_9 = \frac{80}{3.47} = 23.\ \omega$$

1600	2060	485437	0				
20							
40							
60							
80							
1700							
20							
40							
60							
80							
1800	2260	442478	042959	.534	.601	3.42	3.986
20							

$r_a = 33.0^\circ$

$r_8 = 26.8$

	R_a						λ_a	λ_8	λ_9	λ_8
1800	0	2260	384615	442478		0	0	0	1	1
20	4.96	2280	381679	438596	003882		.0484		1.118	
40	11.0	2300	378788	434783	007695		.0957		1.247	
60		20	375940							
80	24	40	373134	427350	015128		.188		1.54	
1900	31.5	60	370370	423729	018749		.233		1.71	
20	39.4	80	367647	420168	022310		.2775		1.891	
40	48.5	2400	364964	416667	025811		.321		2.092	
60	58.2	20	362319	413223	029255		.364		2.31	
80		40	359712							
2000	80.0	60		406504	035974		.448	.504	2.8	3.19
20	97.2	80		403226	039252		.488		3.075	

$P_a = \frac{80}{1.8} = 44.4$

$r_8 = 36.5$

(R) λ_8		R_9				λ_9	λ_8	λ_9	λ_8
0	1400	0	1860	537634					1.0
3.65	20	4.1	1880	531915	005719	.0711	.08	1.178	1.2
9.5	40	8.22	1900	526316	011318	.141	.1582	1.381	1.52
13.1	60		20	520833	016801	.	.2350		1.72
19.0	80	20.4	40	515464	022170	.276	.31	1.885	2.04
25.9	1500		60	510204	027430		.384		2.42
33.8	20	35.4	80	505051	032583	.405	.456	2.54	2.85
43.1	40		2000	500000	037634		.526		3.36
53.6	60	55.0	20	495050	042584	.53	.596	3.385	3.94
65.9	80		40	490196	047438		.664		4.61
80.0	1600	80.0	60	485437	052197	.65	.731	4.47	5.38
96.0	20	94.5	80	480769	056865	.708	.796	5.10	6.25

$r_8 = \frac{80}{4.38} = 18.25^{\omega}$

$r_9 = \frac{80}{3.47} = 23.^{\omega}$

$r_a = 33.0°$
$r_8 = 26.8$

1600	2060	485437	0				
20							
40							
60							
80							
1700							
20							
40							
60							
80							
1800	2260	442478	042859	.534	.601	3.42	3.986
20							

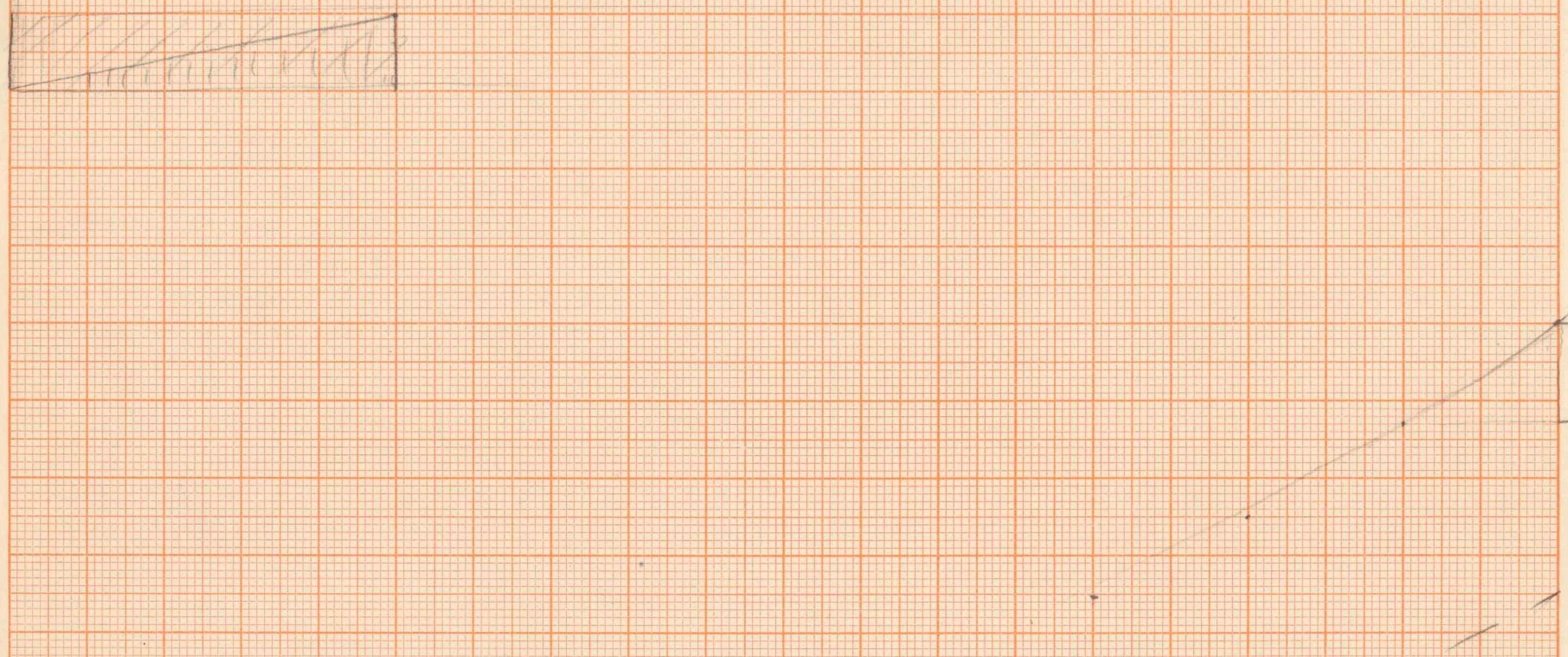
	R_a						λ_a	λ_8	λ_a	λ_8
1800	0	2260	384615	442478		0	0	0	1	1
20	4.96	2280	381679	438596	003882		.0484		1.118	
40	11.0	2300	378788	434783	007695		.0957		1.247	
60		20	375940							
80	24	40	373134	427350	015128		.188		1.54	
1900	31.5	60	370370	423729	018749		.233		1.71	
20	39.4	80	367647	420168	022310		.2775		1.891	
40	48.5	2400	364964	416667	025811		.321		2.092	
60	58.2	20	362319	413223	029255		.364		2.31	
80		40	359712							
2000	80.0	60		406504	035974		.448	.504	2.8	3.19
20	92.2	80		403226	039252		.488		3.075	

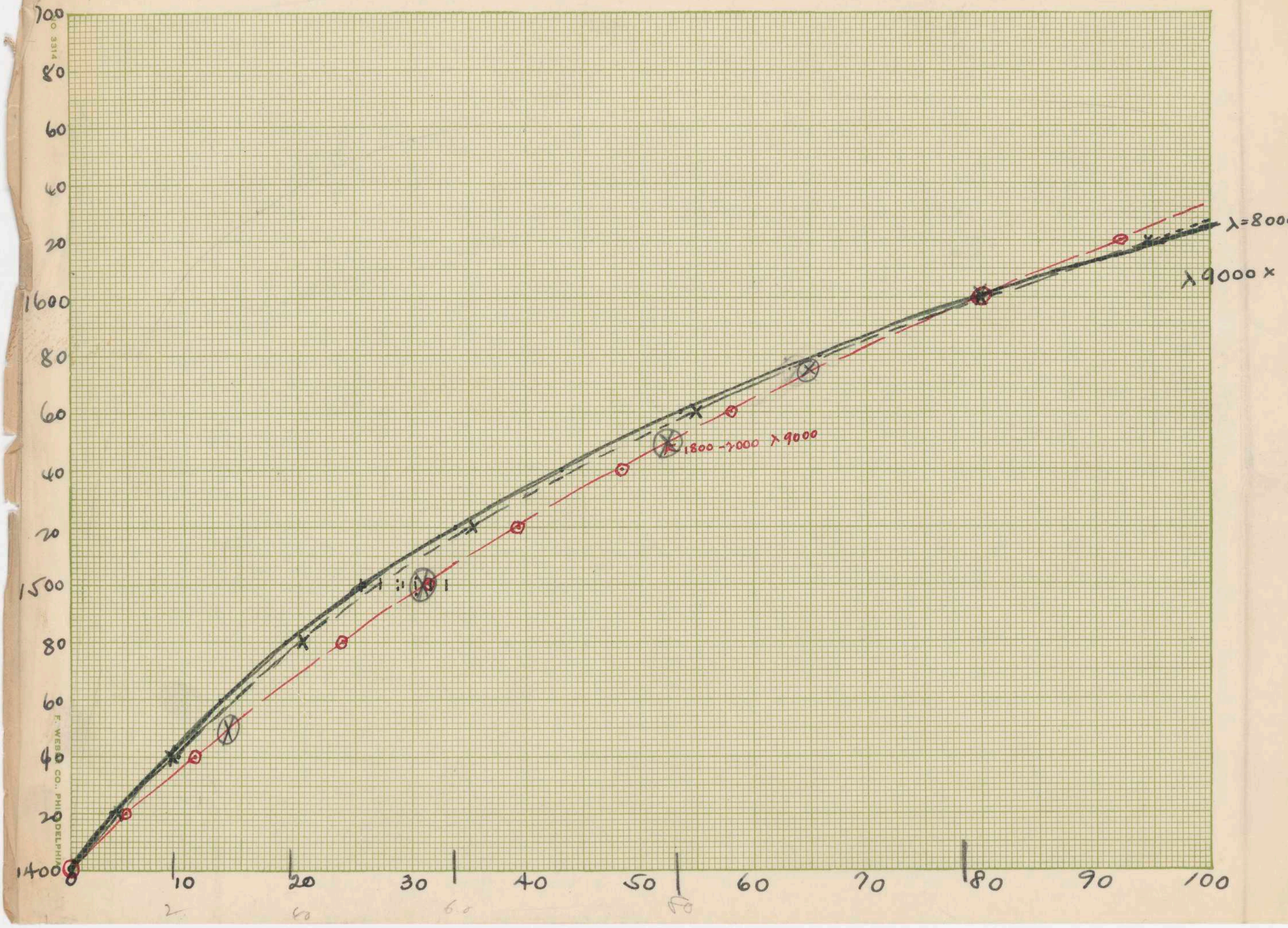
$R_a = \frac{80}{1.8} = 44.4$
$r_8 = 36.5$

