"OPTIMUM Conditions For Maximum Power in Class A Amplifiers" by W.B. Nottingham, 1941

OPTIMUM CONDITIONS FOR MAXIMUM POWER IN CLASS A AMPLIFIERS

BY
WAYNE B. NOTTINGHAM

Reprinted from Proceedings of the I.R.E.
Vol. 29, No. 12, December, 1941
PRINTED IN U.S.A.

OPTIMUM CONDITIONS FOR MAXIMUM POWER IN CLASS A AMPLIFIERS

BY
WAYNE B. NOTTINGHAM

Reprinted from Proceedings of the I.R.E.
Vol. 29, No. 12, December, 1941
PRINTED IN U.S.A.

Optimum Conditions for Maximum Power in Class A Amplifiers*

WAYNE B. NOTTINGHAM†, ASSOCIATE, I.R.E.

Summary—*By following a simple analysis, it is shown that there are three cases for which optimum operating conditions may be established for class A amplifiers. These are for (I) the small signal, (II) the fixed quiescent plate voltage, and (III) the fixed quiescent plate dissipation. There is a "best" operating condition, which might be considered as a fourth case, if both the quiescent plate voltage and the quiescent plate dissipation are fixed. For cases (I) and (II) the results are definite and give $R=r_p$ and $R=2r_p$ but for (III) $R\doteq 20r_p$ and for the fourth condition $R\doteq 8r_p$. In the last two conditions R is not exactly the same for all tube types but depends to some extent on the tube characteristic. The undistorted power delivered to the load for these cases varies by a factor of nearly 3.*

Introduction

THE FIRST optimum condition for class A power amplifiers was established long ago[1] for the case of a small amplitude of grid swing and resulted in the relation $R=r_p$ where R is the load resistance and r_p the plate resistance of the tube. The second relation applies in case the grid swing is limited only by the distortion properties of the tube and the quiescent plate voltage is specified. Brown[2] showed that the maximum power is delivered to the load when $R=2r_p$ assuming that the no-signal plate dissipation is adequate. The third case, which so far as the author is aware, has never been published, applies if the quiescent plate dissipation is specified and no limitations are placed on the plate voltage. The grid swing is assumed to be limited by the distortion properties of the tube just as in the second case above. The optimum plate load for the third case turns out to be of the order of fifteen to twenty times the plate resistance r_p and depends on the quiescent plate dissipation and the minimum plate current permitted by the tube characteristics. Since the treatment of all three of these cases can be unified so easily, it will be presented here as briefly as possible.

* Decimal classification: R132. Original manuscript received by the Institute, September 26, 1940; revised manuscript received, August 25, 1941.

† George Eastman Laboratory of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts.

[1] H. J. Van Der Bijl, "The Thermionic Vacuum Tube," McGraw-Hill Book Company, New York, N. Y., 1920, p. 188.

[2] W. J. Brown, *Proc. Phys. Soc.* (London), vol. 36, p. 218; 1924.

Symbols

The symbols to be used can best be defined in terms of the idealized tube characteristic curves shown in Fig. 1. The static tube characteristics over the range of

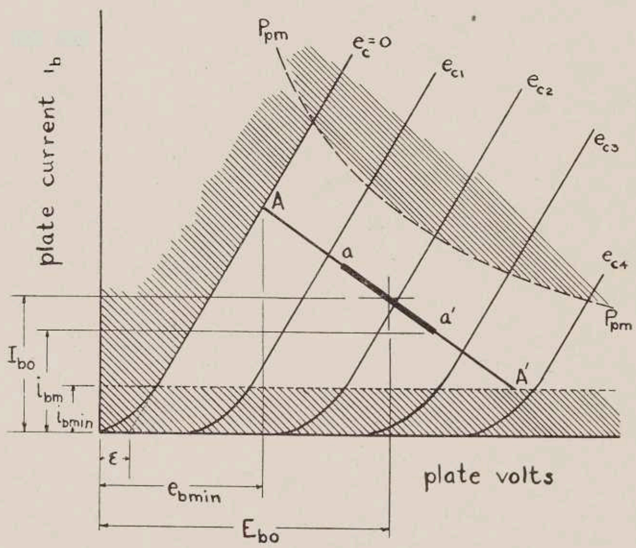

Fig. 1—Idealized tube characteristics to illustrate symbols used. a–a' is load line for small signal; A–A' is load line for large signal always terminated at zero grid volts at A and a minimum plate current at A'. Maximum plate dissipation requires quiescent point to lie on line P_{pm}–P_{pm} or below it.

