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A QUANTITATIVE ANALYSIS OF
A PHOTOEMISSIVE SOLAR ENERGY CONVERTER

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This Report includes the statement of a problem offered to the students in the 8.21 Course on Physical Electronics. Following this statement of the problem is the solution formulated by W. B. Nottingham.

February 23, 1961

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Final Problem of 8:21

Due Date: January 18, 1961

Introduction

This problem is given to test the student's ability to analyze an electronics problem in terms of its basic physics. It is a practical problem in that the electron tube division of the Westinghouse Electric Corp., Baltimore, Md. is working on the project as a Signal Corps Contract No. DA-36-039-SC85248. The title of this Contract is: "Photoemissive Solar Energy Converter."

The device to be described is not accurately the one which is being worked on by Westinghouse but represents so much of the same concept that the fact that it has been simplified somewhat in terms of the necessary analysis does not change it appreciably from the point of view of physics. The problem does not demand that the construction as described be feasible since the system covered here would be more efficient than the Westinghouse plan, but more difficult to build. It is to be expected that you will find the device entirely impractical from the point of view of solar energy conversion when it is realized that other devices are expected to achieve efficiencies in terms of the total energy received in excess of 10 per cent. Even though it is anticipated that you will show this device to be very inefficient, I am forced to assume that neither the Signal Corps nor the Westinghouse Company has analyzed the problem as accurately as I expect you to. Your analysis may very well place this device very far below others that make use of solar energy for the development of electric power.

Some References

One of the best sources for information is an article by G. L. Weissler on "Photoionization in Gases and Photoelectric Emission from Solids". This article begins on p. 304 of Vol. 21 of the Handbuch der Physik, Published by Springer-Verlag, Berlin, 1956. The article has a great deal of useful fundamental information and also summarizes available journal articles. In particular, it refers to the work of Apker, Taft, and Dickey on cesium antimonide. Also references are available there to studies made with silver oxide reacted with cesium. This is not the only source of data but it is a very good one.

There is also a reference on "Optical and Electrical Properties of Thin Films of Tin Dioxide," by V. K. Miloslazskii and S. P. Lyashenko in Optics and Spectroscopy, Vol. 8, p. 455, 1960.

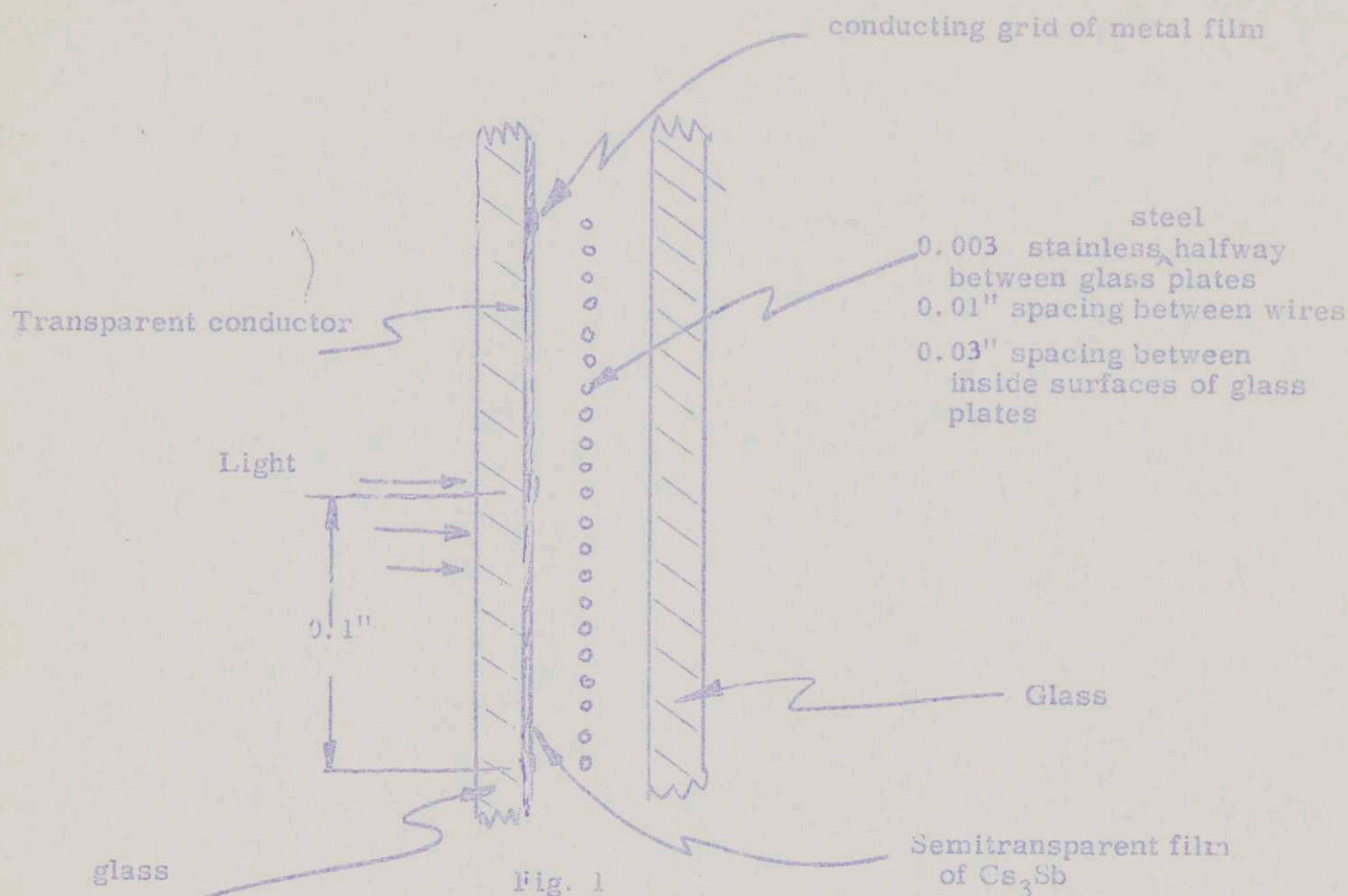
Specifically there is the Apker, Taft, Dickey article in the Journal of the Optical Society of America, Vol. 43, 78 1953. This is on the properties of cesium antimonide. See W. E. Spicer, Phys. Rev. 112, 114 (1958).