R λ_8		R λ_9			Dif	λ_8	λ_9		λ_8	λ_9
	2000		2460	406504	0					
	40		2500							
	80		40							
	2120		80							
	60		2620							
25.5	2200	27.3	60	375940	030564	.427	.38		2.67	2.395
	40		2700							
	80		40							
	2320		80							
	60		2820							
	2400		2860	349650	056854	.795	.706		6.24	5.08
	40		2900							
								r =	15.75	19.6
	2400		2860	349650	0					
	40									
	80									
	2520									
	60									
29.2	2600	30.3	3060	326797	022853	.32	.2845		2.09	1.973
	40									
	80									
	2720									
	60									
	2800		3260	306748	042902	.60	.533		3.98	3.21
	40									
								r	26.8	33.2
	2800		3260	306748	0					
31.6	3000	33.0	3460	289017	017731	.248	.2206		1.77	1.66
	3200		3660	273224	033524	.469	.416		2.94	2.6
	40									
								r =	41.1	50.0

	3200		3660	27 3274		0					
X4 28.7	3600	30.2	4060	246305	026919		.377	.335		×2.38	×2.16
	4000		4460	22 4215	.049 009		.686	.61		4.85	4.07
									r	20.8	26


0
1
2
3
4
5
6
7
8
9
10




λ=8000 •
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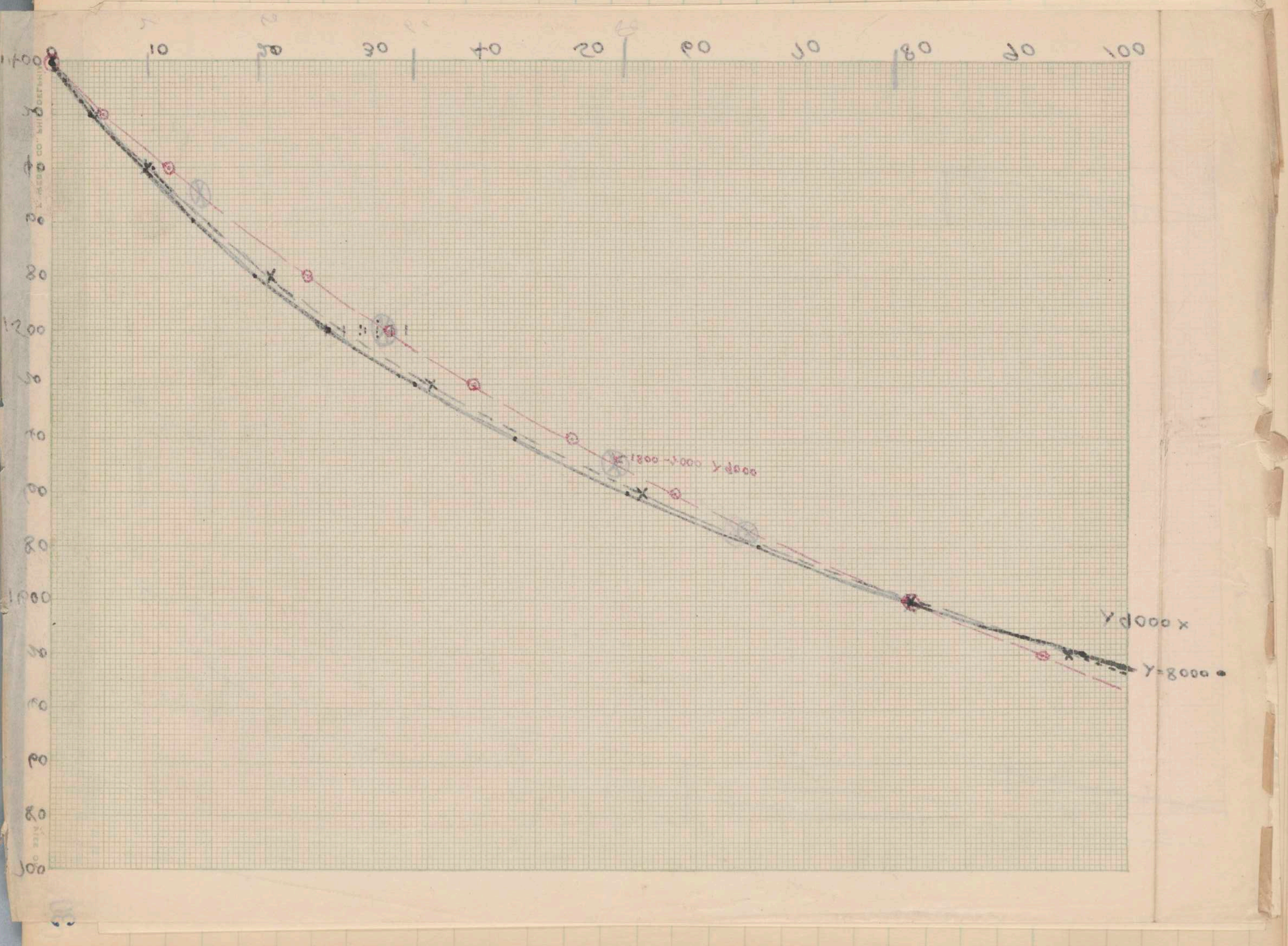
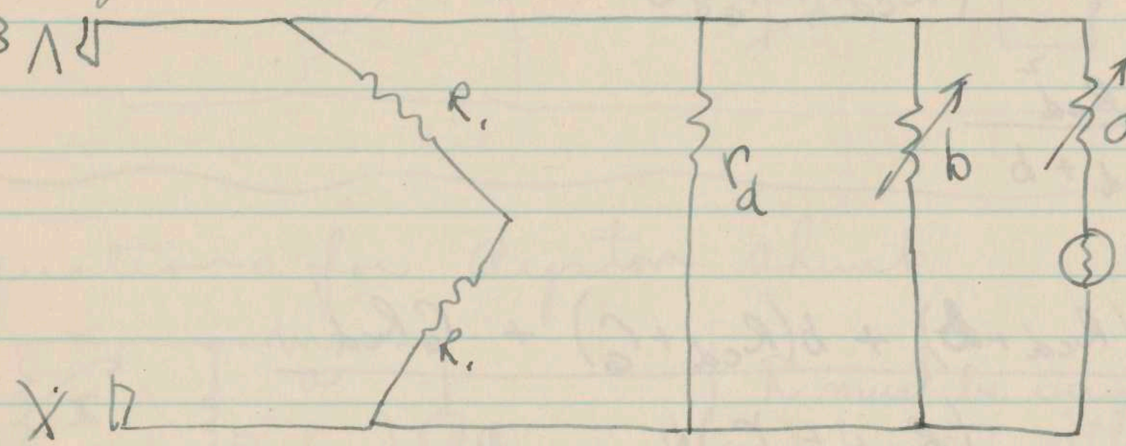


Y=8000

Use of an "L" type network in the plate circuit of a balanced bridge amplifier.



$$\frac{1}{2Z} + \frac{1}{2R_1} + \frac{1}{r_d} = \frac{1}{R_{cd}} = \frac{1}{\text{External damping resistance}}$$