approximate linearity are represented by

$$i_b = \frac{1}{r_p}(\mu e_c + e_b - \epsilon) \qquad (1)$$

where

i_b = instantaneous total plate current
e_c = instantaneous total grid voltage
e_b = instantaneous total plate voltage
r_p = plate resistance over the linear part of the characteristic
μ = amplification factor
ϵ = intercept on the voltage axis of the extrapolated $e_c=0$ curve

In the equations below the following symbols have the meanings:

E_{c0} = quiescent grid voltage
I_{b0} = quiescent plate current
E_{b0} = quiescent plate voltage
$P_{p0} = I_{b0}E_{b0}$ = quiescent plate dissipation
P_{pm} = maximum quiescent plate dissipation
i_{bm} = minimum total plate current for a specified grid signal
$i_{b\min}$ = minimum total plate current permitted as set by distortion
$I_{pm} = I_{b0} - i_{bm}$ = maximum value of varying component of the plate current
E_{gm} = maximum value of the varying component of grid voltage
P = alternating-current component of the power delivered to the load R
R = load resistance.

General Equations

A general equation for the power delivered to the load R is

$$P = \tfrac{1}{2} I_{pm}{}^2 R = \tfrac{1}{2}(I_{b0} - i_{bm})^2 R. \tag{2}$$

If the power is maximized with respect to R then $(dP/dR) = 0$ and we have

$$I_{pm} + 2R(dI_{pm}/dR) = 0 \tag{3a}$$

or

$$(I_{b0} - i_{bm}) + 2R \frac{d(I_{b0} - i_{bm})}{dR} = 0. \tag{3b}$$

The maximum value of the varying component of the plate current is related to the grid signal by

$$\mu E_{gm} = I_{pm}(r_p + R) = (I_{b0} - i_{bm})(r_p + R). \tag{4}$$

Application to Specified Cases

As mentioned above, there are three specific cases of interest which are (*I*) small but fixed grid signal, (*II*) specified quiescent plate voltage, and (*III*) specified quiescent plate dissipation. These will be discussed in this order.

Case I

With a fixed grid signal $(d\,E_{gm}/d\,R) = 0$, and the optimum load resistance is obtained by differentiating (4) with respect to R and combining this with (3a).

$$0 = (r_p + R)(dI_{pm}/dR) + I_{pm} = I_{pm} + 2R(dI_{pm}/dR). \tag{5}$$

From this we obtain

$$R = r_p \tag{6}$$

for the optimum load resistor.[3]

[3] There is a value of I_{b0} for which the greatest amount of power can be delivered to the load of resistance $R = r_p$ without exceeding the quiescent plate power rating P_{pm}. This may be calculated using the equation

$$I_{b0} = [(P_{pm}/3r_p) + (i_{b\min}^2/9)(\epsilon/2r_p i_{b\min} - 1)^2]^{1/2} - (i_{b\min}/3)(\epsilon/2r_p i_{b\min} - 1) \tag{6a}$$

Case II

In order to limit the distortion produced in a power amplifier it is common practice to specify that $-E_{co} = E_{gm}$ (that is the grid must not go positive) and the minimum value of the plate current should never fall below some arbitrary minimum current $i_{b\min}$ which is determined in the practical case by the true characteristics of the tube and the amount of distortion which can be tolerated. If, in addition to these limitations, the quiescent value of the plate potential E_{b0} is specified, then the optimum operating conditions and the load resistance may be determined uniquely.

