MC 0241. "General Theory of the Plasma Diode Energy Converter" by W.B. Nottingham, 1959 - 1960

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TECH. REPORT No. TEE-7002-5

WBN

1959-1960

General Theory of the Plasma Diode Energy Converter

WAYNE B. NOTTINGHAM

THERMO ELECTRON
ENGINEERING CORPORATION

GENERAL THEORY
OF THE
PLASMA DIODE ENERGY CONVERTER

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Thermo Electron Engineering Corporation
Technical Report No. TEE-7002-5

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Foreword

This report has been prepared at the request of the Thermo-Electron Engineering Corporation so that it can make available the results of the theoretical study of the plasma diode. Even though this report should be looked at as a preliminary step, it has been put into this form in order to serve a number of different objectives. Principal among these objectives is the desire to have as quantitative a general understanding of the plasma diode as can be had on the basis of presently available experimental data. In this respect it serves two purposes which are: first, to understand some of the phenomena that characterize this diode; and second, to help point up the new experimental approaches most likely to yield valuable results. Refinement in the theory will be made as rapidly as its deficiencies can be understood and evaluated. The philosophy behind this approach is that the thermionic plasma diode is a device, the operation of which departs so drastically from the generalized concepts associated with thermodynamics that its performance characteristics can only be understood in terms of the correct interpretation of detailed mechanisms.

Shortage of time has made it impossible to apply the results of this study to all of the available experimental data published by the many workers in this field. Throughout this study each of the basic concepts has been tested by a comparison with experimental data whenever it could be found. In all of these tests of the theory, the results have been quantitatively satisfactory. Since this report is definitely preliminary in nature, it is hoped that readers who discover mistakes in it, either arithmetical or mistakes that can be demonstrated to be incorrect in principle, will do the writer a real service to call his attention to them. It is the intention within the next few months to revise this report in such ways as may prove necessary and desirable and to supplement it with a far more complete application to experimental data. In the long run it is hoped that it will serve to guide us toward engineering designs and constructions of practical plasma diodes. It is believed that the full understanding of the general theory of the plasma diode will contribute significantly to the construction of devices that optimize the many parameters that influence the utility of this means of converting heat into electrical energy.

Foreword to Second Edition

This report on the General Theory of the Plasma Diode Energy Converter was largely prepared in late December 1959 and early January 1960, following an oral presentation made at the request of Professor Hatsopoulos of the Mechanical Engineering Department at M.I.T. The manuscript was put in its final form during February and early March of 1960. It was the original intention that by the present time enough additional information would have come to the author to warrant a complete revision and a new presentation. Although some new information has been available, the decision has been made to reprint the original contribution just as it was first prepared except for minor editorial changes to improve the terminology and definition of symbols. It is still the hope of the author that additional experimental evidence will become available so as to warrant a complete revision of this general theory. There are a number of details that have not been treated with as much rigor as will be needed to make the theory applicable to a wider range of experimental conditions. This is one of the reasons why publication through the usual channels has not been used, since by presenting the information as a technical report from the Thermo Electron Engineering Corporation, will still be possible to reach a very large fraction of all readers specifically interested in this subject.

In addition to the report as original prepared, some equations used there, have been adapted to a nomographic chart prepared by an M.I.T. student, James H. Hufford. Although it is obvious that this chart can be improved, specifically as it relates to the drafting, it is thought that it will serve a sufficiently useful purpose to be worth incorporating as an addendum to this technical report. Again let me emphasize that constructive criticism will be most welcome.

Wayne B. Nottingham
September 9, 1960

Glossary of Symbols

E	Surface field in a cavity Eq 3:5
F.L.	Fermi level, Fig. 1
f	Fraction of atoms ionized, Eq. 4:16
f_s	Fractional ionization near collector sheath, Eq. 12:1
f_E	Fractional ionization, Eq. 10:17
f_p	Frequency of electron oscillations, Eq. 8:12
I_1	Electron emission current, Eq. 2:1
I_+	Random ion current, Eq. 12:1
I_{Cs}	Current equivalent of cesium evaporation rate, Eq. 5:4
k	Boltzmann's constant 1.38×10^{-23} joule/deg., Eq. 2:2
m	Mass of electron 9.1×10^{-31} Kg.
M	Mass per atom of gas (2.206×10^{-25} Kg. for cesium)
n	Concentration of neutral atoms of gas. Sect. 4. Also Eq. 4:14
n_-	Electron (or ion concentration) as in Eq. 8:12
n_o	Density of charges at the center of a cavity, Eq. 3:1, and also the density of ions or electrons at sheath boundary as in Eq. 6:1
n_s	Density of charges at the surface of a cavity (Sect. 3) and also at the surface of an ionizer of positive ions. Sect. 4. Eq. 4:3
N_{Cs}	Concentration of cesium atoms (atoms/m^3) Eq. 5:2
N	Concentration of cesium atoms (i.e., both atoms and ions) at a higher temperature, T_g, than T_{Cs}, Eq. 4:16
p	Cesium gas pressure (mm. Hg.) Eq. 5:1
P_c	Collision probability expressed in units of cm^{-1}, Sect. 8
P_i	Ionization efficiency, Eq. 9:1
q	Electron charge 1.60×10^{-19} coulomb, Eq. 2:2
r	Dimensionless parameter defined in Eq. 10:6
R	Proportionality constant, Eq. 10:1
s_1	Emitter surface potential, Fig. 1
s_2	Collector surface potential, Fig. 3
S_1	Extent of positive ion sheath at emitter, Fig. 3
S	Series expansion as defined by Eq. 4:13
S_2	Extent of sheath at the collector, Fig. 3
S'	Series expansion as defined by Eq. 4:10
S'_o	Limiting value of S', Eq. 4:11
T	Emitter temperature in degrees Kelvin, Eq. 2:1
T_{Cs}	Cesium temperature when liquid surface is equilibrium with gaseous cesium, Eq. 5:1
T_g	Temperature of cesium between electrodes. Sect. 4
T_c	Collector temperature
u	Work-function relative to ionization potential, Eq. 4:8
v	Electron velocity, Eq. 10:1

Glossary of Symbols, continued

V	Applied or output voltage (volts) Fig. 1
V'	A potential relative to cavity center, Eq. 3:1, Fig. 5b
V_i	Ionization potential of the gas. Sect. 4.
V_o	Surface potential relative to cavity center, Sect. 3, Fig. 5b
V_-	An electron energy expressed in volts, Eq. 9:1
V_p	Difference in potential across emitter sheath, Fig. 3
V_x	Potential difference between emitter surface and any point on the emitter sheath, Sect. 4, Fig. 6
V_c	Potential difference between maximum at the midline cavity and at any point x, Fig 5
$V_{o.c.}$	Open circuit applied or output voltage (volts), Fig. 4
$\overline{V}$	kT/q voltage equivalent of temperature, Eq. 2:2
$\overline{V}_-$	Voltage equivalent of the temperature of electrons in a plasma, Eq. 8:14
w	Width of a cavity, Fig. 5b, Eq. 3:2
x	Distance from midline of a cavity, Fig. 5b, and also a travel distance in Eq. 8:4
x_o	Half width of cavity, Fig. 5b, Eq. 3:5
y	Parameter used in Eq. 9:6
α	Rate of increase of ionization efficiency, Eq. 9:1
ϵ_o	The permittivity of free space 8.85×10^{-12} Farad/m, Eq. 3:1
ϕ_1	True emitter work-function (volts), Fig. 1
ϕ_2	True collector work-function (volts), Fig. 1
ϕ_m	Space-charge potential minimum, Fig. 1
λ_e	Mean-free path, Eq. 8:3
λ_r	Mean-free path for recombination, Eq. 10:4
λ_p	Mean-free path for recombination, Eq. 10:13
μ	Parameter defined by Eq. 4:6
μ_{Cs}	Rate of cesium atom evaporation, Eq. 5:3
	Proportionality constant, Eq. 10:12
ν ~~ϕ_n~~	Atom arrival rate Eq. 4:1
σ'_c, σ_c	Collision cross section, Eq. 8:1 and Eq. 8:2
σ_i	Ionization cross section, Eq. 9:2
ν_g	Atom production rate, Eq. 4:5
ν_i	Ion arrival rate from within sheath, Eq. 4:3
ν_n	Random current of atoms, Eq. 4:1
ν_{n+}	Ion production rate at hot surface, Eq. 4:2
ν_+	Ion production rate in a plasma, Eq. 9:4
ν_r	Recombination rate, Eq. 10:3
ν_ρ	Recombination rate, Eq. 10:14

Abstract

The new concept which is put forward in this report is that the presence of a suitable pressure of cesium atoms near a hot emitting surface results in an ionization so copious that an ion space-charge sheath can be built up at the surface. If the drop in potential over this sheath is approximately 0.5 volt or more, then the electrons are injected into the interelectrode space with an average energy greater than this. The presence of a large supply of ions in this space essentially equal in number to the electrons per unit volume results in the generation of plasma oscillations. Electrons which entered the region with a more or less monoenergetic energy distribution become randomized both in direction and in energy. With no change in the total energy content, the distribution becomes one that could be characterized as being Maxwellian with a temperature of 4000°K or more. In the presence of neutral atoms, such a distribution can generate such a copious supply of new ions that losses in ion density due to recombination and due to the sweeping of ions across the boundary sheath can be made up.

The detailed analysis concerning these steps is carried through, equations are formulated, and methods of analysis are provided so that good approximations can be made to determine the electron injection energy directly from the known parameters of the problem. These include the work-function of the emitter, its temperature, the condensation temperature of the cesium supply, and an estimation concerning the average temperature of the gas in the space between the emitter and the collector.

In the plasma space, ionization and recombination take place. Again, the equations are formulated so that it is possible to determine the probable fraction of the cesium concentration there that is ionized. The theory shows that if the electron energy distribution is characterized by a temperature of less than 3000°K, the fractional ionization is very small; whereas if it is characterized by a temperature of about 5000°K or more, the gas in equilibrium is very fully ionized. The analysis carries on to a discussion of the formation of the ion sheath at the collector. A calculation is made concerning the rate of loss of ions from the plasma over this sheath and it is compared with the rate of production of ions within the sheath at a distance of a mean-free path from the boundary. Again the critical temperature for the electrons is in the 4000 to 5000°K range in that if the temperature is too low, the supply cannot be maintained and instability is likely to result. On the basis of this analysis a brief discussion of the problem of oscillations generally found in plasma diodes is put forward. Wherever possible throughout the paper, typical numerical results are computed. This presentation should be considered as a preliminary report to be followed later by a more comprehensive one modified wherever necessary and applied to all of the valid experimental data available.

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General Theory of the Plasma Diode Energy Converter

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1. Introduction

The conversion of heat to electrical energy by means of a vacuum diode implies the presence of an electron emitter operating at a high temperature and an electron collector. Any positive ions present are there at such a low concentration that their influence can be neglected. Important to the operation of the "plasma diode," is the combined effect of both electrons and positive ions. There are two broad classifications to be considered that may be identified in terms of the electron mean-free path and its relation to the diode dimensions. The presence of positive ions can be very important even though the electron mean-free path and the ion mean-free path may be larger than the inter-electrode distance. This class of diode is identified as the "low-pressure plasma diode." In some respects, to identify the inter-electrode space by the work "plasma" might be considered an over-statement since it depends only on the concept that parts of this region are essentially free of space charge, and therefore it is only in that sense that the region is a "plasma." The second category of plasma diodes is identified by the phrase "high pressure plasma diode." It is implied here that the mean-free path is shorter than the inter-electrode distance. Under these circumstances the space region may not only contain approximately equal numbers of electrons and ions but these may interact on each other in such a manner as to bring about energy exchange through the medium of plasma oscillations.

There are three distinct regions that make up the total space from the hot emitter surface to the colder collector. These will be identified as: (1) the emitter sheath; (2) the plasma space; and (3) the collector sheath. Details related to each of these regions will be presented and the means for relating them to each other will be discussed.

Very close to the surfaces of both the emitter and the collector, space-charge sheaths will generally be found. These may be either positive ion sheaths or electron sheaths. In the space region conduction will largely take place as a result of the single-stream flow of ions or electrons or both for the low pressure plasma. In the high pressure plasma a drift of electrons or ions will generally be superimposed on a random current flow of electrons or ions or both. By random current flow the implication is that a velocity distribution will exist among the charge carriers which is more or less isotropic and has superimposed upon it a drift velocity which is responsible for carrying the drift current.

2. General Discussion of Emitter Sheath

The emitter generally serves a dual purpose. It emits electrons from its interior and it serves as a generator of positive ions at its surface. All of the discussion that follows will be dependent on the assumption that the gas used in the diode is cesium. Any area of the emitter surface which has a true work-function a few tenths of a volt greater than the ionization potential of cesium which is 3.89 volts will serve as an area which converts practically every neutral atom which arrives at the surface to a cesium ion. If the true work-function of the surface is less than 3.89 volts, then the probability that the valence electron of the neutral atom will make a transition to an empty state among the conduction electrons of the emitter is very small. This probability decreases exponentially with the energy difference between the ionization potential and the true work-function of the emitter at the arrival point of the cesium atom. If the emitter is nonuniform, the greatest electron emission comes from low work-function areas. This could tend to build electron space charge. The high work-function areas will be the most efficient ion generators and will tend to build positive ion space charge in their neighborhood. These two regions of space charge will then tend to neutralize by the fact that the ions will become trapped in the electron space charge and the electrons trapped in the ion space charge to finally equalize the situation. Since it is impossible to generalize concerning the detailed influence of inhomogeneity, this aspect of the problem will not be treated specifically.