When $a = 0$ and $b = \infty$

$$S_{max} = \frac{G_m}{2\left(1 + \frac{r_G}{R_{cd}}\right)}$$

Adjust "a" and "b" to decrease the sensitivity by the ratio $\frac{S_{max}}{S'} = n$ and keep the damping const by making

$$R_{cd} = a + \frac{R_{cd}\, b}{R_{cd} + b} \qquad \therefore\ a = \frac{R_{cd}^2}{R_{cd} + b}$$

$$b = \frac{R_{cd}(R_{cd} - a)}{a}$$

$$S' = \frac{b\, G_m}{2(a+b+r_G)\left(1 + \frac{b(r_G + a)}{(a+b+r_G)R_{cd}}\right)}$$

$$= \frac{b\, G_m}{2\left(a+b+r_G + \frac{b(r_G+a)}{R_{cd}}\right)}$$

$$n = \frac{(a+b+r_G)R_{cd} + b(r_G+a)}{(R_{cd}+r_G)b}$$

$$a = \frac{R_{cd}^2}{R_{cd}+b}$$

$$n = \frac{a(R_{cd}+b) + b(R_{cd}+r_G) + r_G R_{cd}}{(R_{cd}+r_G)b}$$

$$= \frac{R_{cd}(R_{cd}+r_G) + b(R_{cd}+r_G)}{(R_{cd}+r_G)b} = \frac{R_{cd}+b}{b}$$

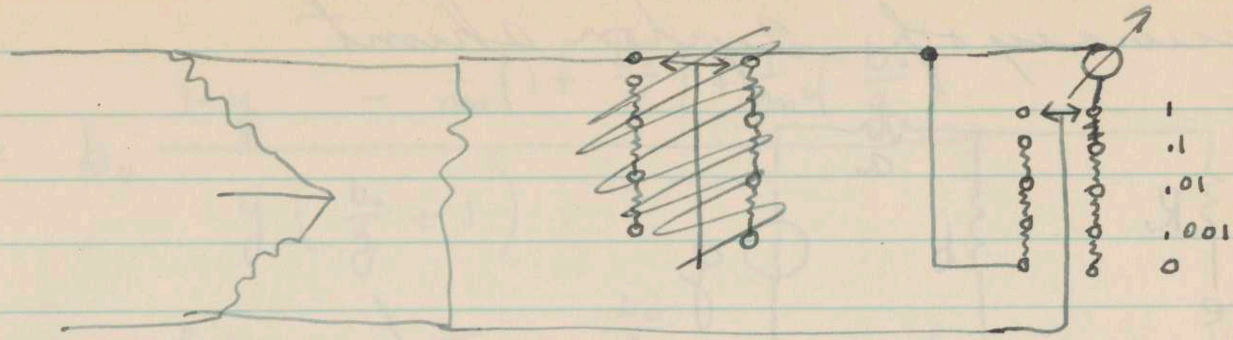
$$b = \frac{R_{cd}}{n-1} \qquad a = \frac{R_{cd}^2}{R_{cd} + \frac{R_{cd}}{n-1}}$$

$$a = R_{cd}\frac{n-1}{n}$$

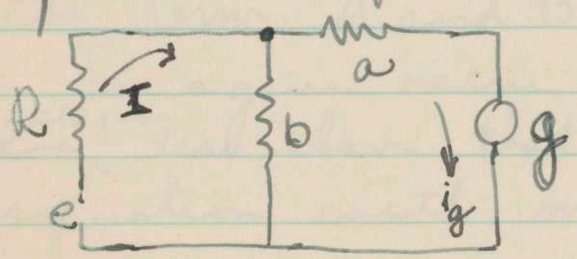
a and b are thus independent of the galvanometer resistance just as ~~was~~ is the case for the ayrton shunt. We do not have the condition in this case that

a + b = constant as is the case

for the Ayrton shunt and therefore a two contact resistance box is necessary.



Equations for Ayrton shunt.



{R must be very great compared with $\frac{b(a+g)}{a+b+g}$}

$a + b = R_{C.D.X.}$

$I = \text{constant}$

$\frac{I}{i_g} = m$

$I b = i_g (a+b+g)$

$\frac{I}{i_g} = \frac{a+b+g}{b} = R_{cDx} +$

$$i_{1g} = \frac{R_{cdx}}{R_{cdx} + g}$$

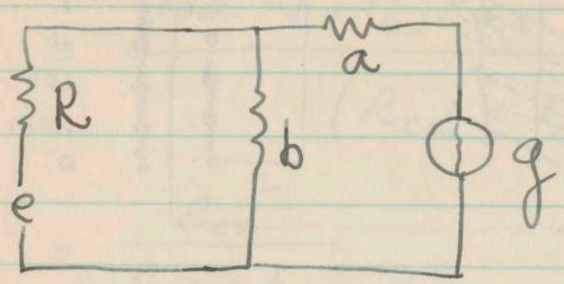
$$i_{2g} = \frac{b}{R_{cdx} + g}$$

$$\frac{i_{1g}}{i_{2g}} = n = \frac{R_{CDx}}{b}$$

$$\therefore b = R_{CDx}\left(\frac{1}{n}\right)$$

$$a = R_{CDx}\left(\frac{n-1}{n}\right)$$

Accuracy of Ayrton shunt.



$$Ib = i_g(a+b+g)$$

$$I = \frac{e}{R + \frac{b(a+g)}{a+b+g}}$$

$$i_g = \frac{eb}{(a+b+g)R + b(a+g)}$$

when $a = 0$

and $b = b_0$

$$i_{g_{max}} = \frac{eb_0}{(b_0+g)R + b_0 g}$$

$$\frac{i_g}{i_{g_{max}}} = n = \frac{b_0}{b} \times \frac{(b_0+g)R + b(a+g)}{(b_0+g)R + b_0 g}$$

Condition of Ayrton shunt.

$\frac{b_0}{b} = m$ $\qquad a+b = b_0$

$\therefore b = \frac{b_0}{m}$ $\qquad a = b_0\left(\frac{m-1}{m}\right)$

Let $n = (1-y)m$ and solve

$$\text{Then } R = b_0 \frac{\overbrace{1-y}^{\text{"A"}} \quad - \overbrace{\frac{1}{m}\left(1+\frac{b_0}{g}\right) + \frac{1}{m^2}\left(\frac{b_0}{g}\right)}^{\text{"B"}}}{y\left(\frac{b_0}{g}+1\right)}$$

for values of m ~~greater~~ $> \dfrac{\frac{b_0}{g}}{1+\frac{b_0}{g}}$ the sum of the term "A" and "B" is a negative number.

∴ For a given value of y the value of R will be slightly less than that for $m = \infty$. Take then the equation with $m = \infty$

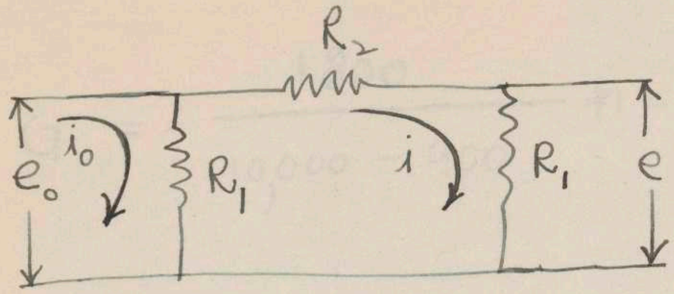
$$R_\infty = b_0 \frac{1-y}{y\left(\frac{b_0}{g}+1\right)}$$

If the current division is to be accurate to 1% or better, then $y = .01$ and we have

$$R = \frac{99 b_0}{\frac{b_0}{g}+1}$$

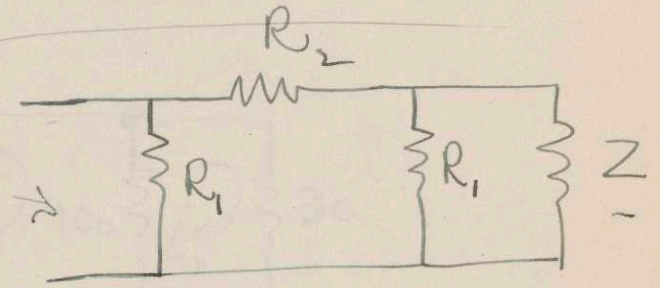
$\frac{b_0}{g}$	R
1	$49.5\ b_0$
2	$33.0\ b_0$
10	$9.0\ b_0$
20	$4.7\ b_0$
40	$2.4\ b_0$

Calculation for symetrical "H" attenuator



$$e_0 = i_0 R_1 = i(R_2 + R_1) = \frac{e}{R_1}(R_2 + R_1)$$

$$e = i R_1$$



$$G = \frac{e_0}{e} = \frac{R_2 + R_1}{R_1}$$

~~$\frac{1}{Z} =$~~

$$\frac{1}{Z} = \frac{1}{R_1} + \frac{1}{R_2 + \frac{R_1 Z}{R_1 + Z}}$$

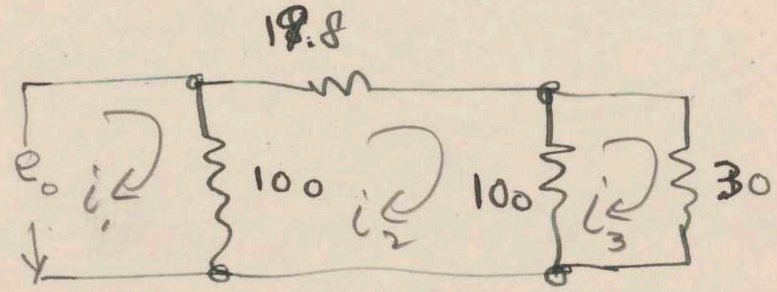
$$\frac{1}{Z} = \frac{1}{R_1} + \frac{R_1 + Z}{R_1 R_2 + R_2 Z + R_1 Z}$$

$Z = 30$

$R_1 = 100$

$$G = \frac{1800}{10,000 - 900} + 1 = \frac{1800}{9100} + 1 = \underline{1.198}$$

$R_2 = \sout{100} = 19.8$



$e_0 = 100 i_1 - 100 i_2$

$0 = -100 i_1 + 219.8 i_2 - 100 i_3$

$0 = \qquad -100 i_2 + 130 i_3$

$$Z = \frac{e_0}{i_1}$$

November 11, 1929

To: Dr. Leo Behr

CC - Dr. W. B. Nottingham

From: I. M. Stein

Subject: X-389 - Characteristics of Vacuum Tubes and Their Applications

I understand that under this job you have produced what appears to be a stable amplifier setup for use with photoelectric cells.