The first and second conditions above yield the following two general equations which may be written directly from an inspection of the curves of Fig. 1 and the use of (1):

$$r_p(2I_{b0} - i_{b\min}) = e_{b\min} - \epsilon \tag{7}$$

since $(2I_{b0} - i_{b\min})$ is the current at point A of Fig. 1.

$$R = \frac{E_{b0} - e_{b\min}}{I_{b0} - i_{b\min}} = \frac{E_{b0} - r_p(2I_{b0} - i_{b\min}) - \epsilon}{I_{b0} - i_{b\min}}. \tag{8}$$

If we differentiate (8) and remember that both $(d\,E_{b0}/d\,R)$ and $(d\,i_{b\min}/d\,R)$ are zero, then

$$(I_{b0} - i_{b\min}) + R\frac{dI_{b0}}{dR} = -2r_p \frac{dI_{b0}}{dR}. \tag{9}$$

With the use of (3b) it follows that the optimum value of the load resistance is given by

$$R = 2r_p, \tag{10}$$

the optimum quiescent plate current is

$$I_{b0} = \tfrac{3}{4} i_{b\min} + \frac{E_{b0} - \epsilon}{4r_p} \tag{11}$$

and the optimum quiescent grid voltage is

$$E_{c0} = -\frac{3r_p}{\mu}(I_{b0} - i_{b\min}) = -\frac{3}{4\mu}(E_{b0} - \epsilon - r_p i_{b\min}). \tag{12}$$

If the conditions of this case are adhered to and R made equal to $2r_p$, then the quiescent plate dissipation P_{p0} may exceed P_{pm} which is the maximum permitted without tube damage. This results if the value of E_{b0} is too large. It is clear that under these conditions the problem is "overspecified" and it is not necessary to determine the optimum load resistance by differentiation since it may be determined directly from (8) because the maximum value of I_{b0} is determined by the specified P_{pm} and E_{b0}. The other constants of the equation may all be determined from the tube characteristics. If the value of R calculated from (8) is greater than $2r_p$, then this is the best value consistent with a specified maximum E_{b0} and P_{pm} but if this value of R is less than $2r_p$ then a choice of $R = 2r_p$ is the optimum and P_{p0} will be less than P_{pm}.

The calculation of the maximum value of E_{b0}, for which *Case II* can be said to apply without exceeding a specified maximum P_{pm}, is straightforward and the result is

$$(\max)\ E_{b0} = \tfrac{1}{2}\{\epsilon - 3i_{b\min}r_p + \sqrt{16r_pP_{pm} + (\epsilon - 3i_{b\min}r_p)^2}\}. \quad (13)$$

A consideration of the usual magnitudes of constants of the above equation leads to the approximation

$$(\max)\ E_{b0} \doteq 2\sqrt{r_pP_{pm}}. \quad (13a)$$

If the maximum value of E_{b0} permitted by the tube manufacturer is greater than that given by (13a), then (8) determines the best value of R unless this value exceeds that determined under *Case III* below. Under those circumstances the value of R determined by the equations of *Case III* is the optimum.

Case III

If the limits set by distortion are again restricted by the two conditions (1) the grid must not go positive and (2) the minimum value of the plate current must not fall below $i_{b\min}$ and in addition the maximum quiescent plate dissipation is specified as P_{pm}, then a unique solution to the problem of finding the optimum operating conditions is available.

Since $(dP_{pm}/dR) = 0$ it follows that

$$\frac{d(I_{b0}E_{b0})}{dR} = E_{b0}\frac{dI_{b0}}{dR} + I_{b0}\frac{dE_{b0}}{dR} = 0. \quad (14)$$

Equation (8) may be differentiated to obtain

$$(I_{b0} - i_{b\min}) + R\frac{dI_{b0}}{dR} = \frac{dE_{b0}}{dR} - 2r_p\frac{dI_{b0}}{dR}. \quad (15)$$

Substitutions from (3b) and (14) result in the relations

$$R = 2r_p + \frac{E_{b0}}{I_{b0}} = 2r_p + \frac{P_{pm}}{I_{b0}^2}. \quad (16)$$

The elimination of R between (8) and (16) gives the final relationship by which the optimum value of I_{b0} may be determined as follows:

$$\frac{P_{pm}i_{b\min}}{I_{b0}^2} = 4r_pI_{b0} - 3r_pi_{b\min} + \epsilon. \quad (17)$$