Figure 1 may be used to define some of the important quantities pertinent to this discussion. Here the emitter and the collector are designated and their Fermi levels indicated by "FL." On this motive diagram the difference between the Fermi levels shown as V is directly measurable as the applied voltage difference between them. The true work-function of the emitter ϕ_1 represents the energy difference between the Fermi level of the emitter and an electron at rest in the immediate neighborhood of the emitter surface that is of the order of 10^{-6} centimeters. The true work-function of the collector is defined in the same way and is represented by ϕ_2. If the electron emission is high, then an electron space-charge sheath may be expected to create a space-charge minimum which lies at ϕ_m with respect to the emitter Fermi level. Electrons emitted within the energy band ϕ_1 to ϕ_m are returned to the emitter whereas those emitted with greater energy than ϕ_m contribute to the observed diode current.

change to 6

If cesium atoms are permitted to enter the space between the emitter and the collector, many of them in their random motion will collide with the emitter. If ϕ_1 is greater than 3.89, most of the atoms that arrive at the surface are ionized and will leave it as though they started at the surface point s_1. Since the ions will have an initial velocity distribution characteristic of the temperature of the emitter, they will occupy ion states within the motive diagram beginning at s_1 and move in states within the space region at energy levels higher than s_1. As long as a net electron space charge exists in front of the emitter area the ions will be accelerated toward the collector but they will not pass all the way to it since the positive surface charge indicated by the motive diagram will bring them to a stop and return them to the emitter. Some ions even though the mean-free path may be long, will collide with atoms and lose some of their kinetic energy. That will put them in energy levels below s_1 and therefore they

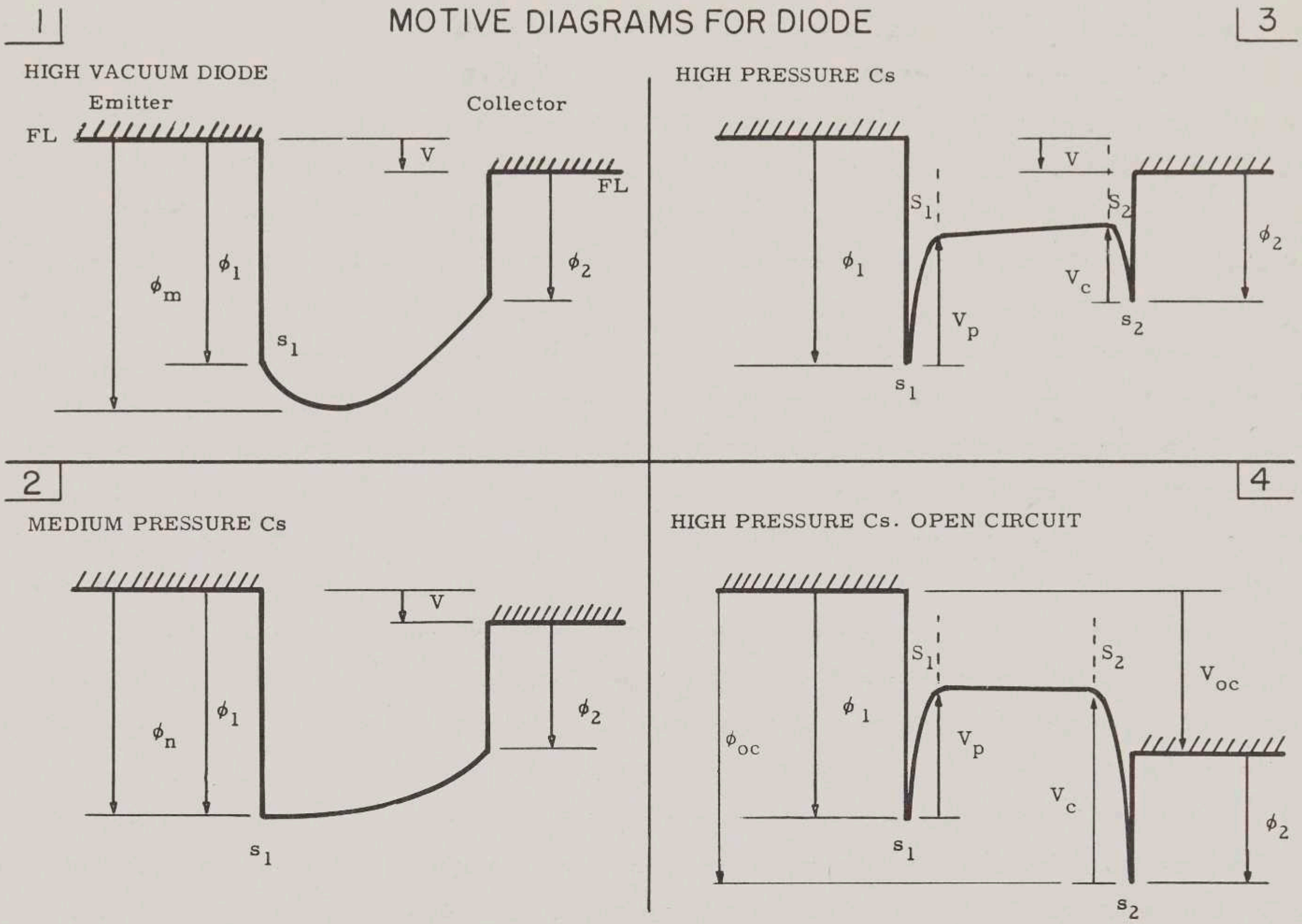


Fig. 1 Motive diagram for high vacuum diode.

Fig. 2 Motive diagram for medium pressure cesium diode.

Fig. 3 Motive diagram for high pressure cesium diode.

Fig. 4 Motive diagram for high pressure cesium diode under open circuit condition.

will be trapped. As more and more ions are trapped the space-charge minimum will disappear.

As the cesium pressure is increased the ion production will also increase until the ion density in the immediate neighborhood of s_1 is exactly equal to the density of the electrons in transit from the emitter to the collector. The motive field at the surface of the emitter will then be zero and the "zero field" emission from the emitter may be obtained. This situation is illustrated in Fig. 2. The zero field electron emission current density may be calculated by the following formula. The equation in this form gives the current density in amps/m^2.

$$I_1 = 1.2 \times 10^6 \, T^2 \, e^{-\phi_1/\overline{V}} \qquad 2:1$$

In this equation T is the emitter temperature, ϕ_1 its true work-function and $\overline{V}$ is defined by

$$\overline{V} = \frac{kT}{q} \qquad 2:2$$

Here q is the electron charge and k is Boltzmann's constant.

As the cesium pressure is increased a positive ion sheath represented as extending over the space S_1 of Fig. 3 will develop. The change in the motive function from the surface potential to that at the end of the sheath is represented by V_p. In the space region between S_1 and the collector sheath at S_2 we can anticipate that there will be a small rise or fall in the motive function. This change is necessary because some electrons that are emitted over the surface barrier at s_1 will lose energy by collision and become trapped in the electron energy levels below the surface potential of the collector at s_2. This trapped negative charge must be compensated for by ions which are brought to rest and returned toward the emitter. Under the conditions shown, the surface potential s_2 of the collector is negative with respect to the motive function at the sheath boundary S_2. In this range the trapped electrons are repelled and the positive ions which enter the sheath at S_2 are accelerated into the collector. Again in this region the net space charge is positive.

If the applied potential V is made more negative than that shown in Fig. 3 then it is possible to set up a condition for which the ion arrival rate is exactly equal to the electron arrival rate. When we assume that there is no electron emission from the collector, then this condition of balancing currents is the "open-circuit" voltage designated as V_{oc} of Fig. 4. Under these conditions there will be nearly twice as much negative charge per unit volume in the space region as there was under the condition illustrated in Fig. 3 because there will be a double stream of electrons across this space. That change will result in a somewhat lower value of V_p to permit more ions to enter the space region for neutralization. Again near the collector a positive ion space charge will develop because the motive function is such as to turn back all electrons with a kinetic energy associated with their motion toward the collector which is insufficient to surmount the barrier at s_2. If there were no electron interactions in the space region, then the electron current received at the collector could be computed by means of Eq. 2:1 by the insertion of ϕ_{oc} which is $V_{oc} + \phi_2$ in place of ϕ_1. A calculation of current density by this formula would in many cases give the minimum current expected at the collector whereas if some electron energy sharing takes place because of their injection into the space by the accelerating potential V_p, then it is possible for more electrons to be received at the surface s_2 than would otherwise have been permitted. In any case the ion current which is accelerated through the collector space-charge sheath S_2 must balance the electron arrival current. It should be clear from this discussion that the open-circuit electron arrival current could very well be less than the "saturation" ion current produced at the hot surface emitter. If the cesium density is sufficiently low, then the ion space-charge sheath at the emitter surface can vanish and the electron arrival at s_2 can, under that condition, equal the saturation ion emission.

3. Calculation of Sheath Potential and Sheath Thickness

Should not this be V_0 here if V_0 is to be output voltage

Fowler(1) formulated the basic equations and showed their method of solution to determine the distribution in electrostatic potential in a plane-parallel cavity-like space containing charged particles. Details are given in Thermionic Emission(2).

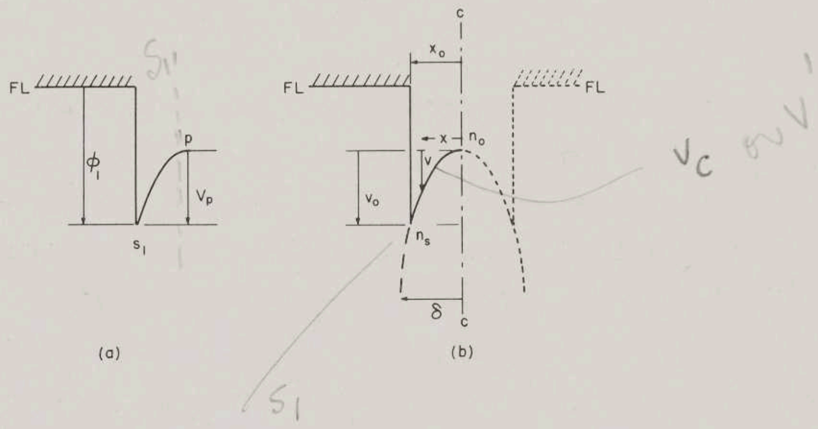


Fig. 5

(a) Motive diagram at emitter space-charge sheath--high density of positive ions at the surface.

(b) Motive diagram for Fowler solution to space charge in a cavity.

Figures 5(a) and 5(b) are drawn to indicate the possible closeness of the conditions in the emitter sheath to those that would obtain in the cavity problem between the surface s_1 and the dividing line c-c. In the cavity problem the density of charges in the center in n_o whereas the density of charges at the surface is n_s. The difference in potential between the surface and the center of the cavity is V_o. The solution of the problem is written in terms of the distance x from the midline c-c and the potential difference between the maximum at the midline and any point on the curve as shown in the diagram as V'. The relations are given by the following equations:

$$x = \left(\frac{2\overline{V}\epsilon_o}{q\,n_o}\right)^{1/2} \arctan\left(e^{\frac{V'}{\overline{V}}} - 1\right)^{1/2} \qquad |3{:}1$$

Here ϵ_o is the permittivity of free space. For values of $(V_o/\overline{V}) > 8$, there is a relation between the half-width of the cavity (w/2) and the charge concentration at the center

$$\frac{w}{2} = \frac{\pi}{2}\left(\frac{2\overline{V}\epsilon_o}{q\,n_o}\right)^{1/2} = 1.651 \times 10^4 \left(\frac{\overline{V}}{n_o}\right)^{1/2} \qquad |3{:}2$$

$$\frac{w}{2} = 1.533 \times 10^2 \left(\frac{T}{n_o}\right)^{1/2} \qquad |3{:}3$$

These equations serve best to evaluate the emitter sheath thickness S_1. When

$(V_o/\overline{V}) < 8$ the thickness will be less and Eq. 3:1 must be used with V_o set in place of V'.

The potential difference is given by:

$$V_o = \overline{V} \ln\left\{ \sec^2\left[x_o\left(\frac{q n_o}{2\overline{V} \epsilon_o}\right)^{1/2} \right]\right\} \qquad |3:4$$

In these expressions the voltage equivalent of the temperature is expressed by Eq. 2:2.

If the plasma case can be assumed to be equivalent to the cavity, it will also be of interest to be able to calculate the surface field once the potential energy difference between the surface and the plasma, or between the surface and the cavity, is known. The exact expression follows:

$$E = \left(\frac{2\overline{V} q n_o}{\epsilon_o}\right)^{1/2} \left(e^{\frac{V_o}{\overline{V}}} - 1\right)^{1/2} \qquad |3:5$$

$$E = 1.897 \times 10^{-4} \left[\overline{V} n_o \left(e^{\frac{V_o}{\overline{V}}} - 1\right)\right]^{1/2} = 1.766 \times 10^{-6} \left[T n_o \left(e^{\frac{V_o}{\overline{V}}} - 1\right)\right]^{1/2} \qquad |3:6$$

For the plasma, use V_p in place of V_o.

These equations are not very useful unless a method is provided for the determination of the ion density (n_o) at the center of the cavity and the effective ion density (n_s) at the surface boundary. It will therefore be necessary to devise a means of evaluating these densities. If this can be done then the value V_p may be used in the above equations in place of V_o and the sheath thickness and surface field may be computed.

4. Determination of Ion Density at the Surface

For the determination of the ion density at the hot surface, slightly different equations must be used depending on whether or not the true work-function of the surface is greater or less than the ionization potential. The first formulation will be on the basis that ϕ_1 is greater than V_i. The arrival rate of atoms (not ions) at the hot surface will depend on the concentration of neutral atoms (n) in the space and their temperature. In the practical tube for which these equations are to be used, the gas temperature should be assumed to be intermediate between the high temperature emitting surface and the lower temperature collector. Although the exact value of the effective temperature will be difficult to know, the analysis will show that there are other factors which make this uncertainty relatively unimportant. The atom arrival from the gas is given by the following equation

Clarify

$$\nu_n = n \left(\frac{k T_g}{2\pi M}\right)^{1/2} \qquad |4:1$$

all values of ϕ (

In this equation n is the concentration of neutral atoms in the gas which is at temperature T_g. The mass per atom is M and its value for cesium is 2.206×10^{-25} kg. With the true work-function higher than the ionization potential, the ion production rate is related to the gas arrival rate by the equation[Δ]

$$\nu_{n+} = n \left(\frac{kT_g}{2\pi M}\right)^{1/2} \frac{1}{\omega e^{-\frac{\phi_1 - V_i}{\overline{V}}} + 1} \qquad 4:2$$

$\omega = 1$ or 2

This equation may also be applied to compute the ion production even though $\phi_1 < V_i$.