Will you please prepare a preliminary report covering this part of your work under X-389, so that this can be turned over to Dr. Dike for his work in connection with the development of the Ives optical pyrometer under X-426.

I understand that the vacuum tube amplifier developed for use with the photoelectric cell is representative of the various problems which require a high current sensitivity D.C. vacuum tube amplifier.

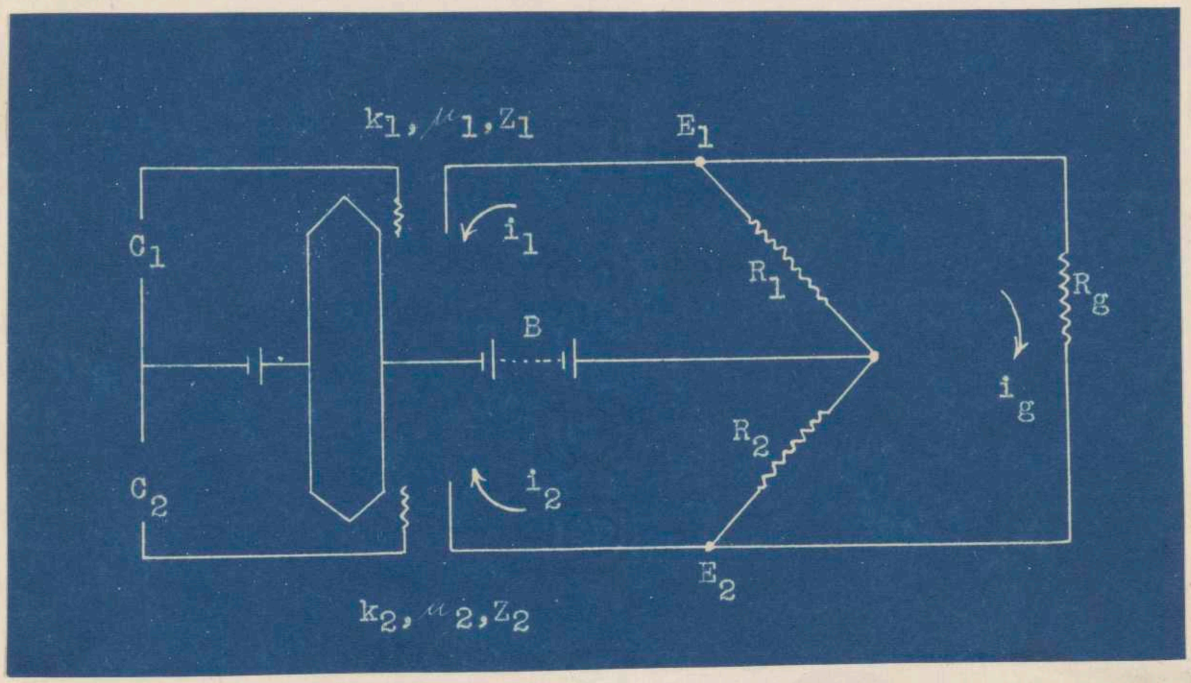
I understand that Dr. Nottingham will take up next the development of a satisfactory vacuum tube amplifier having high voltage sensitivity in D.C. circuits.

HU

I.M.S.

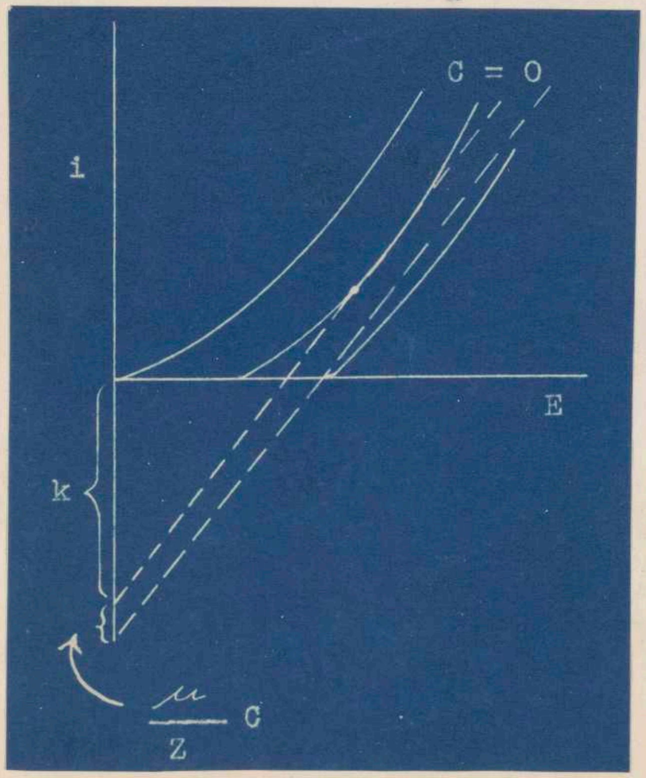
Analysis of Balanced Tube Circuit

for Plate Battery Stability and Amplification



An approximate form of the tube equation is

$$i = k + \frac{1}{Z}(E + \mu C)$$



When i = plate current
E = " voltage
C = impressed grid volts
μ = voltage amplification
Z = tube impedance
$= \frac{\Delta E}{\Delta i}$

k = constant

Over wide ranges of i, E and C, the values of μ, k, and Z are not constant. The following analysis holds only over the range for which these three are constant.

For the balanced circuit, we have

$$\begin{cases} i_1 = k_1 + \frac{1}{Z_1} (E_1 + \mu_1 C_1) & (2) \\ i_2 = k_2 + \frac{1}{Z_2} (E_2 + \mu_2 C_2) & (3) \\ E_1 = B - (i_1 + i_g) R_1 & (4) \\ E_2 = B - (i_2 - i_g) R_2 & (5) \\ 0 = R_1 (i_1 + i_g) + i_g R_g - R_2 (i_2 - i_g) & (6) \end{cases}$$

The solution of these five equations gives:-

$$i_g = \frac{(R_2Z_1 + R_1R_2)(Z_2k_2 + C_2\mu_2 + B) - (R_1Z_2 + R_1R_2)(Z_1k_1 + C_1\mu_1 + B)}{R_1Z_1Z_2 + R_2Z_1Z_2 + R_1R_2Z_1 + R_1R_2Z_2 + R_g (Z_1 + R_1)(Z_2 + R_2)} \quad (7)$$

or

$$i_g = \frac{(R_2Z_1 + R_1R_2)(Z_2k_2 + C_2\mu_2) - (R_1Z_2 + R_1R_2)(Z_1k_1 + C_1\mu_1) + (R_2Z_1 - R_1Z_2) B}{R_1Z_1Z_2 + R_2Z_1Z_2 + R_1R_2Z_1 + R_1R_2Z_2 + R_g (Z_1 + R_1)(Z_2 + R_2)} \quad (8)$$

To make i_g independent of B, we must have

$$R_2Z_1 - R_1Z_2 = 0$$

or

$$\frac{R_1}{R_2} = \frac{Z_1}{Z_2} \quad (9)$$

In case it is required that $i_g = 0$ when the circuit is balanced, there is a second condition on the ratio of $\frac{R_1}{R_2}$ which is

$$i_1R_1 - i_2R_2 = 0 \quad (10)$$

$$\frac{R_1}{R_2} = \frac{i_2}{i_1} \quad (11)$$

The two conditions can always be met and the straightforward way of doing this is the following:-

Choose $E_1 = E_2 = E_o$

After the tubes to be used have been "aged" determine the grid floating potentials V_{10} and V_{20} and the corresponding plate currents i_{10} and i_{20}.

The impedance is determined by $Z_{10} = \dfrac{\Delta E_{10}}{\Delta i_{10}} = \dfrac{E_o - E_1}{i_{10} - i_1}$

The grid voltage on one tube should be made more negative until the condition

$$i_2 = i_{10} \frac{Z_{10}}{Z_2} \tag{12}$$

We then have

$$\frac{i_2}{i_{10}} = \frac{Z_{10}}{Z_2} = \frac{R_1}{R_2} \tag{13}$$

We then have the conditions for balance and for zero galvanometer current met and have the grids equal to or negative with respect to their floating potentials.