This equation is a cubic in I_{b0} depending only on known quantities[4] P_{pm}, $i_{b\min}$, r_p, and ϵ. It may be solved by plotting the left-hand side of the equation as a function of arbitrarily chosen values of I_{b0} and finding the intersection of this curve with the stright line which represents graphically the value of the right-hand side of the equation for the same range in I_{b0}. Since the relations here are so simple, it is also an easy matter, with the help of a slide rule, to find the solution to (17) by trial. After having found the optimum value of I_{b0}, the optimum load resistance R may be determined by (16) and the corresponding value of E_{b0} may be found from the relation

$$E_{b0} = \frac{P_{pm}}{I_{b0}}. \quad (18)$$

Equation (19) serves to determine the optimum value of the quiescent grid voltage E_{c0}

$$E_{c0} = -\frac{(r_p + R)}{\mu}(I_{b0} - i_{b\min}). \quad (19)$$

Numerical Examples

In order to illustrate the use of these equations, a numerical example has been computed for a tube characteristic which is approximately that of the RCA 845 tube. The basic constants are as follows:

Maximum quiescent plate dissipation	P_{pm}	= 100 watts
Plate resistance	r_p	= 1700 ohms
Maximum quiescent plate voltage	E_{b0}	= 1250 volts
Amplification factor	μ	= 5.3
Estimated intercept	ϵ	= 85 volts
Estimated minimum plate current	$i_{b\min}$	= 10×10^{-3} ampere.

TABLE I

Case	E_{b0} Volts	I_{b0} Ampere	P_{pm} Watts	R Ohms	(R/r_p)	P Watts	I_{b0} Equation	R Equation
I	740	135×10^{-3}	100	1700	1.0	13.3	(6a)	(6)
II	840	119×10^{-3}	100	3400	2.0	20.0	(11)	(10)
Limited E_{b0}	1250	80×10^{-3}	100	13,000	7.65	31.9	(18)	(8)
III	1950	51×10^{-3}	100	41,500	24.4	35.3	(17)	(16)

Four examples have been worked out using the above constants and the results are tabulated in Table I. Circuit constants have been chosen so that the quiescent plate dissipation is 100 watts in every case and the power delivered to the load is computed to be the maximum consistent with the two limitations that the grid must not swing positive and that the plate current must not swing to a value less than 10×10^{-3} ampere. In the last two columns of the table, the numbers of the equations used to compute the values of I_{b0} and R are recorded. The value of E_{b0} comes from (18) in every example.

Discussion of Results

The maximum voltage for which *Case II* applies to the 845 tube is 840 volts and yet, since the manufacturer's maximum rating is between 840 volts and 1950 volts, the best operating conditions for this tube are not determined by the true maximization with respect to power delivered. The fact that the power delivered with $E_{b0} = 1250$ volts is less than 10 per cent below the absolute maximum which can be obtained under operating conditions as defined, shows that the manufacturer's rating is a practical and reasonable compromise.

In making the above calculations, $i_{b\min}$ was assumed

[4] A brief discussion of the selection of $i_{b\min}$ comes later in this paper but here it seems desirable to interpret (17) for the indeterminate case of $i_{b\min} = 0$. Here $I_{b0} \to 0$ and $E_{b0} \to \infty$ in such a way that $I_{b0}E_{b0} = P_{pm}$. Of course $R \to \infty$ at the same time.

to have the same value for every case whereas experience shows that the higher the value of the load resistance the lower the value of $i_{b\min}$ for a given distortion. There are no simple rules by which the harmonic distortion can be determined for a given tube characteristic although the graphical methods explained by Reich[5] are helpful and would serve for estimating the harmonic distortion expected after the optimum load resistance has been determined for a definite assumed value of $i_{b\min}$ If the resultant harmonic distortion is greater or less than the amount considered tolerable, then a new value of $i_{b\min}$ would have to be assumed and a new value of load resistance determined using the methods outlined here. These results are applicable to circuits with resistance feed, choke feed, or transformer feed of the plate power as long as the input impedance to the plate load is dominated by its resistance component.

[5] H. J. Reich, "Theory and Application of Electron Tubes," McGraw-Hill Book Company, New York, N. Y., 1939, p. 248.