The ions will acquire a distribution in kinetic energy at the surface characterized by its temperature. Since most of these ions will be trapped in the space-charge field of the previously created ions, they will return to the surface at various energy levels with respect to the surface potential. We may therefore write down the following equation as representing the ion arrival rate from the sheath region as a function of the energy defined as shown in Fig. 6 for the range V_x to $V_x + dV_x$.

$$d\nu_i = n_s \left(\frac{kT}{2\pi M}\right)^{1/2} e^{-\frac{V_x}{\overline{V}}} \frac{dV_x}{\overline{V}} \qquad 4:3$$

In the steady state condition the ion annihilation rate that exists at the surface when the ion density is n_s and the energy distribution given by Eq. 4:3 must be equal to the ion generation rate given by Eq. 4:2[ΔΔ]. The probability that an ion will be reconverted upon collision with the surface into a neutral atom depends on V_x and is thought to be governed by the following relation:

$$\text{Atom production probability} = \left(1 - \frac{1}{\omega^{-1} e^{\frac{V_x - (\phi_1 - V_i)}{\overline{V}}} + 1}\right) \qquad 4:4$$

The form of this equation depends upon the qualitative picture illustrated by Fig. 6. The energy level occupied by the ion is shown to be positive with respect to the surface potential by the amount V_x. The dotted line V_i positive with respect to this level represents an available empty electron level within the cesium ion. When this level is occupied, the ion has converted into an atom and the probability that it will become occupied is taken a dependent on the energy difference between the available state and the Fermi level of the emitter. This energy difference is given by $\left[V_x - (\phi_1 - V_i)\right]$.

[Δ] The Langmuir-Saha equation multiplies the exponential by 2. Neither equation has been verified by experiment.

[ΔΔ] This statement is made as a working approximation since the writer thinks that ion arrival from the plasma will closely equal the ion emission into the plasma.

p. 8.

equation 4:7 can be integrated directly

$$\int_{\mu=-u}^{\infty} e^{-\mu}\left(1-\frac{1}{e^{\mu}+1}\right)d\mu$$

$$= -\ln(1+e^{-\mu})\Big|_{\mu=-u}^{\infty}$$

$$= \ln(1+e^{u}) \qquad \text{for } +\infty > u > -\infty$$

Therefore for $+\infty > u > -\infty$

$$V_g = n_s\left(\frac{kT}{2\pi M}\right)^{\frac{1}{2}} \ln(1+e^{u})$$

and Eqs 4:9, 4:10, 4:11, 4:12, 4:13 can be combined to give

$$\ln n_s = \ln n + \frac{1}{2}\ln T_g - \frac{1}{2}\ln T + u$$

$$- \ln(e^{-u}+1) - \ln[\ln(1+e^{u})]$$

for $+\infty > u > -\infty$

Eq 6:4, 6:5, 6:6, 6:7, 6:8, 6:9 can be combined to give

$$\frac{v_p}{\bar{V}} - \frac{1}{2} \ln\left(\frac{v_p}{\bar{V}} + 1\right)$$

$$= 9.49 - \frac{8910}{T_{co}} + \frac{\phi_1}{\bar{V}} - \ln(1-f)^{-1} + 2\ln \bar{V}^{-1} - \ln T_{co} + H(u)$$

where

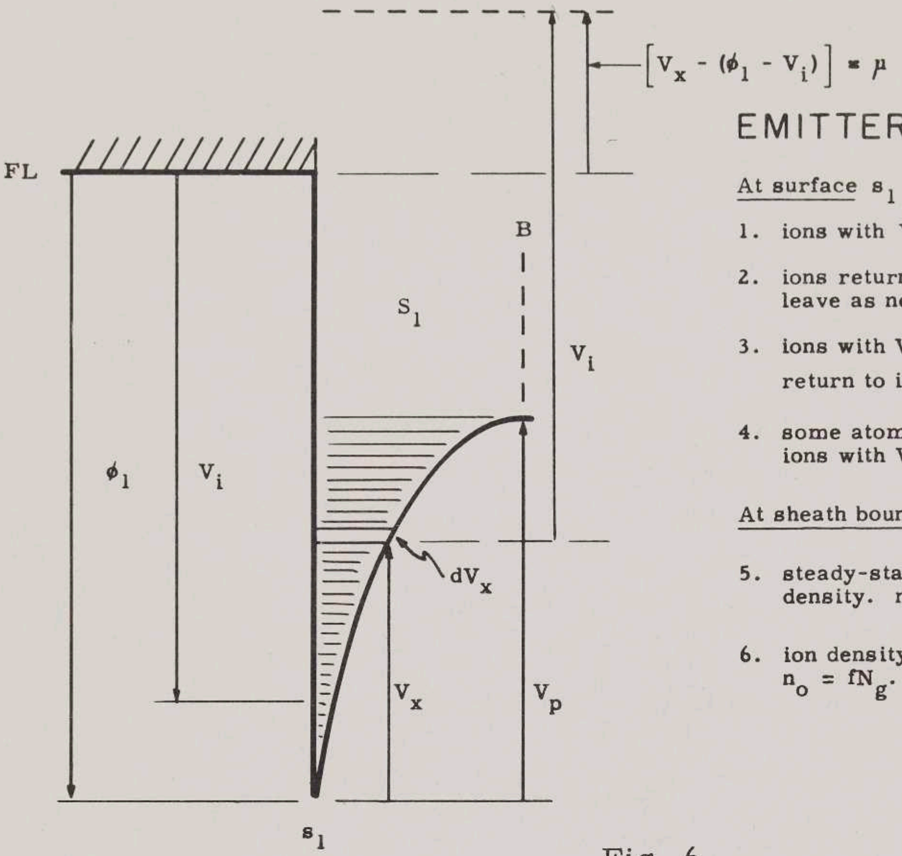
$$H(u) = u - \ln(1 + e^{-u}) - \ln\left[\ln(1 + e^{u})\right]$$

for $+\infty > u > -\infty$

Appendix 1 should now reads

for $+>\infty\ u > -\infty$

$$H(u) = u - \ln(1 + e^{-u}) - \ln\left[\ln(1 + e^{u})\right]$$



EMITTER SHEATH

At surface s_1

1. ions with $V_x < V_p$ are trapped.
2. ions returning to surface with $V_x > (\phi_1 - V_i)$ leave as neutral atoms.
3. ions with $V_x > V_p$ come from plasma and return to it as atoms.
4. some atoms come to surface and leave as ions with $V_x > V_p$ to go into plasma.

At sheath boundary B

5. steady-state ion density equals electron density. $n_o = n_-$.
6. ion density equals fractional gas density. $n_o = fN_g$.

Fig. 6

Detailed diagram for emitter sheath solution.

The combination of Eqs. 4:3 and 4:4 permits the calculation of the total atom production rate. This is also the ion annihilation rate.

$$\nu_g = n_s \left(\frac{kT}{2\pi M}\right)^{1/2} \int_0^\infty e^{-\frac{V_x}{\overline{V}}} \left(1 - \frac{1}{\omega^{-1} e^{\frac{V_x - (\phi_1 - V_i)}{\overline{V}}} + 1}\right) \frac{dV_x}{\overline{V}} \qquad |4{:}5$$

This equation may be rewritten in terms of the following dimensionless parameter.

$$\mu = \frac{V_x - (\phi_1 - V_i)}{\overline{V}} \qquad |4{:}6$$

The following equation results from this substitution:

$$\nu_g = n_s \left(\frac{kT}{2\pi M}\right)^{1/2} e^{-\frac{\phi_1 - V_i}{\overline{V}}} \int_{\mu = -\frac{\phi_1 - V_i}{\overline{V}}}^{\infty} e^{-\mu} \left(1 - \frac{1}{\omega e^{\mu} + 1}\right) d\mu \qquad |4{:}7$$

It is possible to use expansion methods to integrate this equation and inspection shows that this integration can best be done by dividing the range of integration to carry first through the negative values of μ to $\mu = 0$ and from $\mu = 0$ to $\mu = \infty$ as a second step. The final result based on this procedure can be expressed most compactly by defining the parameter u as follows:

$$u = \frac{\phi_1 - V_i}{\overline{V}} \qquad |4:8$$

The present analysis depends upon the condition that the work-function of the surface is equal to or greater than the ionization potential. This condition means that u is positive. The complete solution for this case ($u \geq 0$) is given as:

Case I. $u \geq 0$

$$\nu_g = n_s \left(\frac{kT}{2\pi M}\right)^{1/2} e^{-u} (u + S') \qquad |4:9$$

The series expansion represented by S' is given as follows:

$$S' = e^{-u} - \frac{1}{2} e^{-2u} + \frac{1}{3} e^{-3u} - \frac{1}{4} e^{-4u} + \ldots \qquad |4:10$$

$$\text{for } u = 0; \quad S'_o = \ln 2 \qquad |4:11$$

In the second case for which the value of μ as defined by Eq. 4:6 is always positive subject to the condition that the work-function is always equal to or less than the ionization potential starts again with Eq. 4:7 but requires only one step in integration. The final result expressed in terms of the parameter u of Eq. 4:8 which in this case is numerically negative is given as follows:

Case II ($u \leq 0$)

$$\nu_g = n_s \left(\frac{kT}{2\pi M}\right)^{1/2} e^{-u} S \qquad |4:12$$

In this equation the series S is represented by:

$$S = e^{u} - \frac{1}{2} e^{2u} + \frac{1}{3} e^{3u} - \frac{1}{4} e^{4u} + \ldots \qquad |4:13$$

Equations 4:9 and 4:12 yield identical values as they should for the limiting condition of u = 0. The first formula, however, must be used for the higher work-function emitters and the second formula Eq. 4:12 must be used for low work-function materials.

The next step in the development of this theory depends on the setting of either Eq. 4:9 or Eq. 4:12 equal to the ion generation rate given by Eq. 4:2. We are particularly interested in solving for $\ln n_s$ and without filling in more detailed arithmetical steps the results are given for the two cases in Eq. 4:14 and 4:15.

Case I $(u \geq 0)$

$$\ln n_s = \ln n + \frac{1}{2}\ln T_g - \frac{1}{2}\ln T + u - \ln(e^{-u} + 1) - \ln(u + S') \qquad 4{:}14$$

Case II $(u \leq 0)$

$$\ln n_s = \ln n + \frac{1}{2}\ln T_g - \frac{1}{2}\ln T + u - \ln(e^{-u} + 1) - \ln S \qquad 4{:}15$$

In both of the above equations n is the concentration of neutral atoms at the sheath boundary. These neutral atoms have been assumed to have a random distribution characterized by a gas temperature T_g. Processes difficult to analyze make the precise determination of n very difficult in many cases. When the pressure is moderately high and the ionization in the plasma space over 90 per cent complete, it is believed that many of these ions sweep across the emitter sheath, become neutralized, and return as atoms to create a local situation in which the fractional ionization at the boundary of the sheath is lower than it is in the plasma proper. It is believed that this will often make the fractional ionization in that specific locality close enough to 50 per cent so that in the estimations which will follow the value of f as defined in Eq. 4:16 will generally be taken as 0.5 unless there is some good reason to choose some other value.

$$f = \frac{N - n}{N} \qquad 4{:}16$$

$$n = N(1 - f) \qquad 4{:}17$$

In these equations N is the total number of gas atoms per unit volume including both neutrals and ions; f the fractional ionization, and n the concentration of neutrals.

5. Cesium Pressure and Concentration Relations

References on cesium pressure as a function of condensation temperature are not in precise agreement except as regards the boiling point which is taken to be 963°K.(3, 4) The following formula is a good compromise to match both sets of data.[Δ]

$$p = 2.45 \times 10^8 \, T_{Cs}^{-1/2} \, e^{-\frac{8910}{T_{Cs}}} \text{ mm} \qquad 5{:}1$$

Two additional equations of general usefulness may be derived directly from Eq. 5:1 to obtain the concentration of cesium atoms (N_{Cs} in equilibrium at a cesium surface of temperature T_{Cs} and the evaporation rate expressed by Eq. 5:3 as μ_{Cs}. In the first of these equations the concentration is given in atoms/m^3 and in the second,

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[Δ] See computational chart attached to this report.

it is atoms evaporated from the liquid surface in atoms/m^2 - sec.

$$N_{Cs} = 2.37 \times 10^{33} \, T_{Cs}^{-3/2} \, e^{-\frac{8910}{T_{Cs}}} \quad \text{atoms/m}^3 \qquad |5:2$$

$$\mu_{Cs} = 7.48 \times 10^{33} \, T_{Cs}^{-1} \, e^{-\frac{8910}{T_{Cs}}} \quad \text{atoms/m}^2\text{ - sec} \qquad |5:3$$

This equation expressed in "current equivalent" is:

$10^{-3870/T_{Cs}}$

$$I_{Cs} = 1.2 \times 10^{15} \, T_{Cs}^{-1} \, e^{-\frac{8910}{T_{Cs}}} \quad \text{amp/m}^2 \cancel{\text{/sec}} \qquad |5:4$$

If the concentration of atoms at the cesium liquid surface is known, then the corresponding concentration of atoms at a connected region characterized by a higher temperature T_g is given by

$$N = N_{Cs} \left(\frac{T_{Cs}}{T_g}\right)^{1/2} \qquad |5:5$$

The combination of Eqs. 4:17, 5:2 and 5:5 gives the sought-for value of ln n.

$$\ln n = 76.86 - \ln T_{Cs} - \frac{8910}{T_{Cs}} - \frac{1}{2} \ln T_g - \ln (1 - f)^{-1} \qquad |5:6$$

6. Analysis To Obtain the Sheath Difference in Potential

Although it is impossible on the basis of present knowledge to obtain an incontravertible determination on a theoretical basis of the potential difference across the emitter sheath, the method given below is thought to be sufficienly accurate to be worth considering. This method depends on the assumption that the ion density at the boundary surface between the sheath and the plasma will be equal to the density of the electrons which are injected into the plasma at this point by the strong sheath field which extends over a range of distance small compared to the mean-free path. In terms of this difference in potential over the sheath the electron density is given in terms of the emission current as follows:

$$n_o = 1.052 \times 10^{13} \frac{I_1}{\overline{V}^{1/2} \left(\frac{V_p}{\overline{V}} + 1\right)^{1/2}} \qquad |6:1$$

This expression may be combined with Eq. 2:1 to obtain the ion density at the sheath boundary based on the assumption of the equality of electron density and ion density. The combined expression is:

$$\ln n_o = 43.97 + 2 \ln T + \frac{1}{2} \ln \overline{V}^{-1} - \frac{\phi_1}{\overline{V}} - \frac{1}{2} \ln \left(\frac{V_p}{\overline{V}} + 1\right) \qquad |6:2$$

Another simplification implicit in the entire analysis is that the relation between the densities is given by the Boltzmann equation as follows:

$$\ln n_s - \ln n_o = \frac{V_p}{\overline{V}} \qquad |6:3$$

In order to arrive at a determination of $(V_p / \overline{V})$ Eq. 4:14 may be combined with Eqs. 6:2 and 6:3 to obtain the relation

Case I High work-function $(u \geq 0)$

$$\frac{V_p}{\overline{V}} - \frac{1}{2} \ln \left(\frac{V_p}{\overline{V}} + 1\right) = 9.49 - \frac{8910}{T_{Cs}} + \frac{\phi_1}{\overline{V}} - \ln (1 - f)^{-1} + 2 \ln \overline{V}^{-1} - \ln T_{Cs} + H_1(u) \qquad |6:4$$

In this equation the symbol H_1 (u) is a function of u only and given by Eq. 6:5 for its exact value or Eq. 6:6 for an excellent approximation for the larger values of u.

$$H_1(u) = u - \ln (1 + e^{-u}) - \ln (u + S') \qquad |6:5$$

For $u > 2.5$

$$H_1(u) = u - \ln u - \left(1 + \frac{1}{u}\right) e^{-u} \qquad |6:6$$

In order to simplify calculations, a table of values of H(u) good for both positive and negative values of u is given in Appendix 1. A plot of this function is shown in Fig. 7.