Another reference is "The Resistance of Semitransparent Photocathodes." by W. J. Harper and W. J. Choyke, Journal of Applied Physics, 27, 1358 (1956).

There are undoubtedly many other pertinent articles related to these problems.

Description of a Proposed Photoemissive solar Energy Converter

The converter proposed may be best described with the help of Fig. 1. Here in cross section is a test device which is similar to the one proposed by Westinghouse to establish experimentally the feasibility of this converter.



It is, of course, not the type of converter that would actually be used in practice, but one essentially designed to prove their principles. On a glass surface a conducting grid work of squares of metallic deposit is placed with a spacing between the parallel lines making up the squares of 0.1 inch. Each conducting bar is approximately 0.01 inch in width. Although the semi-transparent cesium antimonide to be placed on this surface has some conductivity, it has been assumed that it may be necessary to place under the cesium antimonide a better conductor such as tin dioxide or a thin evaporated film of gold. Between the two glass plates, we have a set of grid wires made of stainless steel; each wire is of the diameter 0.003 inch and the spacing between wires is 0.01 inch. The spacing between the glass surfaces is 0.03 inch. In the actual Westinghouse proposal they describe a woven mesh of stainless steel wires, 100 to the inch. In terms of construction, the mesh is obviously more practical. In terms of a theoretical analysis, the two-dimensional features of Fig. 1 will be much easier to handle. The result will be "optimistic" compared with the woven mesh system.

The method of assembly calls for the deposit of evaporated silver on the backside of each of the wires, that is, the righthand side as you look at Fig. 1. This silver film is to be oxidized and reacted with cesium to produce a very low work-function surface of moderately low yield expressed in terms of photoelectrons per quantum.

The semi-transparent film of cesium antimonide is proposed since it has a very high yield and can be used to deliver electrons into the vacuum space when light falls on the glass surface and is transmitted through the film with some absorption but the concept is that the major absorption takes place in the p-n junction very near the superficial vacuum surface.

Although it would be practically impossible to build this device with the hoped-for high quantum-yield cesium antimonide surface and maintain the high work-function stainless steel on the lefthand side of each of the wires and the very low work-function silver oxide-cesium surface on the righthand side of the wires, you may analyze this problem in the most optimistic sense, by your assuming the published very high quantum yield values for the photoemitter and the low work-function values for the backside of the grid system.