OPTIMUM CONDITIONS FOR MAXIMUM POWER IN CLASS A AMPLIFIERS

BY
WAYNE B. NOTTINGHAM

Reprinted from Proceedings of the I.R.E.
Vol. 29, No. 12, December, 1941
PRINTED IN U.S.A.

OPTIMUM CONDITIONS FOR MAXIMUM POWER IN CLASS A AMPLIFIERS

BY
WAYNE B. NOTTINGHAM

Reprinted from Proceedings of the I.R.E.
Vol. 29, No. 12, December, 1941
PRINTED IN U.S.A.

Optimum Conditions for Maximum Power in Class A Amplifiers*

WAYNE B. NOTTINGHAM†, ASSOCIATE, I.R.E.

Summary—*By following a simple analysis, it is shown that there are three cases for which optimum operating conditions may be established for class A amplifiers. These are for (I) the small signal, (II) the fixed quiescent plate voltage, and (III) the fixed quiescent plate dissipation. There is a "best" operating condition, which might be considered as a fourth case, if both the quiescent plate voltage and the quiescent plate dissipation are fixed. For cases (I) and (II) the results are definite and give $R=r_p$ and $R=2r_p$ but for (III) $R \doteq 20r_p$ and for the fourth condition $R \doteq 8r_p$. In the last two conditions R is not exactly the same for all tube types but depends to some extent on the tube characteristic. The undistorted power delivered to the load for these cases varies by a factor of nearly 3.*

Introduction

THE FIRST optimum condition for class A power amplifiers was established long ago[1] for the case of a small amplitude of grid swing and resulted in the relation $R=r_p$ where R is the load resistance and r_p the plate resistance of the tube. The second relation applies in case the grid swing is limited only by the distortion properties of the tube and the quiescent plate voltage is specified. Brown[2] showed that the maximum power is delivered to the load when $R=2r_p$ assuming that the no-signal plate dissipation is adequate. The third case, which so far as the author is aware, has never been published, applies if the quiescent plate dissipation is specified and no limitations are placed on the plate voltage. The grid swing is assumed to be limited by the distortion properties of the tube just as in the second case above. The optimum plate load for the third case turns out to be of the order of fifteen to twenty times the plate resistance r_p and depends on the quiescent plate dissipation and the minimum plate current permitted by the tube characteristics. Since the treatment of all three of these cases can be unified so easily, it will be presented here as briefly as possible.

* Decimal classification: R132. Original manuscript received by the Institute, September 26, 1940; revised manuscript received, August 25, 1941.

† George Eastman Laboratory of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts.

[1] H. J. Van Der Bijl, "The Thermionic Vacuum Tube," McGraw-Hill Book Company, New York, N. Y., 1920, p. 188.

[2] W. J. Brown, *Proc. Phys. Soc.* (London), vol. 36, p. 218; 1924.

Symbols

The symbols to be used can best be defined in terms of the idealized tube characteristic curves shown in Fig. 1. The static tube characteristics over the range of

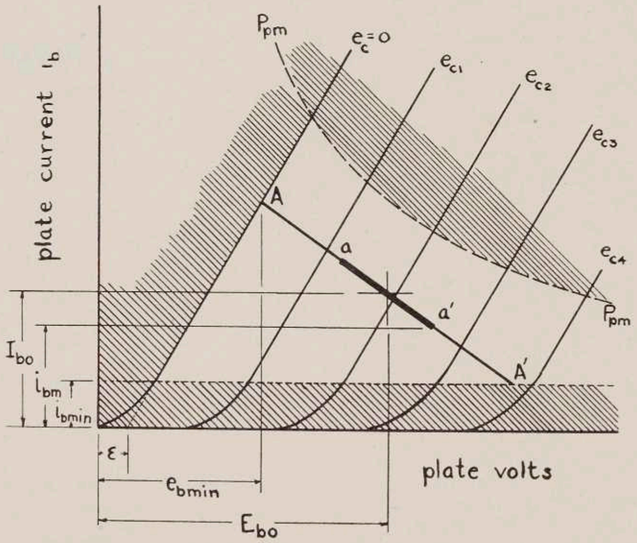

Fig. 1—Idealized tube characteristics to illustrate symbols used. a–a' is load line for small signal; A–A' is load line for large signal always terminated at zero grid volts at A and a minimum plate current at A'. Maximum plate dissipation requires quiescent point to lie on line P_{pm}–P_{pm} or below it.