Note that in Eq. 6:4 the essential specifications of the problem are found on the right-hand side of this equation. It remains only to choose the value of $(V_p / \overline{V})$ required on the left-hand side of the equation to give the same value and thus lead to the correct solution. Figure 8 is a plot of the functional relation and Fig. 9 has been prepared to serve as a graphical means of relating the corresponding values. The procedure is to insert all of the essential facts into the right-hand side of the equation, find its value, locate it on the lower scale of Fig. 9, and identify the corresponding solution on the upper scale. Numerical examples will be given in the next section. See Appendix 2.

The analysis applicable to the low work-function case identified here as Case II follows exactly the same lines and yields the following set of equations.

Case II Low work-function $(u \leq 0)$

$$\frac{V_p}{\overline{V}} - \frac{1}{2} \ln \left(\frac{V_p}{\overline{V}} + 1\right) = 9.49 - \frac{8910}{T_{Cs}} + \frac{\phi_1}{\overline{V}} - \ln (1 - f)^{-1} + 2 \ln \overline{V}^{-1} - \ln T_{Cs} + H_2 (u) \qquad |6:7$$

$$H_2(u) = u - \ln (1 + e^{u}) - \ln (1 - \frac{1}{2} e^{u} + \frac{1}{3} e^{2u} - \frac{1}{4} e^{3u} + \frac{1}{5} e^{4u} - \ldots \qquad |6:8$$

For $u < - 2$

$$H_2(u) = u - \frac{1}{2} e^{u} \qquad |6:9$$

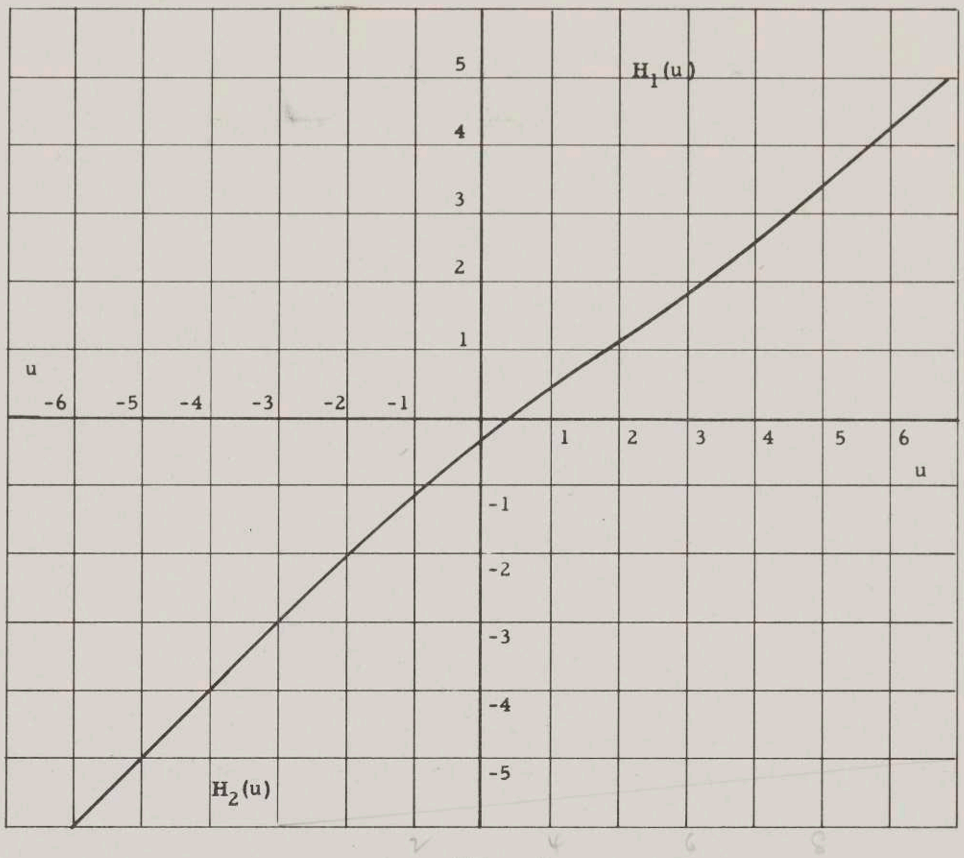


Fig. 7

Universal function needed in Eqs. 6:4 and 6:7. See Appendix 1 for the tabulated values.

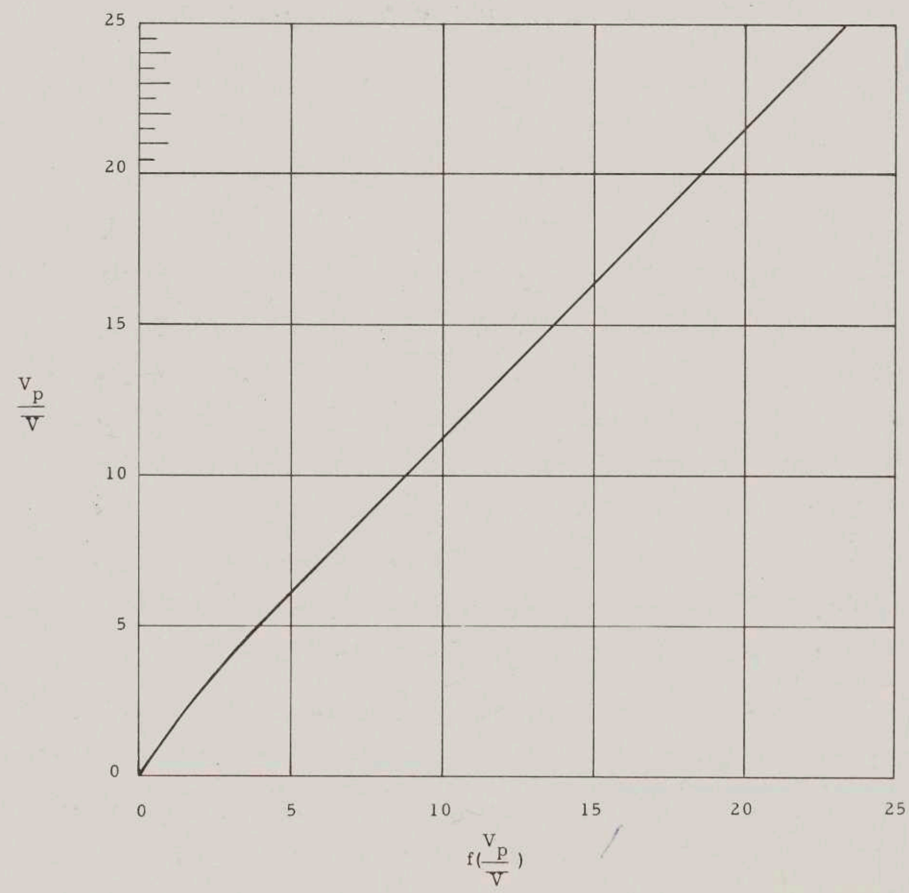
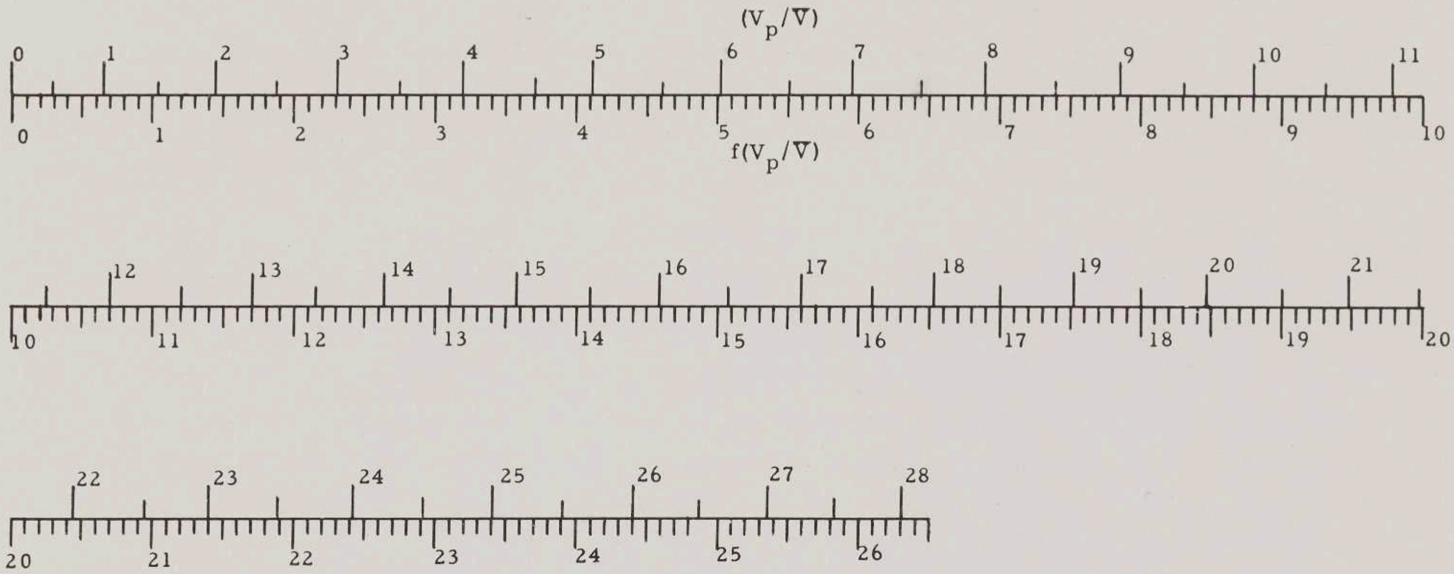


Fig. 8

Correlation function used in Eqs. 6:4 and 6:7.



$$f(V_p/\overline{V}) = \frac{V_p}{\overline{V}} - \frac{1}{2} \ln \left(\frac{V_p}{\overline{V}} + 1\right)$$

Fig. 9

Linear correlation chart of function used in Eqs. 6:4 and 6:7.

7. Numerical Examples

The first example to be discussed numerically may best be described by the data recorded here as Table 1

Table 1

Specifications Related to a Numerical Example

Cesium Temperature	T_{Cs}	500°K
Surface Temperature	T	2320°K
Volt Equivalent	$\overline{V}$	0.2 ev
Emitter work-function	ϕ_1	4.09 ev
Ionization Potential	V_i	3.89 ev
$(\phi_1 - V_i)/\overline{V}$	u	1.0

This table supplies all of the necessary information to use Eq. 6:4 for the calculation of the value of $f(V_p/\overline{V})$. The result of this calculation is a value of 8.76 and yields the result $(V_p/\overline{V})$ of 9.95 which in turn gives the value of $V_p = 2.0$ volts.

Equation 6:2 may be used to compute the value of n_o to be used in Eq. 3:1. The value of n_o for this example is 8×10^{16} ions/m^3 and with this value inserted into Eq. 3:2 the thickness of the sheath is 26×10^{-6} meters. Equation 3:7 may be used to calculate the surface field and from it we obtain $E = 3.6 \times 10^6$ volt/m.

With all of the conditions the same as those described by Table 1 except for the true work-function of the emitting surface, it is possible to compute by Eq. 6:7 approximately the lowest value of ϕ_1 for which sufficient ionization will still be available to neutralize the space charge. This value is $\phi_1 = 3.2$ ev.

8. Collision Phenomena

As electrons or ions traverse the space between the electrodes they will encounter the cesium atoms. Under low pressure conditions many electrons and ions will traverse the entire distance without collision, whereas others will experience such collisions. Experimental data on collision phenomena are usually presented by curves that show the "collision probability" P_c as a function of the electron energy. Numerically P_c^{-1} is the average distance, usually measured in centimeters, that the electron will travel through a gas at that concentration which gives a pressure of 1 mm of mercury at 0°C. This concentration of gas atoms is 3.536×10^{16} atoms/cm^3. Thus if P_c is 10 then the electron, on the average, will travel 1 mm between collisions in the gas at this pressure. More useful for the present purpose is the collision cross-section which is simply another way of expressing the required information by which the mean-free path of an electron can be computed applicable to a particular experimental situation. The relations in equation form are the following:

$$\sigma'_c = \frac{P_c}{3.536 \times 10^{16}} \quad \mathrm{cm}^2 \qquad |8:1$$

$$\sigma_c = \frac{P_c \times 10^2}{3.536 \times 10^{22}} \quad \mathrm{m}^2 \qquad |8:2$$

$$\lambda_c = \frac{1}{\sigma_c N} \quad \mathrm{m} \qquad |8:3$$

Note that the numerator in Eq. 8:2 is the "collision probability" expression that would be used in the mks system, with the denominator the number of atoms/m^3 under standard conditions specified above. In Eq. 8:3, λ_c represents the mean-free path of an electron in the gas which has an actual concentration of N. This quantity λ_c is very useful in that it permits the writing of expressions for the <u>true</u> probability that an electron will travel a certain specified distance x <u>without</u> having made a collision. This probability is expressed as:

$$\text{Travel probability} = e^{-\frac{x}{\lambda_c}} \qquad |8:4$$

The probability that a collision will take place in a travel distance dx at x is:

$$\text{True collision probability} = e^{-\frac{x}{\lambda_c}} \frac{dx}{\lambda_c} \qquad |8:5$$

A suitable equation by which the atom concentration in the space between the electrodes can be calculated is

$$\ln N = 76.86 - \ln T_{Cs} - \frac{8910}{T_{Cs}} - \frac{1}{2}\ln T_g \qquad 8{:}6$$

$$\log_{10} N = 33.38 - \log_{10} T_{Cs} - \frac{3870}{T_{Cs}} - \frac{1}{2}\log_{10} T_g \qquad 8{:}7$$

Equations 8:2 and 8:6 may be combined to permit a direct calculation of the mean-free path for any chosen value of P_c. Although the answer will be given in meters, the conventional centimeter units described above for P_c must be applied. The equations are

$$\ln \lambda_c = \frac{8910}{T_{Cs}} + \ln T_{Cs} + \frac{1}{2}\ln T_g - \ln P_c - 29.54 \qquad 8{:}8$$

$$\log_{10} \lambda_c = \frac{3870}{T_{Cs}} + \log_{10} T_{Cs} + \frac{1}{2}\log_{10} T_g - \log_{10} P_c - 12.83 \qquad 8{:}9$$

The only data available for the electron mean-free path in cesium gas seem to be those reported by Brode(5). In the very low range of electron energy these data seem to indicate a value of P_c of 1200. Between 1 electron volt and 3 it rises to 2000 and then between 3 and 5 it falls again to about 1000. For the present purpose it seems as though an average of about 1400 would be most suitable. If this value is used then Eq. 8:8 can be simplified[Δ] to be

$$\ln \lambda_c = \frac{8450}{T_{Cs}} + \frac{1}{2}\ln T_g - 29.61 \qquad 8{:}10$$

$$\log_{10} \lambda_c = \frac{3670}{T_{Cs}} + \frac{1}{2}\log_{10} T_g - 12.86 \qquad 8{:}11$$

The mean-free path calculated by these methods may be used to estimate the distance which an electron that leaves the hot surface will travel after its injection into the space before its direction of motion is randomized without appreciable change in the energy distribution. In case the injection energy exceeds about 0.5 volt, it will be shown in a later section that the plasma will be very highly ionized. Many electrons will be trapped and plasma oscillations will randomize the electron energy distribution without much loss in total energy if any.