In order to determine the sensitivity of the circuit, let

$$\left.\begin{array}{l} R_1 = R_2 = R \\ Z_1 = Z_2 = Z \\ k_1 = k_2 \\ \mu_1 = \mu_2 = \mu \end{array}\right) \tag{14}$$

$$i_g = \frac{\left(1 + \frac{R}{Z}\right) (C_1 - C_2)\, \mu}{2R + 2Z + R_g \left(\frac{R}{Z} + \frac{Z}{R} + 2\right)} \tag{15}$$

Define sensitivity as

$$\frac{i_g}{C_1 - C_2} = S = \frac{\mu\left(1 + \frac{R}{Z}\right)}{2R + 2Z + R_g \left(\frac{R}{Z} + \frac{Z}{R} + 2\right)} \tag{16}$$

Certain points of interest can be seen by making the following substitutions:-

$$\left.\begin{aligned} \text{Let } L &= \frac{R}{R_g} \\ M &= \frac{Z}{R_g} \\ N &= \frac{L}{M} = \frac{R}{Z} \end{aligned}\right\} \qquad (17)$$

then

$$S = \frac{\mu}{2Z + R_g \left(\frac{N+1}{N}\right)} \qquad (18)$$

from which we see that with Z and R_g given, the value of S is increased by increasing N. We have for $N = \infty$ i.e. for R very large compared with Z.

$$S_\infty = \frac{\mu}{2Z + R_g} \qquad (19)$$

and with $Z \gg R_g$ we have for the maximum possible sensitivity

$$S_\infty = \frac{1}{2}\frac{\mu}{Z} = \frac{1}{2}\, G_m \qquad (20)$$

where G_m = mutual conductance.

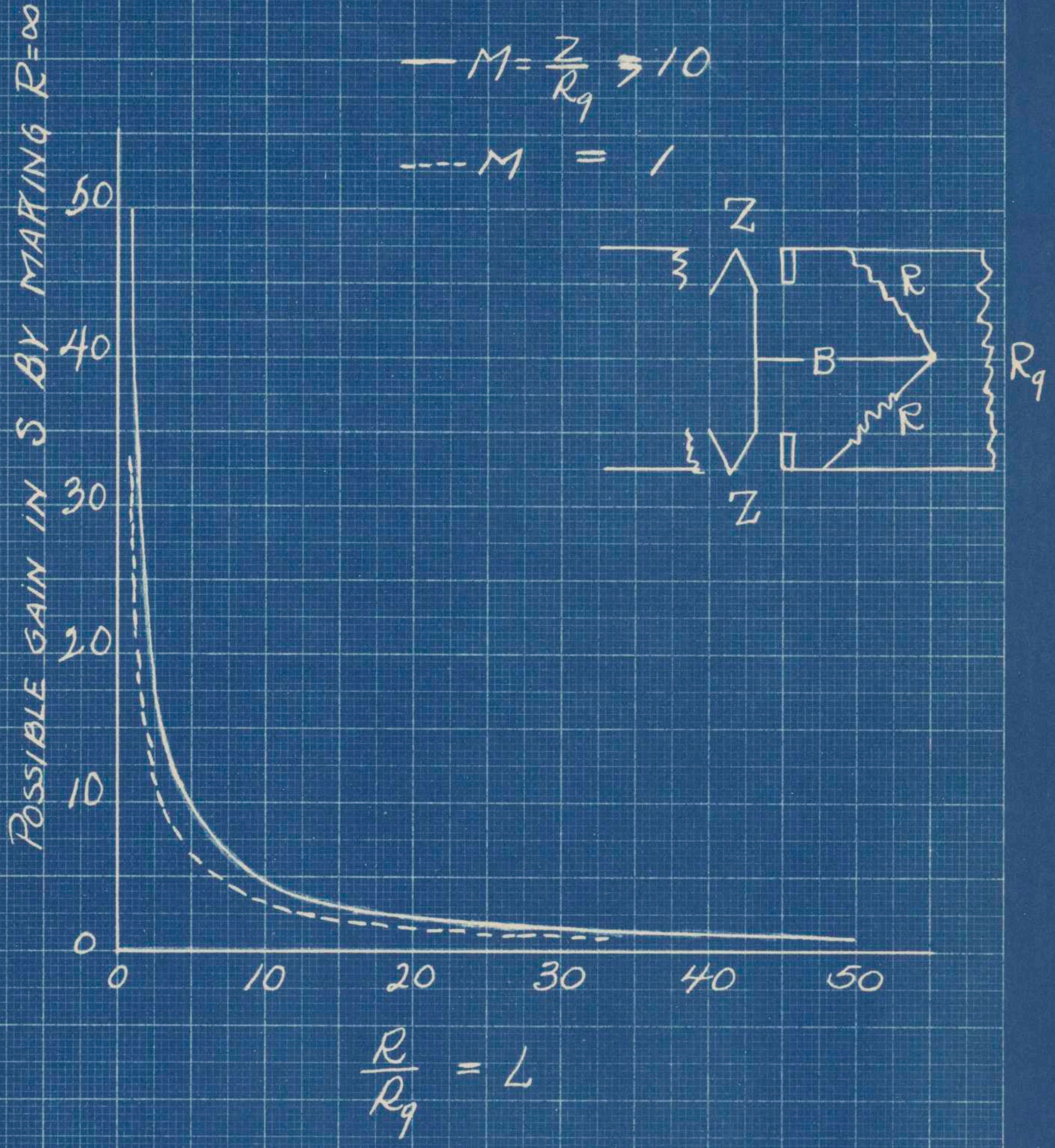
Consider the percentage increase in sensitivity which would always be possible in any given case by allowing R to go from $R = R_1$ to $R = \infty$. This would be

$$P = 100\,\frac{S_\infty - S_1}{S_1} \qquad (21)$$

where S_1 = the sensitivity with $R = R_1$.

Using equations (17) and (18), we get

$$P = \frac{100}{L\left(2 + \frac{1}{M}\right)} \qquad (22)$$


— M = Z/Rq = 10
---- M = 1
POSSIBLE GAIN IN S BY MAKING R=∞
0
10
20
30
40
50
R/Rq = L
Z
R
B
Rq
R
Z

This represents the percentage by which the sensitivity can be increased by making $R = \infty$ compared with $R = R_1$; for example, take

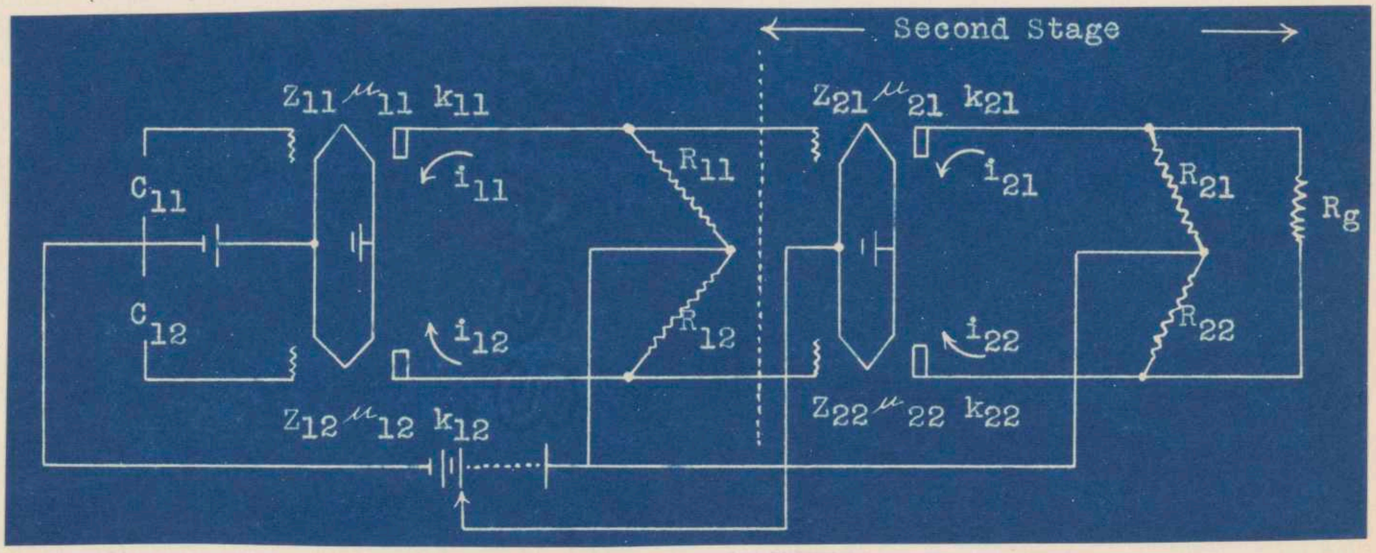
$$L = \frac{R}{R_g} = \frac{5000}{1000} = 5$$

$$M = \frac{Z}{R_g} = \frac{10,000}{1000} = 10$$

$$P = \frac{100}{5\ (2 + 0.1)} = 9.5\%$$

By increasing R from 5000$^{\omega}$ to 5 megs, we could not gain more than 9.5% in sensitivity.