approximate linearity are represented by

$$i_b = \frac{1}{r_p}(\mu e_c + e_b - \epsilon) \tag{1}$$

where

i_b = instantaneous total plate current
e_c = instantaneous total grid voltage
e_b = instantaneous total plate voltage
r_p = plate resistance over the linear part of the characteristic
μ = amplification factor
ϵ = intercept on the voltage axis of the extrapolated $e_c=0$ curve

In the equations below the following symbols have the meanings:

E_{c0} = quiescent grid voltage
I_{b0} = quiescent plate current
E_{b0} = quiescent plate voltage
$P_{p0} = I_{b0}E_{b0}$ = quiescent plate dissipation
P_{pm} = maximum quiescent plate dissipation
i_{bm} = minimum total plate current for a specified grid signal
i_{bmin} = minimum total plate current permitted as set by distortion
$I_{pm} = I_{b0} - i_{bm}$ = maximum value of varying component of the plate current
E_{gm} = maximum value of the varying component of grid voltage
P = alternating-current component of the power delivered to the load R
R = load resistance.

General Equations

A general equation for the power delivered to the load R is

$$P = \tfrac{1}{2}I_{pm}^2R = \tfrac{1}{2}(I_{b0} - i_{bm})^2R. \quad (2)$$

If the power is maximized with respect to R then $(dP/dR) = 0$ and we have

$$I_{pm} + 2R(dI_{pm}/dR) = 0 \quad (3a)$$

or

$$(I_{b0} - i_{bm}) + 2R\frac{d(I_{b0} - i_{bm})}{dR} = 0. \quad (3b)$$

The maximum value of the varying component of the plate current is related to the grid signal by

$$\mu E_{gm} = I_{pm}(r_p + R) = (I_{b0} - i_{bm})(r_p + R). \quad (4)$$

Application to Specified Cases

As mentioned above, there are three specific cases of interest which are (*I*) small but fixed grid signal, (*II*) specified quiescent plate voltage, and (*III*) specified quiescent plate dissipation. These will be discussed in this order.

Case I

With a fixed grid signal $(d\,E_{gm}/d\,R) = 0$, and the optimum load resistance is obtained by differentiating (4) with respect to R and combining this with (3a).

$$0 = (r_p + R)(dI_{pm}/dR) + I_{pm} = I_{pm} + 2R(dI_{pm}/dR). \quad (5)$$

From this we obtain

$$R = r_p \quad (6)$$

for the optimum load resistor.[3]

[3] There is a value of I_{b0} for which the greatest amount of power can be delivered to the load of resistance $R = r_p$ without exceeding the quiescent plate power rating P_{pm}. This may be calculated using the equation

$$I_{b0} = [(P_{pm}/3r_p) + (i_{bmin}^2/9)(\epsilon/2r_pi_{bmin} - 1)^2]^{1/2} - (i_{bmin}/3)(\epsilon/2r_pi_{bmin} - 1) \quad (6a)$$

Case II

In order to limit the distortion produced in a power amplifier it is common practice to specify that $-E_{co} = E_{gm}$ (that is the grid must not go positive) and the minimum value of the plate current should never fall below some arbitrary minimum current i_{bmin} which is determined in the practical case by the true characteristics of the tube and the amount of distortion which can be tolerated. If, in addition to these limitations, the quiescent value of the plate potential E_{b0} is specified, then the optimum operating conditions and the load resistance may be determined uniquely.