In order to give Eq. 8:11 numerical significance we may compute the following results all applicable to a mean temperature in the gas of 1500°K. With a cesium temperature of 500°K the mean-free path is 118 microns. At 400°K it is 0.81 centimeters, and at 355°K it is 11.2 centimeters. The dividing line in cesium temperature therefore between the "high pressure" and the "low pressure" cesium diode is approximately 400°K. Certainly at 500°K with anticipated practical dimensions the

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[Δ] See chart for nomographic calculation of these relations. p40 mean-free path will probably be smaller than the dimensions, whereas at the lower cesium temperatures it will either be equal to or much longer than the inter-electrode spacing.

Before the properties of the plasma space in the diode are considered in still more detail, some attention must be given to the motion and behavior of the positive ions in this neighborhood. If the emitter work-function is so low that ionization is not copious enough to neutralize the electron space charge, then it is to be anticipated that ions will be injected into the space region in a manner not unlike that associated with the injection of electrons when a positive ion space-charge sheath exists at the emitter. Just as electrons can be trapped in a positive ion region, the ions can be trapped in a region normally occupied by an excess of electrons such as is illustrated by Fig. 1. The ions originate at the surface potential s_1 and they are injected into the space-charge region. Here they may encounter collisions with atoms and share their energy so as to fall into energy states between the motive line and the energy level at s_1. Since data are not available upon which we can base an accurate choice of the value of P_c for this particular collision process, a guess will have to be made on the basis of other experience and it seems reasonable to assume a value close to 1000. Because of this uncertainty in the choice, Eq. 8:11 will be used to calculate the mean-free path of an ion for this type of energy change since any re-adjustment in the value of the term which is 12.86 would be very arbitrary. Under the circumstances illustrated in all the figures other than Fig. 1, the kinetic energy associated with the ions will not be sufficiently different from that of the neighboring gas to make it necessary to consider the question of ion energy distribution. Collisions with the gas atoms will randomize the direction of motion without any appreciable change in the energy distribution.

Whenever electrons are injected into a region that is more or less free of space charge such as that illustrated by Fig. 3 a small localized deviation from complete space-charge neutrality is likely to result in very localized "plasma oscillations." We must visualize these plasma oscillations in terms of groups of electrons which very locally oscillate back and forth in the field of the ions which because of their very high mass are essentially stationary. If we put this description in other words we have very locally a kind of grouping of "short range" order of the electrons that partake in this oscillation. Whereas at distances within the plasma which are larger than the probable amplitude of the oscillation, there is no coherence. This concept of the presence of short range order permits one to write a set of equations which may be simplified by assuming that a small group of electrons are oscillating back and forth in a particular direction. The frequency of such oscillations has been shown to be related to the electron density by the equation:

$$f_p = \frac{1}{2\pi} \left(\frac{n_- q}{\epsilon_o m}\right)^{1/2} \qquad |8:12$$

Direct calculations can best be made as follows:

$$f_p = 8.979\ n_-^{1/2} \qquad |8{:}13$$

With electron densities of particular interest in the plasma diode, the frequency of these plasma oscillations is likely to be hundreds or even thousands of megacycles and the fact that they are so completely incoherent both with respect to phase and direction, means that they are very difficult to detect in most experiments.

This high frequency disturbance within a region that is nearly free of space charge because of the equality of the density of electrons and ions has been shown to exert a strong influence in the direction of randomizing any injected beam of electrons in such a manner as to destroy their monoenergetic character. Since the electron group as a whole will not actually lose energy rapidly in this process, a good approximation would be to take the average energy of a "Maxwell-Boltzmann" energy distribution to be equal to the average energy of injection. If we define $\overline{V}_-$ as the voltage equivalent of the temperature of the Maxwell-Boltzmann distribution among these electrons, we can relate it to the injection energy and the temperature of the emitter by the following:

$$\overline{V}_- = \frac{2}{3}\,(V_p + 2\overline{V}) \qquad |8{:}14$$

Expressed in temperature units, this equation may be written

$$T = 7850\ (V_p + 2\overline{V}) \qquad |8{:}15$$

In the example shown in the previous section, the value of V_p was 2.0 volts. This would indicate a possible maximum temperature of the electrons under these circumstances of 17,000°K. It will be shown in the next section that an electron distribution characterized by this high temperature is one in which there are a sufficient number of high energy electrons to contribute substantially to the ionization process and thus maintain an adequate ion density to neutralize space charge in that part of the plasma space beyond the emission sheath S_1.

9. Ionization by a Maxwell-Boltzmann Distribution of Electrons

As a consequence of the above-mentioned redistribution in the energy of the electrons, many are able to acquire sufficient energy to ionize cesium. The energy required is 3.89 volts. There are no data available by which the ionization probability can be deduced with accuracy. Electrons with less energy than the ionization potential do not ionize in a single step, but those above the ionization potential are known to ionize with an increasing efficiency as the electron energy increases. This statement applies only within the first three or four volts of the onset of ionization. The increase in efficiency is certainly not precisely linear with the excess electron energy and yet when thought of in terms of a power series expansion of the more complex curve, the linear term is the most important. Expressed in equation form:

$$P_i = \alpha\,(V_- - V_i) \qquad 9:1$$

In this equation P_i is the ionization efficiency expressed as was the case for the collision problem by the statement that P_i^{-1} is the average distance travelled by an electron of energy V_- before an ionizing collision takes place. The constant α is a direct measure of the rate of increase in ionization efficiency as V_- exceeds V_i, the ionization potential. Because of the total lack of experimental data on this subject, I choose the value of α as 35 when P_i is expressed in the units previously described. In the mks system we would write an α of 35×10^2. This assumption yields the following formula for the ionization cross-section σ_i.

$$\sigma_i = 10^{-19}\,(V_- - V_i) \qquad m^2 \qquad |9:2$$

The next problem to be solved is the determination of the rate of ion production when we assume that the electron distribution can be characterized as a Maxwell-Boltzmann distribution of known temperature T_- or voltage equivalent of this temperature $\overline{V}_-$. From the statistics of a Maxwell-Boltzmann distribution it is possible to show that the number of electrons in a unit volume, which have kinetic energy expressed in electron volts between V and V + dV, is given by the following formula

$$dn_- = \frac{2}{\sqrt{\pi}}\, n_- \left(\frac{V_-}{\overline{V}_-}\right)^{1/2} e^{-\frac{V_-}{\overline{V}_-}}\, \frac{dV_-}{\overline{V}_-} \qquad |9:3$$

The number of ions that these electrons can produce per unit time will be proportional to their speed and inversely proportional to the mean-free path expected for an ionizing collision. The ionization cross-section and the gas concentration are related to the mean-free path by Eq. 8:3. With these facts in mind, the following equation can be written to give the differential ionization rate expressed as the number per unit volume created in unit time

$$d\nu_+ = \frac{2}{\sqrt{\pi}} \left(\frac{2q}{m}\right)^{1/2} \frac{\alpha\,\overline{V}_-^{\,3/2}}{3.5 \times 10^{20}} \cdot n\, n_- \frac{V_-\,(V_- - V_i)}{\overline{V}_-^{\,2}}\, e^{-\frac{V_-}{\overline{V}_-}}\, \frac{dV_-}{\overline{V}_-} \qquad |9:4$$

In this equation all of the symbols have been defined previously except for n which is now defined as the concentration of neutral atoms in the plasma space. It is to be anticipated that a very substantial fraction of all of the atoms in the plasma space will be ionized and therefore the number of atoms available for ionization will be only a fraction of the total number existing there. This fractional relation can be expressed as follows:

$$n = (1 - f)\,N \qquad |9:5$$

Here N represents the concentration of atoms including the ions in the plasma space. A fraction f of these atoms will be ionized and (1 - f) will be the fraction of the total number of atoms which are neutral and therefore available for ionization. It is this quantity that belongs in Eq. 9:4 and must be used to compute the rate of ionization. Equation 9:4 can be prepared for integration by making the substitution

$$y = \frac{V_- - V_i}{\overline{V}_-} \qquad |9:6$$

The expression that gives the rate of ionization expressed as new ions produced per unit volume in a period of one second is

$$\nu_+ = \frac{2}{\sqrt{\pi}} \left(\frac{2q}{m}\right)^{1/2} \frac{\alpha \overline{V}_-^{\,3/2}}{3.5 \times 10^{20}} \cdot (1 - f)\, N\, n_-\, e^{-\frac{V_i}{\overline{V}_-}} \int_{y=0}^{y=\infty} y \left(y + \frac{V_i}{\overline{V}}\right) e^{-y}\, dy \qquad |9:7$$

This equation is readily integrated and the constants may be assembled together to give:

$$\nu_+ = 1.9 \times 10^{-15}\, \alpha (1 - f)\, N\, n_-\, \overline{V}_-^{3/2}\; e^{-\frac{V_i}{\overline{V}_-}} \left(2 + \frac{V_i}{\overline{V}_-}\right) \qquad |9:8$$

There are three principle means by which ions are removed from the plasma space. These are: (1) the loss of ions to the collector electrode, (2) the loss of ions to the emitter surface, and (3) the loss of ions due to space recombination. Since the electron emitter is a generator of ions as well as an absorber, the loss of ions from the plasma at that surface will be different from that loss at the collector. In case the geometrical configuration is not that of concentric cylinders, other ion loss mechanisms may have to be taken into consideration. Since the geometry involved in those diode designs has many possibilities it is impossible to generalize concerning these ion losses. Detailed discussion of them will be omitted therefore.

Before additional details concerning the properties of the plasma are worked out, some general remarks concerning it are in order. In the high pressure diode, an approximate figure for the sheath thickness at either electrode is 20 to 100 microns. If we take the interelectrode spacing to be of the order of 1 or 2 thousand microns, it is evident that only a small fraction of the interelectrode space is occupied by the sheaths and that the rest of the space is largely field-free. This means that the total number of electrons per unit volume at all points in the space is very closely (within 1 or 2 per cent) equal to the ion density. In the central part of the plasma space taken to be 2 or 3 mean-free paths away from the emitter or collector surfaces, the principal ion loss mechanism is that of recombination. There the ion production rate becomes equal to the ion loss rate which is therefore smaller than that close to the electrodes. Since the electrodes are in effect sources of neutral atoms created by the absorption of ions, the value of (1 - f) which is the fraction of the total atoms present that are neutral will be greater there by a considerable factor even though the value of f in the plasma space may still be very close to 1.

10. Recombination in the Plasma Space

The ion loss mechanisms that ultimately determine the concentration of ions and electrons in the plasma space cannot be generalized to take into account all possible diode configurations. The three most important mechanisms are: (1) the loss of ions to the electron collector, (2) the loss of ions to the electron emitter, and (3) the loss of ions through the process of recombination. If the linear dimensions required to bound the plasma space region are large compared with the mean-free path, then recombination can be the principle ion loss mechanism. The fractional ionization in this space can be computed subject to necessary assumptions concerning the applicability of the meager experimental information available. This fraction will therefore be interpreted as an upper bound since other loss mechanisms acting near the plasma boundaries will always reduce the fractional ionization present there. The specific procedure to be used here therefore is to find an expression for the recombination rate and set it equal to the ionization rate given by Eq. 9:8.

The recombination process assumed in this analysis is that associated with radiative recombination in which a slow electron comes into the immediate neighborhood of a positive cesium ion to recombine with the ion while a light quantum is emitted to dissipate the excess energy of the system. The experimental data applicable to this recombination mechanism for a complex ion such as cesium are extremely meager. According to Mohler(6) theoretical analyses underestimate the actual recombination probability by an order of magnitude. In describing his studies on mercury, Mohler states: "This work gives no evidence of any large change in the recombination coefficient as the mean electron temperature changes from 2800°K to 1800°K. . . "

It will be shown here that the recombination coefficient is independent of the electron temperature if the probability of recombination is inversely proportional to the velocity of the electron as it approaches the ion, with which it recombines by the radiative process. A second analysis of the problem will be made on the assumption that the probability of recombination is inversely proportional to the kinetic energy of the approaching electron. Since the difference in the overall results is not great and the neglect of other ion loss mechanisms down grade the fractional ionization, I believe that either of these probability functions may be chosen without the introduction of significant error.