Analysis for the Two-Stage Amplifier



Assuming $i_{11} \gg$ grid current of tube 21 and $i_{12} \gg$ grid current of tube 22, we have

$$\left\{\begin{aligned} i_{11} &= k_{11} + \frac{1}{Z_{11}}\left(E_{11} + \mu_{11} C_{11}\right) \\ i_{12} &= k_{12} + \frac{1}{Z_{12}}\left(E_{12} + \mu_{12} C_{12}\right) \\ E_{11} &= B - i_{11} R_{11} \\ E_{12} &= B - i_{12} R_{12} \end{aligned}\right\} \qquad (23)$$

$$Z_{11} i_{11} = Z_{11} k_{11} - i_{11} R_{11} + \mu_{11} C_{11} + B$$

$$i_{11} = \frac{B + Z_{11} k_{11} + \mu_{11} C_{11}}{R_{11} + Z_{11}}$$

$$i_{12} = \frac{B + Z_{12} k_{12} + \mu_{12} C_{12}}{R_{12} + Z_{12}}$$

The potential applied to the grids of the second stage is:

$$V_{c2} = i_{12}\, R_{12} - i_{11}\, R_{11}$$

$$= R_{12}\,\frac{B + Z_{12}\, k_{12} + \mu_{12}\; C_{13}}{R_{12} + Z_{12}} - R_{11}\,\frac{B + Z_{11}\, k_{11} + \mu_{11}\, C_{11}}{R_{11} + Z_{11}}$$

For V_{c2} to be independent of B, we must have

$$\frac{R_{12}}{R_{12} + Z_{12}} - \frac{R_{11}}{R_{11} + Z_{11}} = 0 \qquad (24)$$

that is

$$\frac{R_{11}}{R_{12}} = \frac{Z_{11}}{Z_{12}} \qquad (25)$$

which is the same condition as was found for the single-stage amplifier.

Letting $R_{11} = R_{12} = R_1$

$$Z_{11} = Z_{12} = Z_1$$

$$k_{11} = k_{12}$$

$$\mu_{11} = \mu_{12} = \mu_1$$

and $S_1 = \dfrac{V_{c2}}{C_{12} - C_{11}}$ (26)

we have

$$S_1 = \frac{R_1}{R_1 + Z_1}\quad \mu_1 \qquad (27)$$

This shows that the maximum possible amplification of the first stage is μ and is realized only when $R_1 \gg Z_1$.

Let $N = \dfrac{R_1}{Z_1}$ as before

and define $P = 100 \quad \dfrac{S_R = R}{S_R = 0}$

$$P = \frac{N}{N + 1}\; 100$$

we have

$$S_1 = \frac{R_1}{R_1 + Z_1} \mu_1 \tag{27}$$

This shows that when $R_1 \gg Z_1$ we have the maximum possible amplification, which then gives

$$S_{R=\infty} = \mu$$

Let $N_1 = \frac{R_1}{Z_1}$ as before

we have

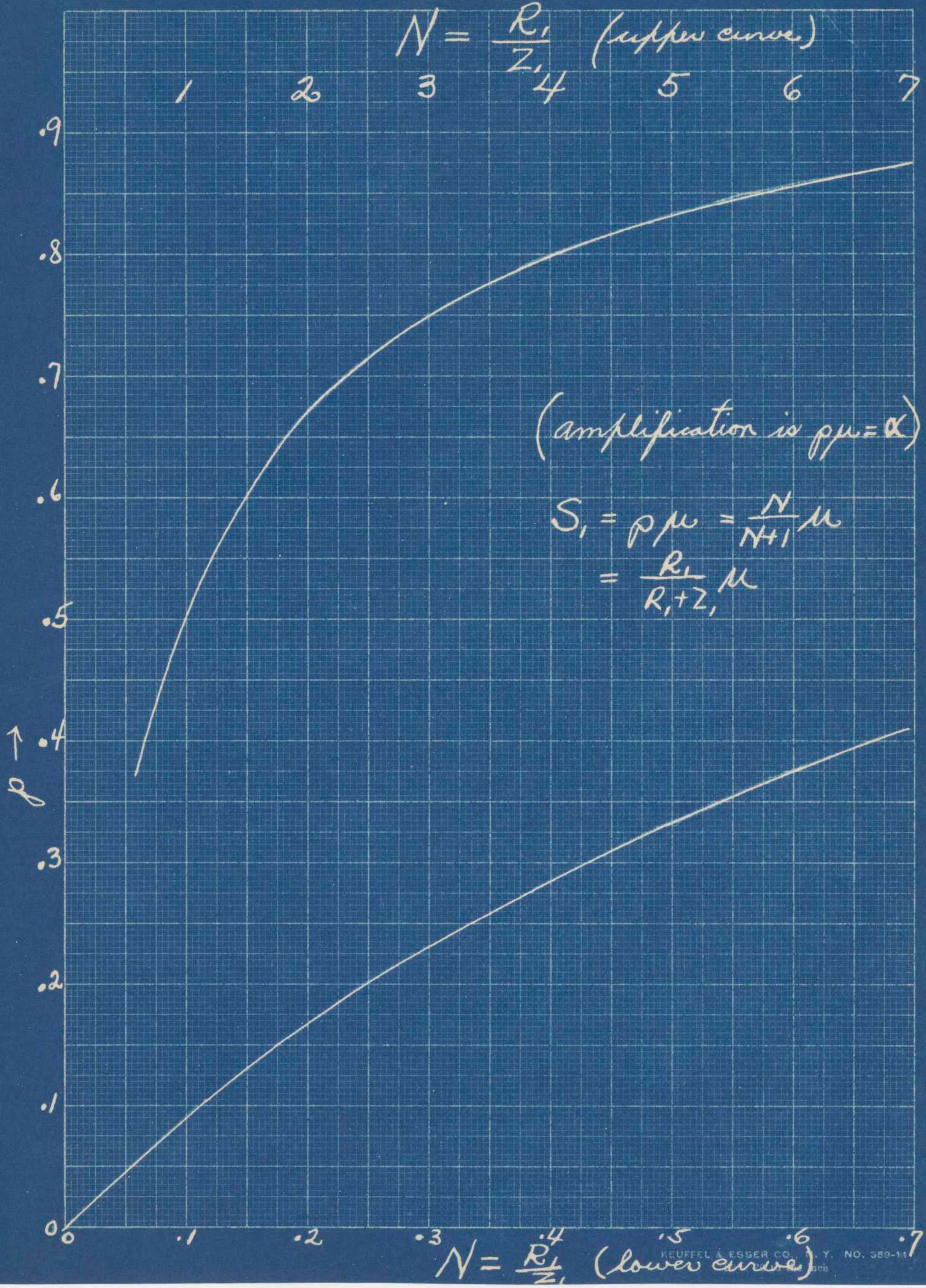
$$S_1 = \frac{N}{N + 1} \mu \tag{28}$$

$$= P\mu$$

$$P = \frac{N}{N + 1} \tag{29}$$

This is shown by attached curve.

HU W. B. Nottingham



N = R1/Z1 (upper curve)
1 2 3 4 5 6 7
.9 .8 .7 .6 .5 .4 .3 .2 .1 0
(amplification is ρμ = α)
S1 = ρμ = N/(N+1) μ
= R1/(R1+Z1) μ
ρ ↑
0 .1 .2 .3 .4 .5 .6 .7
N = R1/Z1 (lower curve)
KEUFFEL & ESSER CO., N. Y. NO. 359-11

Mulder and Razek[1] have shown that the effective mutual conductance of a three element "vacuum" tube can be greatly increased by operating the tube in a grid circuit containing a high resistance. Their treatment of the problem does not show clearly the conditions which must be met in order to obtain this high sensitivity without a discontinuous and non-reversible part in the characteristic.

With the plate voltage and the filament current in a three element tube constant, the plate current and the grid current are related to the grid voltage as shown by Figs. 1 and 2. The first of these is of course perfectly familiar and needs no explanation. The second is a composite of three currents. For very small values of grid voltage the current is predominating by electron current from the filament. This of course falls off rapidly as the grid is made more and more negative with respect to the filament.

There is the leakage current over the base of the tube and the other external parts. This current is usually negligible if the surfaces are clean and the tube is kept in a dry atmosphere. This leakage current <u>increases</u> as the grid voltage is made more negative. The third and important component depends on the presence of gas in the tube. It is this component which plays the all important part in the type of amplifier under discussion. The positive ion current to the grid <u>decreases</u> as the grid is made more and more negative because the number of positive ions decreases as the electron current from the filament to the plate is decreased. As the subsequent discussion will show, it is the point of inflection (A) in the grid current characteristic that is of primary interest. This characteristic in the immediate neighborhood of the point of inflection can be represented very accurately by the equation

$$i_g = -i_o - mE \qquad (1)$$

where i_o = constant

m = a constant (the slope at (A))

i_g = the grid current

and E = " " voltage

1.