The first and second conditions above yield the following two general equations which may be written directly from an inspection of the curves of Fig. 1 and the use of (1):

$$r_p(2I_{b0} - i_{bmin}) = e_{bmin} - \epsilon \quad (7)$$

since $(2I_{b0} - i_{bmin})$ is the current at point A of Fig. 1.

$$R = \frac{E_{b0} - e_{bmin}}{I_{b0} - i_{bmin}} = \frac{E_{b0} - r_p(2I_{b0} - i_{bmin}) - \epsilon}{I_{b0} - i_{bmin}}. \quad (8)$$

If we differentiate (8) and remember that both $(d\,E_{b0}/d\,R)$ and $(d\,i_{bmin}/d\,R)$ are zero, then

$$(I_{b0} - i_{bmin}) + R\frac{dI_{b0}}{dR} = -2r_p\frac{dI_{b0}}{dR}. \quad (9)$$

With the use of (3b) it follows that the optimum value of the load resistance is given by

$$R = 2r_p, \quad (10)$$

the optimum quiescent plate current is

$$I_{b0} = \tfrac{3}{4}i_{bmin} + \frac{E_{b0} - \epsilon}{4r_p} \quad (11)$$

and the optimum quiescent grid voltage is

$$E_{c0} = -\frac{3r_p}{\mu}(I_{b0} - i_{bmin}) = -\frac{3}{4\mu}(E_{b0} - \epsilon - r_pi_{bmin}). \quad (12)$$

If the conditions of this case are adhered to and R made equal to $2r_p$, then the quiescent plate dissipation P_{p0} may exceed P_{pm} which is the maximum permitted without tube damage. This results if the value of E_{b0} is too large. It is clear that under these conditions the problem is "overspecified" and it is not necessary to determine the optimum load resistance by differentiation since it may be determined directly from (8) because the maximum value of I_{b0} is determined by the specified P_{pm} and E_{b0}. The other constants of the equation may all be determined from the tube characteristics. If the value of R calculated from (8) is greater than $2r_p$, then this is the best value consistent with a specified maximum E_{b0} and P_{pm} but if this value of R is less than $2r_p$ then a choice of $R = 2r_p$ is the optimum and P_{p0} will be less than P_{pm}.

The calculation of the maximum value of E_{b0}, for which *Case II* can be said to apply without exceeding a specified maximum P_{pm}, is straightforward and the result is

$$(\max)\ E_{b0} = \tfrac{1}{2}\{\epsilon - 3i_{b\min}r_p + \sqrt{16r_pP_{pm} + (\epsilon - 3i_{b\min}r_p)^2}\}. \quad (13)$$

A consideration of the usual magnitudes of constants of the above equation leads to the approximation

$$(\max)\ E_{b0} \doteqdot 2\sqrt{r_pP_{pm}}. \quad (13a)$$

If the maximum value of E_{b0} permitted by the tube manufacturer is greater than that given by (13a), then (8) determines the best value of R unless this value exceeds that determined under *Case III* below. Under those circumstances the value of R determined by the equations of *Case III* is the optimum.

Case III

If the limits set by distortion are again restricted by the two conditions (1) the grid must not go positive and (2) the minimum value of the plate current must not fall below $i_{b\min}$ and in addition the maximum quiescent plate dissipation is specified as P_{pm}, then a unique solution to the problem of finding the optimum operating conditions is available.

Since $(dP_{pm}/dR) = 0$ it follows that

$$\frac{d(I_{b0}E_{b0})}{dR} = E_{b0}\frac{dI_{b0}}{dR} + I_{b0}\frac{dE_{b0}}{dR} = 0. \quad (14)$$

Equation (8) may be differentiated to obtain

$$(I_{b0} - i_{b\min}) + R\frac{dI_{b0}}{dR} = \frac{dE_{b0}}{dR} - 2r_p\frac{dI_{b0}}{dR}. \quad (15)$$

Substitutions from (3b) and (14) result in the relations

$$R = 2r_p + \frac{E_{b0}}{I_{b0}} = 2r_p + \frac{P_{pm}}{I_{b0}^2}. \quad (16)$$

The elimination of R between (8) and (16) gives the final relationship by which the optimum value of I_{b0} may be determined as follows:

$$\frac{P_{pm}i_{b\min}}{I_{b0}^2} = 4r_pI_{b0} - 3r_pi_{b\min} + \epsilon. \quad (17)$$