The inverse velocity assumption permits the writing of the following formula for the recombination cross-section σ_r

$$\sigma_r = \frac{R}{v} = \frac{R}{\left(2 \frac{q}{m} V_-\right)^{1/2}} \qquad 10:1$$

p 24

eq 10:14 should read

$$\nu_\rho = \frac{2\rho}{\sqrt{\pi}}\left(\frac{2}{mq}\right)^{\frac{1}{2}} \frac{1}{\sqrt{\bar{V}_-}}\, n_+ n_- \int_0^\infty e^{-\frac{V_-}{\bar{V}_-}}\, \frac{dV_-}{\bar{V}}$$

eq 10:15 should read

$$\nu_\rho = \left[\frac{2\rho}{\sqrt{\pi}}\left(\frac{2}{mq}\right)^{1/2} \frac{1}{\sqrt{\bar{V}_-}}\right] n_+ n_-$$

p 24 line 13

wrong	right
$\rho = 9 \times 10^{-12}$	$\rho = 2.56\ 10^{-41}$ M^2-joule

eq 10:16 should read

$$\nu_\rho = \frac{1.07\ 10^{\boxed{-6}}}{(\bar{V}_-)^{1/2}}\, n_+ n_-$$

eq 10:17 should read

$$\frac{f_E}{1-f_E} = \left[1.96\ 10^4\ \bar{V}_-^{\frac{3}{2}}\ e^{-\frac{3.89}{\bar{V}_-}}\left(2 + \frac{3.89}{\bar{V}_-}\right)\right]_{\sqrt{10\bar{V}_-}}$$

eq 10:18 should read

$$\left[\frac{f_E}{1-f_E}\right]_{\text{energy}} = \left[\frac{f}{1-f}\right]_{\text{velocity}}^{\sqrt{10\bar{V}_-}}$$

In this equation R is a proportionality constant, v the velocity of the electron which is related to the V_- by the relation shown. The symbol V_- is the corresponding electron energy equivalent for the velocity v. In a Maxwellian distribution characterized by its voltage equivalent $\overline{V}_-$, the number of electrons which have energy between V_- and $V_- + dV_-$ is given by:

$$dn_- = \frac{2}{\sqrt{\pi}} n_- \left(\frac{V_-}{\overline{V}_-}\right)^{1/2} e^{-\frac{V_-}{\overline{V}_-}} \frac{dV_-}{\overline{V}_-} \qquad |10{:}2$$

These two equations may be combined to give the rate of recombination for these electrons and the negative for this expression is the time rate of loss of either ions or electrons for this electron energy group when the concentration of ions is n_+. The equation is:

$$d\nu_r = \frac{2\, n_+ n_-\, R}{\sqrt{\pi}} \left(\frac{V_-}{\overline{V}_-}\right)^{1/2} e^{-\frac{V_-}{\overline{V}}} \frac{dV_-}{\overline{V}_-} \qquad |10{:}3$$

The arithmetic procedure in the formulation of Eq. 10:3 involves the following two relations that are practically self-evident:

$$\lambda_r = \frac{1}{\sigma_r n_+} \qquad |10{:}4$$

$$d\nu_r = \frac{v\, dn}{\lambda_r} \qquad |10{:}5$$

Equation 10:3 can be put into the form for direct integration by substitution of the dimensionless parameter

$$r = \frac{V_-}{\overline{V}} \qquad |10{:}6$$

With the integration limits put in, Eq. 10:3 takes on the form:

$$\nu_r = \frac{2}{\sqrt{\pi}} n_+ n_- R \int_0^\infty r^{1/2} e^{-r} dr \qquad |10{:}7$$

The value of the definite integral is known to be $(\sqrt{\pi}/2)$ and we recognize that the proportionality constant R is the conventional recombination coefficient expressed in the following form

$$\frac{dn_-}{dt} = -\nu_r = -R\, n_+ n_- \qquad |10{:}8$$

" This explanation does not hold on "attachment" picture without additional statement that attachment goes as $\frac{1}{v}$.

To summarize this result, we see that subject to the assumption that the probability of recombination is inversely proportional to the electron velocity and that the electron energy distribution is characterized by a Maxwellian distribution, the recombination coefficient is independent of the electron temperature. This result is consistent with Mohler's experience and serves to make his determination of the recombination coefficient useful for the present purposes. His experimentally determined value of R for cesium is 3.4×10^{-16} m^3/sec when expressed in units suitable for use in the equations given here. Mohler's determination applied to the temperature of 1200°K. It is unfortunate that no additional data exists that will serve as an experimental determination of this coefficient for a much higher temperature electron distribution.

Inspection of Eqs. 9:8 and 10:8 leads directly to the conclusion that at a specific fractional ionization f, the recombination rate will be precisely equal to the ion generation rate. The ion concentration required in Eq. 10:8 is (fN) since N is the total atom concentration and f the fractional ionization. The steady state value of f is obtained from a solution of the following equation.

$$\frac{f}{1-f} = 1.9 \times 10^{-15} \frac{a}{R} \; \overline{V}_{-}^{3/2} \; e^{-\frac{V_i}{\overline{V}_-}} \left(2 + \frac{V_i}{\overline{V}_-}\right) \qquad |10:9$$

This general equation when applied to cesium converts to one specifically applicable by the introduction of the values of a, R, and V_i mentioned previously and specified here as:

$$a = 35 \times 10^{2}; \; R = 3.4 \times 10^{-16}; \quad V_i = 3.89 \text{ volts} \qquad |10:10$$

The equation prepared for direct numerical calculation is given as:

$$\frac{f}{1-f} = 1.96 \times 10^{4} \; \overline{V}_{-}^{3/2} \; e^{-\frac{3.89}{\overline{V}_-}} \left(2 + \frac{3.89}{\overline{V}_-}\right) \qquad |10:11$$

If the voltage equivalent of the electron temperature is 0.36 ev, the fractional ionization calculated by Eq. 10:11 is 0.52. Since the exponential factor has such a dominating influence, the lowering of the electron temperature by only 30 per cent reduces the fractional ionization to less than 1 per cent, whereas the increase in electron temperature 25 per cent raises the fractional ionization to over 90 per cent.

In view of the uncertainty as to whether or not the recombination coefficient should be inversely proportional to the velocity of the electron or inversely proportional to its energy, an analysis should be made based on this second assumption in order to show quantitatively that the final conclusions drawn are only slightly dependent on this choice except in the electron temperature range between 3000 and 4000°K. Since the analysis follows closely along the same line of reasoning as is given by Eqs. 10:1 to 10:11, less detail will be given here. In place of Eqs. 10:1 and 10:4 we write:

$$\sigma_\rho = \frac{\rho}{q\, V_-} \tag{10:12}$$

$$\lambda_\rho = \frac{q\, V_-}{\rho\, n_+} \tag{10:13}$$

The recombination rate is expressed by ?

$$\nu_\rho = \frac{2\rho}{\sqrt{\pi}} \left(\frac{2m}{q}\right)^{1/2} \frac{n_+ n_-}{\overline{V}_-} \int_0^\infty e^{-\frac{V_-}{\overline{V}_-}} \frac{dV_-}{\overline{V}} \tag{10:14}$$

In Eq. 10:15 which is obtained upon integration of Eq. 10:14, the quantity in the square brackets is the recombination coefficient.

$$\nu_\rho = \left[\frac{2\rho}{\sqrt{\pi}} \left(\frac{2m}{q}\right)^{1/2} \frac{1}{\overline{V}_-}\right] n_+ n_- \tag{10:15}$$

The only available data for this coefficient are given by Mohler(6) whose experiments indicate the coefficient to be 3.4×10^{-16} for an electron temperature of $\overline{V}_- = 0.1$. The value of ρ obtained by setting the accommodation coefficient of Eq. 10:15 equal to Mohler's value is $\rho = 9 \times 10^{-12}$. Thus the final equation for the recombination rate is given by:

$$\nu_\rho = \frac{3.4 \times 10^{-17}}{\overline{V}_-} n_+ n_- \tag{10:16}$$

After this rate is set equal to the generation rate, the solution is the following:

$$\frac{f_E}{1 - f_E} = \left[1.95 \times 10^4\, \overline{V}_-^{3/2}\, e^{-\frac{3.89}{\overline{V}_-}} \left(2 + \frac{3.89}{\overline{V}_-}\right) \right] 10\, \overline{V}_- \tag{10:17}$$

This equation has been written in this form to make it quite evident that the quantity inside of the square brackets is precisely the same set of factors as we have in Eq. 10:11. This justifies the writing of the equation in the form shown as Eq. 10:18:

$$\left[\frac{f_E}{1 - f_E}\right]_{\text{Energy}} = \left[\frac{f}{1 - f}\right]_{\text{Velocity}} 10\, \overline{V}_- \tag{10:18}$$

A table of values of the functions represented in Eqs. 10:11 and 10:17 has been computed and the corresponding values of f and f_E are given in Appendix 3. It is these numbers that serve to establish the location of the two curves in Fig. 10. On the basis of these curves and the theory given above, the statement may be made that if the electron temperature is less than 3000^oK, the fractional ionization will be extremely low, whereas if the electron temperature is above 5000^oK, the fractional ionization in the plasma can very well approach unity and the plasma space may be considered to be fully ionized.

11. Collector Sheath

Figures 3 and 4 are representative of extremes in motive diagrams of most interest in connection with the plasma diode. The open circuit condition is described qualitatively by Fig. 4. For applied voltages less than V_{oc} motive diagram is much the same and the current-voltage characteristic expected is generally similar to that of Fig. 11. Here the observed circuit current is plotted as a function of the potential applied to the collector relative to that of the emitter. This is plotted as a negative voltage since the Fermi level of the collector is negative with respect to the Fermi level of the emitter by the amount shown as V. For applied voltages more negative than V_{oc}, the current observed is also negative in that it may increase indefinitely as the voltage is made more and more negative. For the proper interpretation of the curve for applied voltages between V_{oc} and 0, it is necessary to obtain detailed information concerning the current flow as a function of the voltage with the greater values of V. The observed currents are likely to come from four important current components. Of these, probably the most important is the delivery of positive ions from the plasma across the collector sheath S_2. The second component is the leakage current from this electrode and any other conductors connected to it which may depend on the applied potential. A third component is the thermionic emission which may come from the collector if it is not held at a sufficiently low temperature to make this component negligible. A fourth component is the photoelectric yield from the collector which results from the absorption of light from the emitter and radiation excited in the plasma and absorbed at the collector surface. The electron current from the plasma should cut off very rapidly and applied voltages greater than V_{oc}.

The observed net current is zero at V_{oc} because the electron current able to surmount the barrier at s_2 is exactly equal in magnitude and opposite in sign to the sum of the above mentioned components that flow at that same applied voltage. In order to determine this electron current it is therefore necessary to understand the characteristic curve at more negative voltages so that an accurate estimate of the reverse current existing at V_{oc} can be obtained and set equal to the electron current. As the magnitude of the applied potential is made less than V_{oc}, the rise in current is dominated by the electron current but again it is important to estimate the reverse current so that the electron current itself can be analyzed. A typical curve of this type is shown in Fig. 12. The dotted line there joins the observed points together by

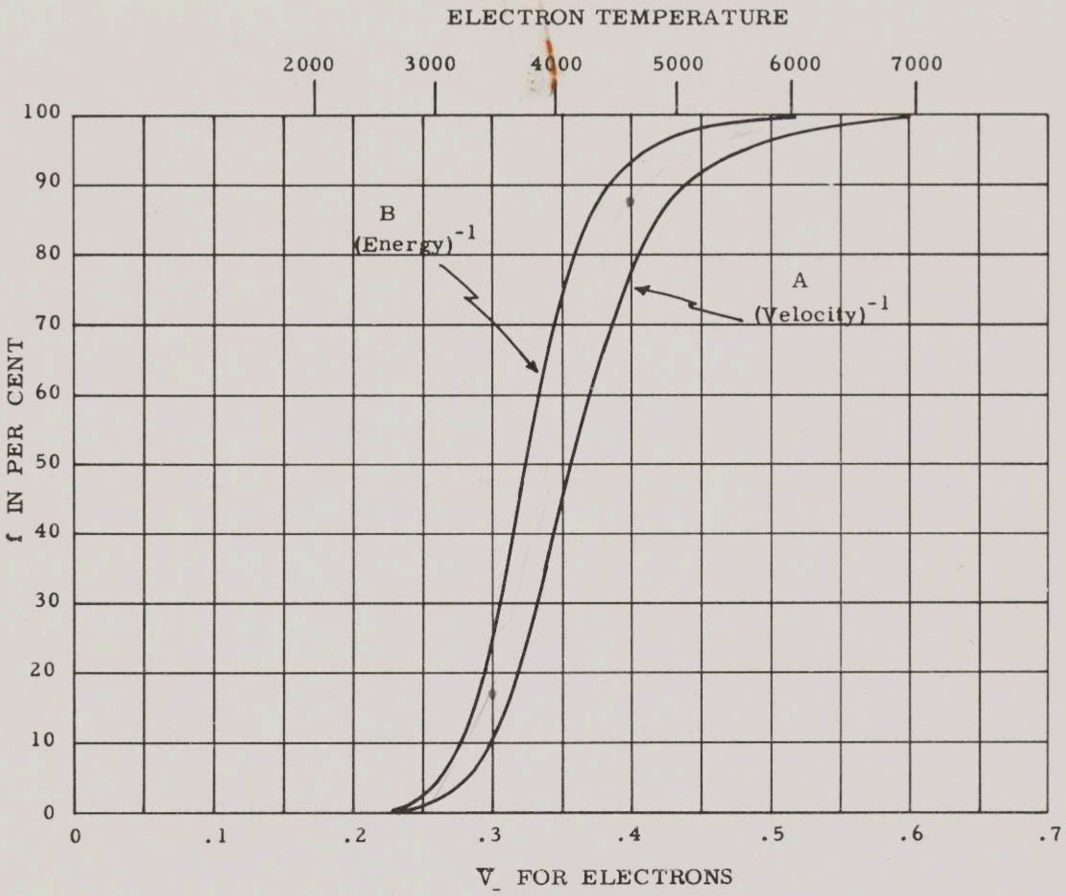

Fig. 10

Fractional ionization in plasma space, computed by balance of ionization rate vs. recombination rate. Curve A for recombination inversely proportional to electron velocity. Curve B recombination probability proportional to inverse electron energy.