The corresponding part of the plate current characteristic can also be represented by a straight line

$$I = I_o + gE \qquad (2)$$

where

I_o = constant

g = " (mutual conductance or slope at A' of Fig. 1)

I = plate current

E = grid voltage

Referring to the circuit sketched in Fig. 3, we have a third relationship given by

$$V = Ri_g + E \qquad (3)$$

These three equations can be solved to give

$$I = I_o - \frac{gRi_o}{1-mR} + \frac{g}{1-mR}V \qquad (4)$$

which expresses the relation between the plate current and the "C" battery voltage V. We see at once from the equation

$$\frac{\Delta I}{\Delta V} = \frac{g}{1-mR} \qquad (5)$$

that the effective mutual conductance

$$g^1 = \frac{g}{1-mR} \qquad (6)$$

can be made very large if we make

$$R = \frac{1}{m} \qquad (7)$$

It is at once obvious that in order to make use of this high effective mutual conductance severe requirements must be met as regards filament, plate and grid batteries and also the tube must be so constructed that the amount and kind of gas does not change with time. It is thought that this requirement can be met if the tube is so designed that the metal and glass parts can be thoroughly baked out under good vacuum conditions and purified argon or some other inert gas is introduced to produce the required gas pressure. The "balanced bridge" circuit could probably be used to make battery maintenance a little easier.

Applications:

The most important application to which a tube of this character can be applied is to the amplification of thermocouple potentials since this type of amplifier is in a sense really "voltage sensitive", that is, its sensitivity does not necessarily depend on producing the voltage as an "IR" drop as in the usual photoelectric cell application which has become so popular.

From the preceding discussion, it is not at once obvious that there would be any advantage in this type of tube for photoelectric or other high resistance problems. The following analysis shows that here too an advantage can be realized under certain conditions.

Referring to Fig. 4, we can write the following equations:

$$V = E + R\,(i_g - i_x) \quad \text{(From Fig. 4)} \qquad (8)$$

$$I = I_o + gE \quad \text{(From Fig. 1)} \qquad (9)$$

$$i_g = -i_o - mE \quad \text{(From Fig. 2)} \qquad (10)$$

These equations are essentially the same as (1), (2) and (3) above. In this case, V is to be constant and the relation between i_x (the photoelectric current) and I (the plate current) is of interest. Solving these equations, we have

$$I = I_o + \frac{g\,(V + Ri_o)}{1-mR} + \frac{g}{1-mR}\,Ri \qquad (11)$$

This can be more simply written

$$I = K + g'\,Ri \qquad (12)$$

where K = constant

$$\text{and } g^1 = \frac{g}{1-mR} \qquad (13)$$

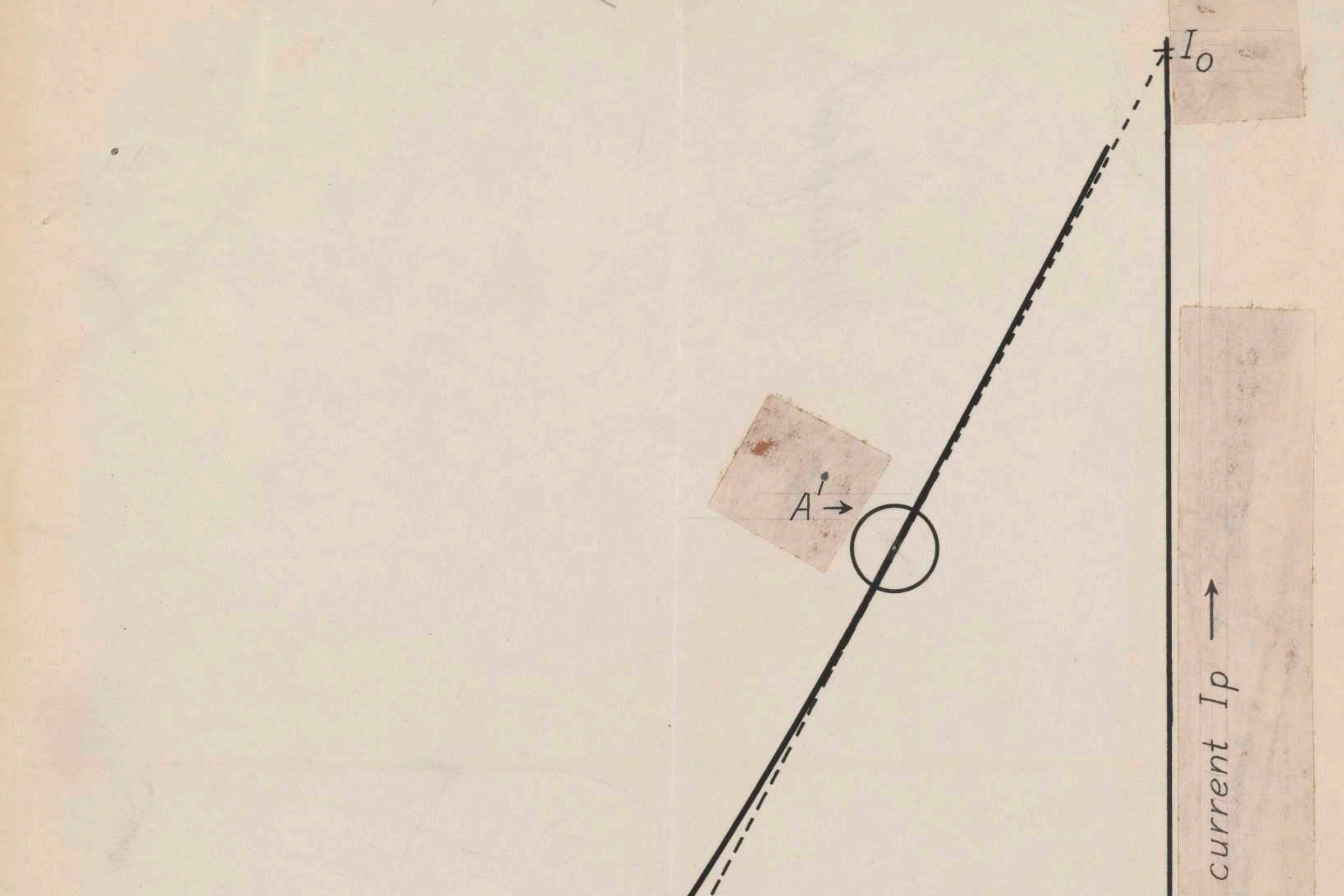
Again the sensitivity is represented by

$$\frac{\Delta I}{\Delta i_x} = \frac{gR}{1-mR} \qquad (14)$$

and this is very large when

$$R = \frac{1}{m}$$

From the equations, we see that for values of R less than $\frac{1}{m}$ the circuit has no discontinuity and is stable for all values of V or i_x as the case may be. On the other hand, if $R > \frac{1}{m}$ there will be a discontinuous region which can be marked out on the grid current characteristic by locating the two points of tangency of the line of slope $\frac{1}{R}$ with the grid current curve. The region between these points of tangency is that in which a discontinuity is certainly to be found.