This equation is a cubic in I_{b0} depending only on known quantities[4] P_{pm}, $i_{t\min}$, r_p, and ϵ. It may be solved by plotting the left-hand side of the equation as a function of arbitrarily chosen values of I_{b0} and finding the intersection of this curve with the stright line which represents graphically the value of the right-hand side of the equation for the same range in I_{b0}. Since the relations here are so simple, it is also an easy matter, with the help of a slide rule, to find the solution to (17) by trial. After having found the optimum value of I_{b0}, the optimum load resistance R may be determined by (16) and the corresponding value of E_{b0} may be found from the relation

$$E_{b0} = \frac{P_{pm}}{I_{b0}}. \quad (18)$$

Equation (19) serves to determine the optimum value of the quiescent grid voltage E_{c0}

$$E_{c0} = -\frac{(r_p + R)}{\mu}(I_{b0} - i_{b\min}). \quad (19)$$

Numerical Examples

In order to illustrate the use of these equations, a numerical example has been computed for a tube characteristic which is approximately that of the RCA 845 tube. The basic constants are as follows:

Maximum quiescent plate dissipation	$P_{pm} = 100$ watts
Plate resistance	$r_p = 1700$ ohms
Maximum quiescent plate voltage	$E_{b0} = 1250$ volts
Amplification factor	$\mu = 5.3$
Estimated intercept	$\epsilon = 85$ volts
Estimated minimum plate current	$i_{b\min} = 10\times10^{-3}$ ampere.

TABLE I

Case	E_{b0} Volts	I_{b0} Ampere	P_{pm} Watts	R Ohms	(R/r_p)	P Watts	I_{b0} Equation	R Equation
I	740	135×10^{-3}	100	1700	1.0	13.3	(6a)	(6)
II	840	119×10^{-3}	100	3400	2.0	20.0	(11)	(10)
Limited E_{b0}	1250	80×10^{-3}	100	13,000	7.65	31.9	(18)	(8)
III	1950	$\cdot51\times10^{-3}$	100	41,500	24.4	35.3	(17)	(16)

Four examples have been worked out using the above constants and the results are tabulated in Table I. Circuit constants have been chosen so that the quiescent plate dissipation is 100 watts in every case and the power delivered to the load is computed to be the maximum consistent with the two limitations that the grid must not swing positive and that the plate current must not swing to a value less than 10×10^{-3} ampere. In the last two columns of the table, the numbers of the equations used to compute the values of I_{b0} and R are recorded. The value of E_{b0} comes from (18) in every example.

Discussion of Results

The maximum voltage for which *Case II* applies to the 845 tube is 840 volts and yet, since the manufacturer's maximum rating is between 840 volts and 1950 volts, the best operating conditions for this tube are not determined by the true maximization with respect to power delivered. The fact that the power delivered with $E_{b0} = 1250$ volts is less than 10 per cent below the absolute maximum which can be obtained under operating conditions as defined, shows that the manufacturer's rating is a practical and reasonable compromise.

In making the above calculations, $i_{b\min}$ was assumed

[4] A brief discussion of the selection of $i_{b\min}$ comes later in this paper but here it seems desirable to interpret (17) for the indeterminate case of $i_{b\min}=0$. Here $I_{b0}\to0$ and $E_{b0}\to\infty$ in such a way that $I_{b0}E_{b0}=P_{pm}$. Of course $R\to\infty$ at the same time.

to have the same value for every case whereas experience shows that the higher the value of the load resistance the lower the value of $i_{b\min}$ for a given distortion. There are no simple rules by which the harmonic distortion can be determined for a given tube characteristic although the graphical methods explained by Reich[5] are helpful and would serve for estimating the harmonic distortion expected after the optimum load resistance has been determined for a definite assumed value of $i_{b\min}$ If the resultant harmonic distortion is greater or less than the amount considered tolerable, then a new value of $i_{b\min}$ would have to be assumed and a new value of load resistance determined using the methods outlined here. These results are applicable to circuits with resistance feed, choke feed, or transformer feed of the plate power as long as the input impedance to the plate load is dominated by its resistance component.

[5] H. J. Reich, "Theory and Application of Electron Tubes," McGraw-Hill Book Company, New York, N. Y., 1939, p. 248.