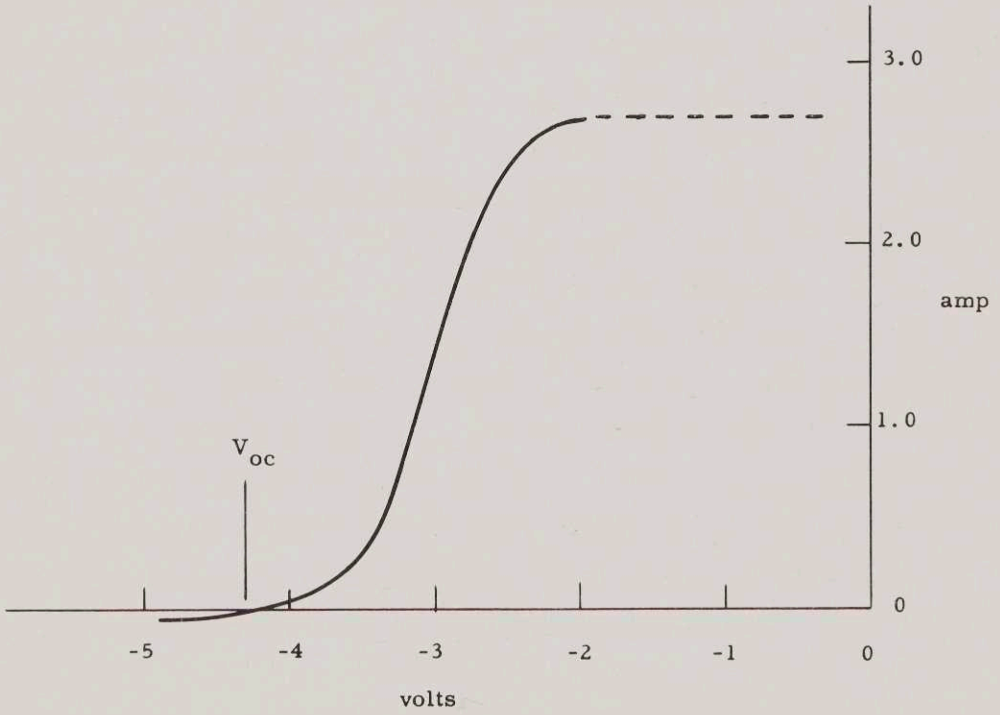

Fig. 11

Normal current voltage curve for a plasma diode. Solid line over range of steady state values. Dotted line over range of reported oscillations.

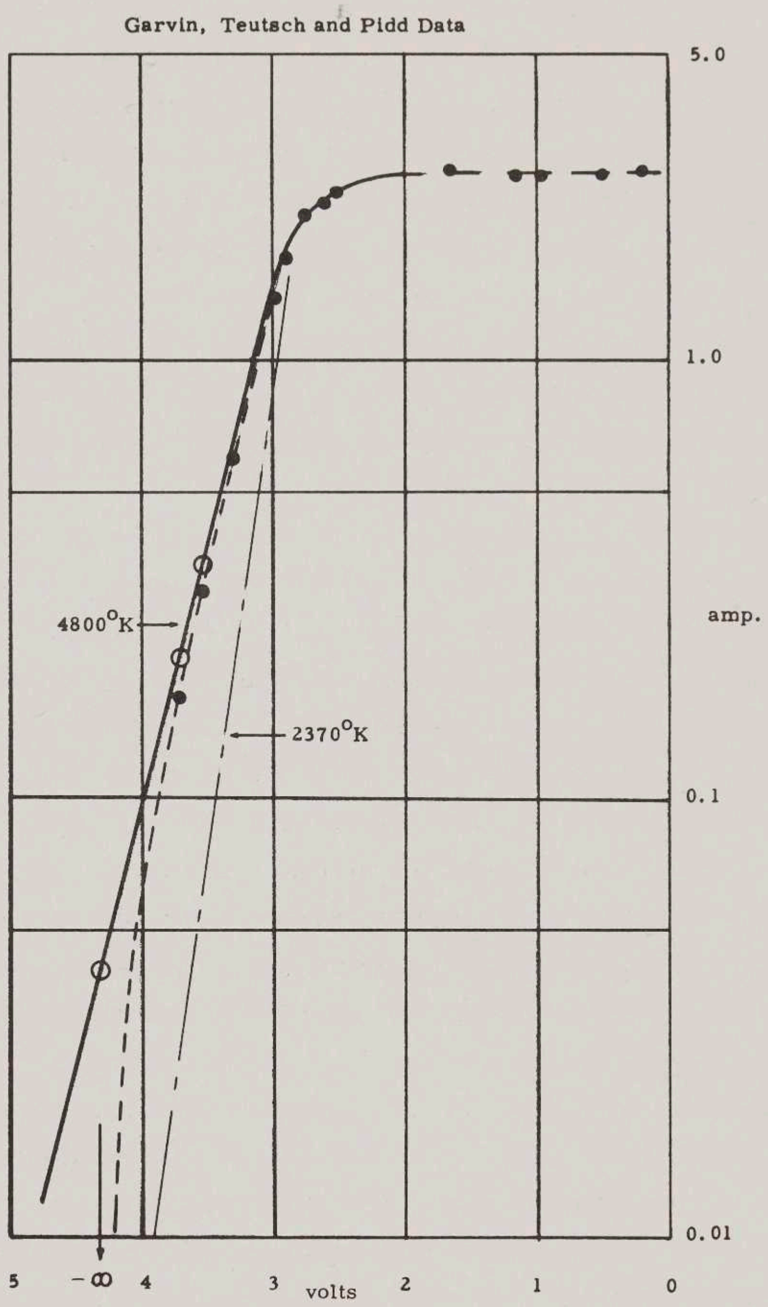

Fig. 12

Log plot to determine electron temperature. Data furnished by Dr. W. B. Teutsch of General Atomic.

a smooth line, whereas the solid line joins the corrected observed points to give on this plot a relatively good straight line which in turn can be interpreted as the measure of the electron energy distribution of those electrons which arrive at the collector surface s_2.

As the applied potential is made more and more positive, that is less and less negative, it is generally found that the rate of change of current with voltage changes rapidly and there is a strong tendency for saturation. The sharp change-over which is seen to occur between 2.7 volts applied and 2.3 volts is to be identified with a surface potential s_2 such that electrons are permitted to leave the plasma space at exactly the same rate at which they are introduced into the plasma space. Thus this saturation current is dominated by the thermionic emission current from the emitter. The observed current is generally less than the true electron current that arrives at the surface because electron emission, photoelectric or thermionic, from the collector and ion arrival at the collector give "negative" current which subtracts from the true electron current entering the collector over the barrier at s_2. These reverse currents are going to be dependent on the cesium pressure, the collector temperature, the collector potential and the collector material. All of these factors influence the collector work-function which establishes the potential location of the surface s_2 with respect to the Fermi level in the collector.

It is essential to the maintenance of the high saturation current to the collector that electrons are not removed from the plasma region at such a high rate that it stops the process of plasma maintenance, which in turn would sharply reduce the current which can be conducted from an emitter to a collector. A number of observers(7) have studied the transition region that occurs at the beginning of the saturation range by measuring the IR drop over a series resistance, which in turn is varied systematically in order to explore the range of constant current and low voltage. These observers report that strong oscillations develop which are interpreted as a repetitive cut-off of the net conduction current at a frequency related to the diode spacing.

On the basis of this discussion concerning the need to maintain plasma conditions, we would anticipate that the true current-voltage characteristic of a plasma diode would follow qualitatively the line shown in Fig. 13. Three typical "load lines" are shown by the dashed lines: OL_1, OL_2, OL_3. It is the intersection of this load line with the actual curve which establishes both the voltage and the current flow at points 1, 2, and 3. The intersection points 1 and 2 are stable in their operation. The plasma is well developed and the drain of electrons from the plasma region at the collector is amply supplied by the emitter. As saturation sets in the supply of electrons is limited to the emission capability of the emitter and yet as the surface potential of the collector is made less and less negative, more electrons are swept out of the plasma region and if there are an insufficient number of high energy electrons there, then the ionization in the plasma will collapse and the current will cut off. This is shown by the dotted line in Fig. 11 and indicates at the critical point 3 that an oscillation can set in.

Although a number of observers have found oscillations, none of them has given sufficient data to establish the basic physical phenomena involved. Therefore the present opinion concerning the mechanism is offered without adequate experimental support. Studies now underway will establish the extent to which this mechanism seems to be the important one. If we consider an instant of time at which the current-voltage point is illustrated at point 3 of Fig. 13, it is thought that the motive diagram will be represented qualitatively by Fig. 14a. Note that in this figure, the surface potential at the collector is slightly more positive than the surface potential at the emitter.

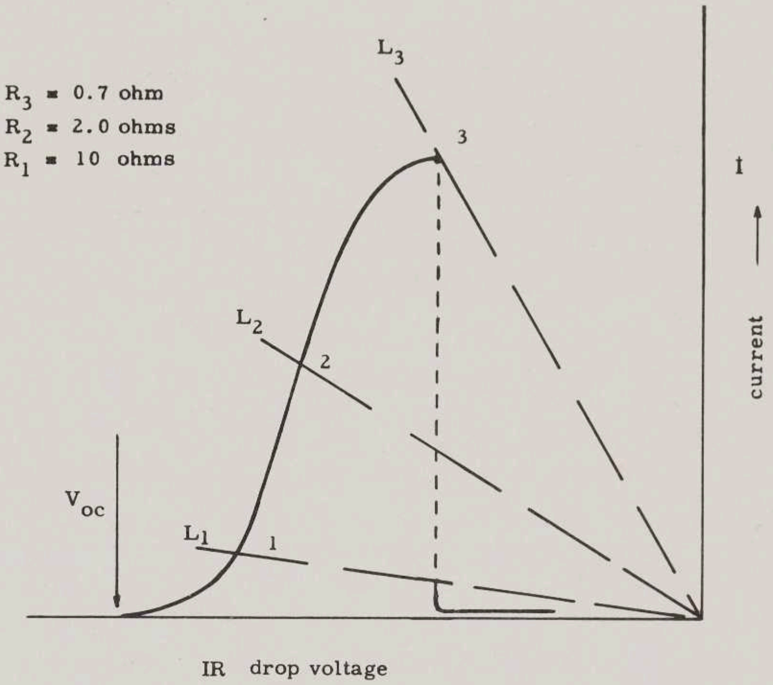

Fig. 13

Current voltage characteristic as observed by measuring IR drop over resistances of various values. Lines L_1, L_2, and L3 correspond to decreasing resistance. Stable points at 1 and 2. Critical point at 3.

Under this condition, electrons are injected into the plasma region here shown as "plasma space." Provided the electron energy distribution can be characterized as having a temperature of approximately 5000^{o}K, new ionization will compensate for electron-ion recombination. The low surface potential at s_2 permits some of the high energy electrons in the immediate neighborhood of the collector sheath to be absorbed into the collector and thus reduce the ionization capability of the electron distribution there. If this loss of high energy electrons is maintained for a time of the order of a microsecond, the ions will be swept out at a rate greater than their production rate, and the space-charge distribution will change drastically. The first step in this change is illustrated by Fig. 14b indicating that an electron space charge is beginning to develop at the surface of the collector. Up to this point, the current flow over the surface barrier at s_2 can continue at practically a constant value since the surface barrier will still control the flow at the energy level s_2. A fraction of a microsecond later this negative space charge will start to sweep toward the emitter and get still deeper. This in turn cuts off the current and since the Fermi level of the collector was established to be at voltage point 3 of Fig. 13 by the IR drop in the load resistance, the next step is illustrated by Fig. 14c. Because of the development of this space-charge minimum at ϕ_m negative with respect to the Fermi level of the emitter, the current available to the collector is drastically reduced. This minimum serves as an effective ion trap since ions generated in the greatly reduced plasma space and those generated at the hot surface can pour into the minimum and lose kinetic energy by collision with neutral atoms to drop into the trap and initiate the

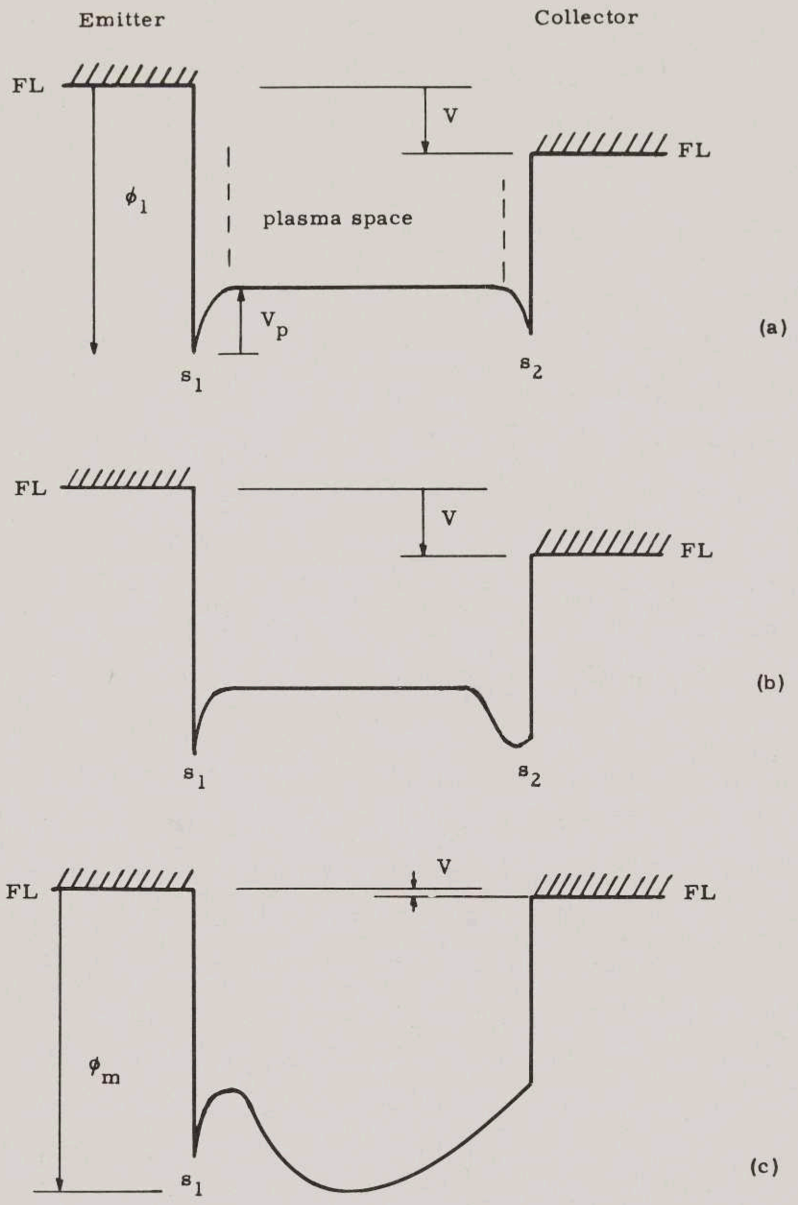

Fig. 14

(a) Motive diagram for plasma diode at critical condition.