4"
reletter

FIG 1 PLATE CURRENT CHARACTERISTIC OF VACUUM TUBE

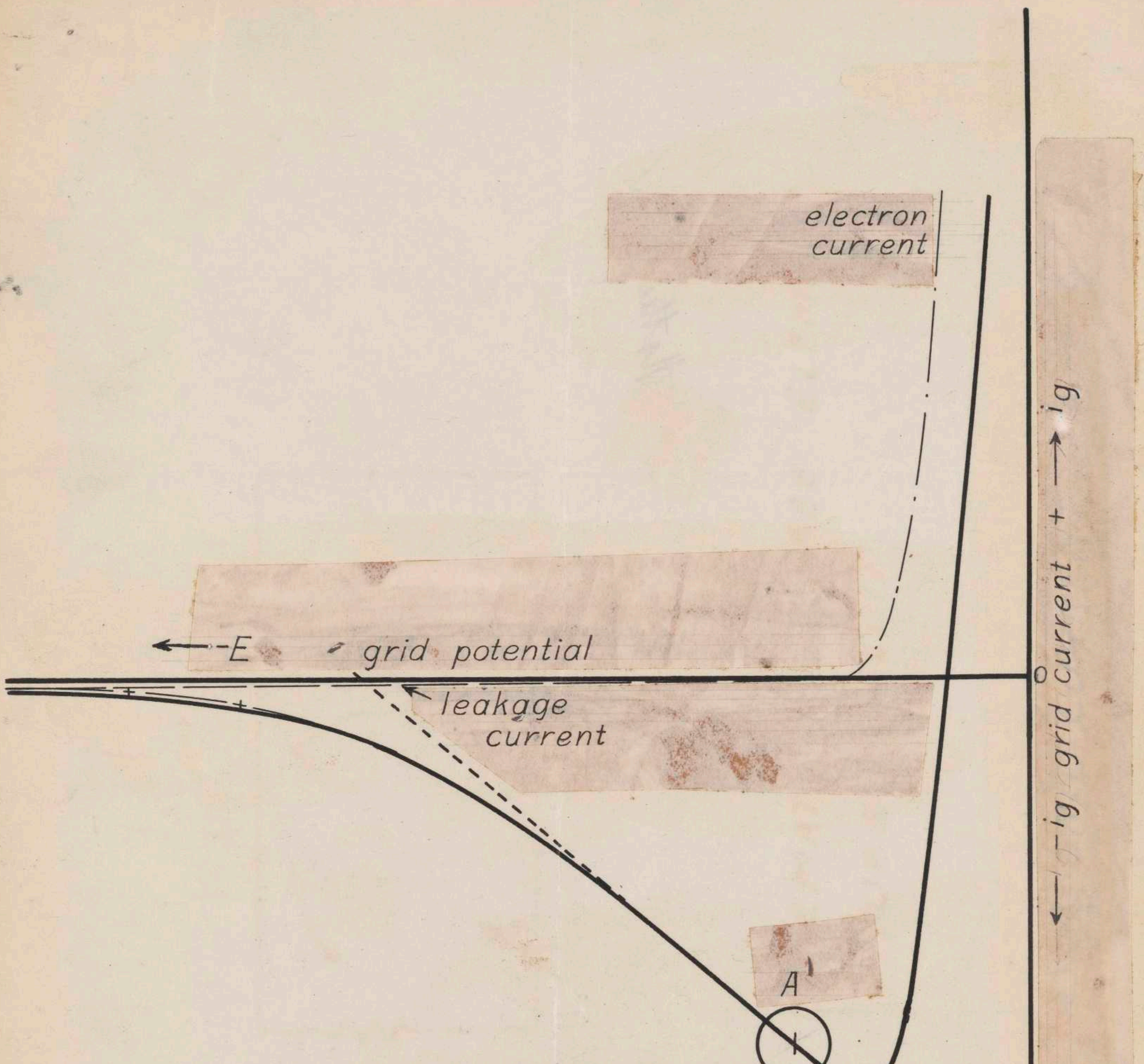


FIG 2. GRID CURRENT CHARACTERISTIC OF VACUUM TUBE

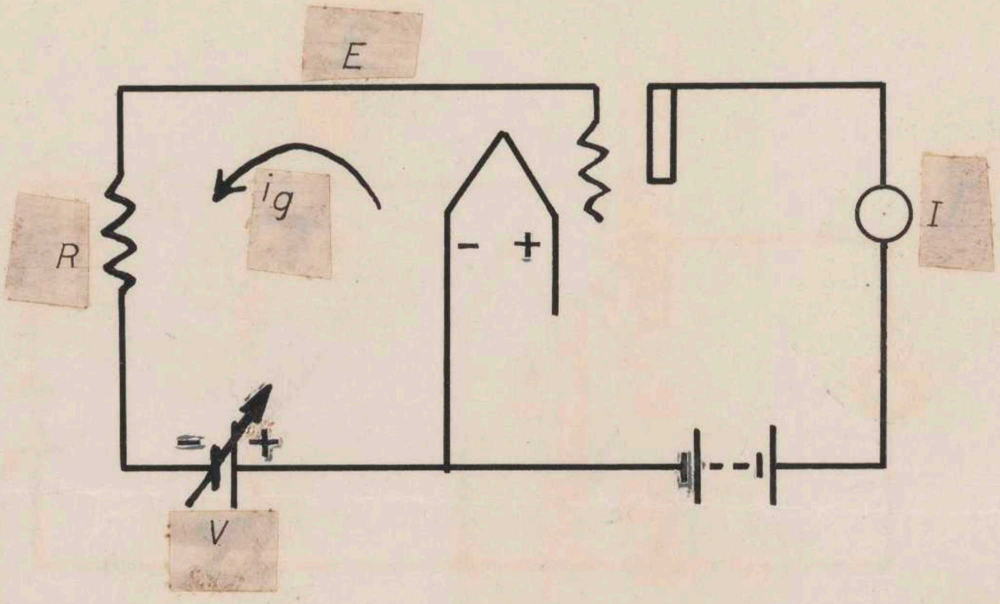


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FIG. 3. SCHEMATIC OF THREE ELEMENT TUBE CIRCUIT

Nottingham

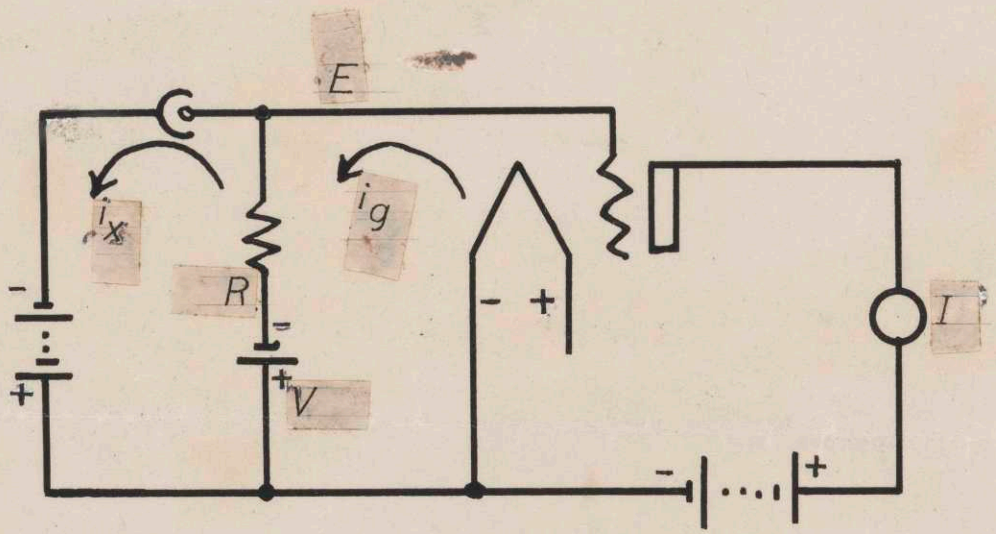

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FIG. 4. SCHEMATIC OF PHOTOELECTRIC CELL AND THREE ELEMENT TUBE CIRCUIT.

8) Is there any systematic ~~work~~ work on the study of photoelectric cells that would be of interest? Some of this work can be worked in along with the testing of the present amplifier.

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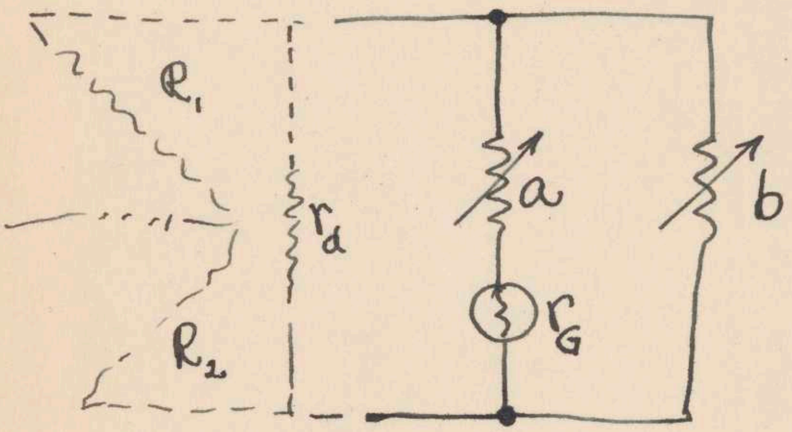

Fig 8

September 26, 1929

To: Mr. E. D. Doyle

c/o Mr Behr.
Dev. Dept

CC. Mr. Notingham

From: F. H. Wyeth

Mr. C. J. McCarthy of Electrical Research Products mentioned to me on his recent visit that an article appeared in the QST for June, 1929 entitled "Photo-electric Cells and Methods of Coupling to Vacuum Tubes" by Thornton P. Dewhirst, and that the information contained might be of some interest to us in our application of the photoelectric cell as a radiation pyrometer.

The style of the article is a typical QST wording but the subject matter in regard to the use of a tube may be of some slight interest to you.

F. H. W.

F

F. H. W.

Sensitivity
(see card.)

Stability:-

1) Tubes (201-A or 101-F satisfactory
246-A and 245-A under test)

2) Power

a. Plate ~~[illegible]~~ battery adjustment can be made so that 1% change will not produce a step.

b) Filament current must be constant to better than 0.1% for no step.

b.1) compensation can be introduced to make this tolerance greater, method under test.

c) Slide-wire current requirements are the same as in any recorder.

3) Resistances:-

Nothing has been done to study the effect of temperature and humidity on the resistances in the grid circuit. A systematic study should be undertaken to select the best type of resistance for the range .5 to 100 megs and these should be thoroughly tested.

Questions:

1) What form of power supply is preferred?

2) When should a model be attempted?

3) Who can study the resistance question and when?

4) Would it be advisable to select one or two applications which are of greatest interest and carry through the development completely so as to give an actual test on the over all system?

5) What are the application in which "voltage sensitivity" as such is wanted and what is lacking in the present methods of recording in these fields?

6.) Should the attention be divided and some work started to show the possibilities and limitations along these lines?

7) Is there any work with A.C. ~~an~~ amplifiers which should be undertaken and for what purpose?