(b) Motive diagram the instant insufficient ionization is produced at the collector.

(c) Motive diagram after electron space charge has set up a limiting space-charge minimum. Sequence follows back from 14c to 14a at the end of a period of oscillation.

first steps in a sequence that returns the plasma space to one more like Fig. 14a. Since the plasma development and trapping depend on the actual flow of ions across the space, it is to be expected that the repetition frequency will be approximately inversely proportional to the diode spacing when other conditions are maintained more or less constant. Dr. R. J. Zollweg(8) has reported that the period of oscillation increases approximately 1 microsecond for each increase in diode spacing of 1 mm. This suggests a sweep speed of 10^3 m/sec. If the ions are injected into the space-charge minimum with an average energy corresponding to 0.7 ev, their speed will equal that of 10^3 m/sec.

In general maximum dc power conversion takes place when the motive diagram similar to Fig. 14a exists except that the surface potential of the collector is equal to or very slightly negative with respect to the surface potential of the emitter. This can be a stable situation.

12. Collector Sheath Potential and Dimensions

It seems unlikely that a simple method will become available for an exact solution of the potential distribution and charge distribution in the collector sheath that will be adaptable to the great variety of boundary conditions likely to be encountered. This section will therefore deal with the general problem and offer equations by which approximate results can be obtained. The discussion will apply to that range in the current-voltage characteristic and to cesium pressures for which a stable mode of operation is possible. The analysis therefore applies to the range in voltage between the open circuit voltage and that associated with maximum power. Maximum efficiency is likely to be close to the condition of maximum power and will be within the voltage range included here.

Typical of the motive function applicable is the open circuit condition of Fig. 4. At the boundary between the sheath S_2 and the plasma space, equal concentrations of electrons and ions are to be found. The surface charge on the collector is strongly negative as is indicated by the potential function. In order for zero field to exist in the plasma, an excess of ions must be in transit between the sheath boundary and the collector surface to exactly balance the negative surface charge on the collector. Thus the electron density in the sheath falls off rapidly whereas the positive ion density falls only slightly because of the acceleration of these ions. The number of ions that cross any imaginary boundary within the sheath is constant and is equal to the arrival rate of ions at the collector. The easiest approximation to use to obtain an estimate of the sheath thickness and its configuration in potential is that of Nottingham's(9) formulation of the Langmuir solution to the space-charge problem between the space-charge minimum and the collector. To use this solution is the equivalent of stating that the exponential decrease in electron density reduces the contribution of the electrons to the space charge sufficiently so that Langmuir's theory for a single stream of charges is adequate. It is on this basis that we visualize the sheath boundary is practically independent of the voltage applied to the collector. Thus the ion current across to the collector will remain practically constant and be approximately equal to the random current of ions close to the boundary surface. This random current would be expressed as follows:

$$I_+ = q\, f_s\, N \left(\frac{kT_g}{2\pi M}\right)^{1/2} \qquad |12:1$$

$$I_+ = 5.05 \times 10^{-19}\, f_s\, N\, T_g^{1/2} \qquad |12:2$$

In this equation f_s is the fractional ionization in the neighborhood of the sheath boundary. In a highly ionized plasma f_s approaches 1, but close to the boundary it might be better to take this fraction close to 0.5 since practically every ion that goes to the collector will return as an atom at this boundary. It will not have had an opportunity to become re-ionized until the atom stream has penetrated appreciably away from the boundary. The total atom concentration is N and the current is expressed in amp/m^2. The calculation to follow will show that the maintenance of good plasma conditions in the immediate neighborhood of the collector sheath depends on the presence of a Maxwellian electron distribution characterized by a temperature of at least 4300°K. With temperatures as high as 5000°K the rate of production of ions within the plasma, one mean-free path away from the sheath boundary, is more than adequate to supply the ion current that flows out of this small volume of the plasma across the sheath to become neutralized at the collector surface. The following equation expresses this balance between the rate of loss of ions across the sheath and the rate of production of ions within a mean-free path.

$$N f_s \left(\frac{kT_g}{2\pi M}\right)^{1/2} = 1.9 \times 10^{-15}\, \alpha(1 - f_s)\, N^2 f_s\, \overline{V}_-^{3/2}\, e^{-\frac{V_i}{\overline{V}_-}} \left(2 + \frac{V_i}{\overline{V}_-}\right) \lambda_c \qquad |12:3$$

With the mean-free path expressed as in Eq. 12:4 and the constants suitable for cesium given just below, it is possible to solve Eq. 12:3 for the fractional ionization at the sheath-plasma boundary near the surface of the collector

$$\lambda_c = \frac{1}{N\,\sigma_c} \qquad |12:4$$

Use $\alpha = 35 \times 10^2$ and $\sigma_c = 4 \times 10^{-18}$ m^2

$$f_s = 1 - \frac{1.9 \times 10^{-6} \times T_g^{1/2}\, e^{\frac{V_i}{\overline{V}_-}}}{\overline{V}_-^{3/2} \left(2 + \frac{V_i}{\overline{V}_-}\right)} \qquad |12:5$$

If it is assumed that the gas temperature T_g is 1500°K then the introduction of a value of $\overline{V}_-$ of 0.37 gives the result that if more than 3 per cent of the gas atoms are ionized, then the flow to the collector will exceed the generation rate within a mean-free path. In contrast, the use of $\overline{V}_- = 0.5$ results in a solution which indicates that 94 per cent of the atoms could be ionized and still the ion generation rate would equal the loss of ions across the sheath. The conclusion that one may draw is that if the conditions at the emitter are such that the plasma potential is 0.5 volt or more positive with respect to the surface potential of the emitter, then under the condition that the surface potential at the collector is equal to or more negative than that of the emitter, the ion production in the plasma will be adequate for the maintenance of a stable discharge.

13. Some Considerations Related to Conversion Efficiency

In addition to the heat lost from the emitter by radiation and direct thermal conduction of heat, consideration must be given to that taken away from the emitter by the electrons and the ions. The volt equivalent of the energy for each electron emitted will be $\phi_1 + 2\overline{V}$, if the cesium ion density happens to be precisely that necessary to neutralize the space charge of the electrons. Under this condition the electron current delivered to the collector would be the "zero-field saturation" emission from the emitter and the electron energy distribution at the collector would be characterized by the temperature of the emitter. As the cesium temperature increases, the emitter sheath develops and electrons are injected. The minimum sheath potential which will result in the development of a well-ionized plasma is probably greater than 0.4 volt. Depending on geometrical configurations, this could very well take place at a pressure such that the mean-free path was long compared with the dimensions of the diode. Under this condition, the total energy delivered to the collector would in all probability be a minimum. An increase in cesium pressure beyond this point would do very little to increase the electron emission. Such an increase in pressure might have a favorable influence on the work-function of the collector depending on the temperature at which it is maintained. In this respect cesium pressure and the collector temperature play an important secondary role in that efficiency is always improved by the reduction of the collector work-function. Any unnecessary increase in cesium pressure is likely to lower efficiency since every cesium ion that neutralizes at the collector delivers heat to it approximately equal to the plasma potential relative to the collector surface plus the ionization potential of the cesium. Although the actual ion current delivered at the collector is generally very much smaller than the electron current, this delivery of so much energy with each ion represents a heat-transfer mechanism which should not be overlooked.

14. Concluding Remarks

All of the relations of importance presented in this report have been tested to some extent by comparison with experiments on a quantitative basis. This does not mean that all experiments have been analyzed in terms of these relations. It is my hope to carry through a program of systematic application of this general theory of the plasma diode to all existing data and to use the theory to interpret results of new experiments which probably will become available in the near future. The most important features of this report include the concept that in the presence of a hot surface at a sufficiently high temperature and having a sufficiently high work-function, ionization of cesium can be so copious that an ion space-charge sheath will form in the immediate neighborhood of the hot surface and serve as a means of injecting electrons into the plasma space. If, after injection, the nearly monoenergetic electron distribution becomes randomized so that it can be characterized as an electron temperature of approximately 5000° or more, a fractional ionization exceeding 90 per cent can be maintained in the diode space. If the electron temperature is significantly lower that this value, the ionization of the cesium will be so slight that its function will be largely that of altering the work-functions of the conducting materials and to some extent the ions produced will neutralize space charge. The full development of an adequate plasma that will be stable requires this mechanism of injection and energy redistribution to obtain electron temperatures far in excess of those characterized by the heated surface itself.

The numerical relations given to prove these points have depended upon the interpretation of basic ionization and collision cross section data available in the literature. A close reading of this report shows that some choices has to be made based on a personal interpretation of the existing data. Criticism of these choices backed up by better data or more conclusive arguments would be very welcome.

In conclusion I want to express my appreciation to Professors Allis and Hatsopoulos for the opportunity to discuss with them many of the results of this calculation as it developed.

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9. W. B. Nottingham, loc. cit. See Sect. 25 and Table 3c for the Langmuir analyses. Equation for sheath thickness is:

$$x_c = \chi_c x_1 = \sqrt{2}\left[\frac{kT_g \epsilon_o}{q^2 f_s N}\right]^{1/2} \chi_c = 97.6\left[\frac{T_g}{f_s N}\right]^{1/2} \chi_c$$

Express $\psi_c = (V_c / \overline{V}_g)$ with $\overline{V}_g = T_g/11600$;
take ψ_c corresponding to this ψ_c from Table 3c and solve for x_c. The actual sheath thickness will be greater than x_c.

Appendix 1

H(u) functions used in Eqs. 6:4 and 6:6. See Fig. 7 for plot.

For $$u = \frac{\phi_1 - V_i}{\overline{V}_-} \leq 0$$

ln

$$H_2(u) = u - \ln(1 + e^u) - \mathrm{lh}\,(1 - \tfrac{1}{2} e^u + \tfrac{1}{3} e^{2u} - \tfrac{1}{4} e^{3u} + \tfrac{1}{5} e^{4u} - \ldots)$$

$$u = \frac{\phi_1 - V_i}{\overline{V}_-} \geq 0$$

$$H_1(u) = u - \ln(1 + e^{-u}) - \ln(u + e^{-u} - \tfrac{1}{2} e^{-2u} + \tfrac{1}{3} e^{-3u} - \tfrac{1}{4} e^{-4u} + \ldots)$$

u	H(u)
- 4	- 4.018
- 3.5	- 3.530
- 3	- 3.050
- 2.5	- 2.580
- 2.0	- 2.063
- 1.5	- 1.600
- 1.0	- 1.152
- 0.5	- 0.729
0	- 0.327
+ 0.5	+ 0.053

u	H(u)
1.0	+ 0.414
1.5	+ 0.767
2.0	+ 1.118
2.5	+ 1.474
3.0	+ 1.835
3.5	+ 2.208
4.0	+ 2.591
4.5	+ 2.982
5.0	+ 3.383

Correct formula from K. Luke

$$H(u) = u - \ln(1 + e^{-u}) - \ln\ln(1 + e^{u})$$

all values of u.

Appendix 2

Correlation function. See Eqs. 6:4 and 6:7

$$f(V_p/\overline{V}) = \frac{V_p}{\overline{V}} - \frac{1}{2} \ln \left(\frac{V_p}{\overline{V}} + 1\right)$$

See Figs. 8 and 9

$\frac{V_p}{\overline{V}}$	$f(\frac{V_p}{\overline{V}})$
0	0.297
0.5	0.653
1.0	1.042
1.5	1.451
2.0	2.307
3.0	3.195
4.0	4.104
5.0	

$\frac{V_p}{\overline{V}}$	$f(\frac{V_p}{\overline{V}})$
6.0	5.017
8.0	6.891
10.0	8.801
12.0	10.718
14.0	12.646
16.0	14.583
18.0	16.528
20.0	18.478
24.0	22.410

Appendix 3

Fractional ionization from Eqs. 10:11 and 10:18.

See Fig. 10 for plot.

$\overline{V}$	f	f_E
0.20	1.35×10^{-4}	2.7×10^{-4}
0.24	3.8×10^{-3}	9×10^{-3}
0.28	4.11×10^{-2}	0.107
0.30	0.101	0.251
0.32	0.208	0.456
0.34	0.359	0.656
0.36	0.523	0.798
0.38	0.668	0.885
0.40	0.776	0.933
0.42	0.852	0.96
0.44	0.900	0.974
0.48	0.949	0.99
0.52	0.975	0.997

ADDENDUM

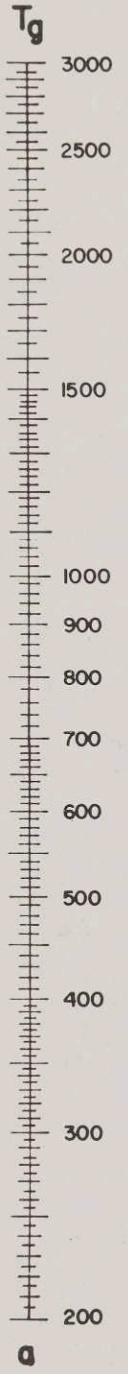
Nomographic Chart

of

Temperature Pressure Mean-Free Path Relations For Cesium

from

Equations 5:1 and 8:11



TEMPERATURE PRESSURE MEAN-FREE PATH RELATIONS FOR CESIUM

prepared by

JAMES H. HUFFORD

from equations by

WAYNE B. NOTTINGHAM

$$p = 2.45 \times 10^{8}\, T_{Cs}^{-1/2}\, e^{-\frac{8910}{T_{Cs}}} \quad \text{mm}$$

$$p = 2.45 \times 10^{8}\, T_{Cs}^{-1/2}\, 10^{-\frac{3870}{T_{Cs}}} \quad \text{mm}$$

$$\lambda_c = 1.38 \times 10^{-13}\, T_g^{1/2}\, 10^{\frac{3670}{T_{Cs}}} \quad \text{m}$$

p = pressure in mm

T_{Cs} = cesium condensation temperature °K

T_g = average temperature of cesium gas °K

λ_c = mean-free path in meters

Use scales a, b and c for high temperature.

Use scales a, d and e for low temperature.

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