Method of Operation

The theoretical method of operation is that light is transmitted and finally absorbed in the photosensitive surface. Electrons generated there pass between the collecting grid wires and some of them will go over to the glass surface and charge it up negatively. This negative charge on the glass surface will tend to drive the electrons that do go through the "saddle point" between the wires back to the low work-function surface that is maintained in the dark. It is this electron current that is supposed to operate to make this device a solar energy converter. No power can be extracted from the device unless the Fermi level of the collecting grid system is negative with respect to the Fermi level of the conducting grid system of the emitter.

If the Fermi level of the collector system is made very negative with respect to the emitter, then no current can flow to it and the power output will be zero. As the Fermi level is made less and less negative, current can flow and finally when the Fermi level is made to coincide with that of the emitter surface grid system, the power output will again be zero because the product of this maximum in the current multiplied by the zero voltage output gives zero power. It is, therefore, the problem to evaluate as best you can the current-voltage characteristic of this device when it is illuminated by sunlight as found outside of the earth's atmosphere, but in the neighborhood of the earth. The first calculation should be simplified by making optimistic assumptions. State your assumptions clearly.

The surface potential acquired by the uncoated glass wall at the right of the figure is likely to be quite negative with respect to the surface potential of the emitter. The first tendency upon operation will be for this glass wall to charge up negatively very quickly. This trapped electric charge on the wall will respond to the light and the wall potential will be established by the equality of the arrival rate of photoelectrons and the emission rate of photoelectrons from the glass wall itself.

Some Suggested Approximations

Assume that the surface potential of the glass wall is always equal to the surface potential of the uncoated side of the stainless steel wires. You may not find any determinations of the photoelectric work-function of stainless steel. I am assuming that its work-function is approximately 4.7 ev. You may find information that would justify some other choice. I would take the work-function of the silver-oxide-cesium surface as 1.2 ev. Again you may justify another choice.

Use any means at your disposal to estimate the nature of the "saddle potential" that will be found between the collecting grid wires. To get some feeling for the nature of the potential distribution, draw on an expanded scale just two adjacent grid wires and the corresponding photoelectric and glass surfaces. Work out as best you can the nature of the equipotentials

of the system for a given bias potential on the grid wires with respect to the emitter. If by this means you can estimate the location of the saddle point and its value at various applied potentials, you may find it convenient to use a simple parabolic function to join the saddle point to the high work-function surface potential of the stainless steel. Unless you can work out some well-justified distribution of potential there, such an approximation would be good enough for the present purposes.

Assume optimistic quantum yield data for the emitter which you can support by the literature. Assume that the quanta available as a function of wavelength are those given by F. S. Johnson, *Journal of Meteorology*, 11, 431 (1954) December. His data are plotted on the attached graph. Note that this gives the number of quanta per square centimeter for a micron range located at a specific wavelength. This means that the number $H_{\lambda} d\lambda$ with λ expressed in microns gives you the number of quanta available from the sunlight in this narrow range. Note that the abscissa scale of λ is not microns but millimicrons. Note also that you will probably find it more convenient to convert this scale to one which gives you the number of light quanta per square centimeter in a unit range expressed as the electron-volt equivalent of $h\nu$. You must therefore use the proper analysis to convert the data from one system to the other.

For a given value of $h\nu$ there will be an energy distribution among the emitted electrons that will extend from 0 up to a high kinetic-energy limit well-approximated by the Einstein photoelectric equation. Observed energy distributions are generally quite complex, and therefore, it would be quite impractical to try to use these in this analysis. Instead, it is advised that you use the following, simplifying approximation which is that the number of electrons per unit range in kinetic energy associated with their motion perpendicular to the emitting surface will be uniform from 0 out to the Einstein limit, and that the total area under this energy distribution curve for a given $h\nu$ will be that associated with the quantum yield. Quantum yield measurements are usually made with a small accelerating field so that all electrons generated for a given $h\nu$ can be collected.

Clearly, this analysis calls for integrations which, because of the complexity of the yield curves and the energy distribution found in the solar radiation, must be done graphically. Of course, some of the factors in the integrand can be expressed analytically and full use should be made of such simplifications that come this way.

Expected Results

A. Give a word analysis of the problem being analyzed with generalized equations that will then show the steps which you consider necessary to follow to obtain specific numerical results.

B. Obtain an estimated voltage-current curve showing the anticipated output current at selected voltage values. This may very well be started by taking the voltage output of 0 and determining the maximum current that you can expect to deliver. Then increase the voltage in the negative direction in small steps and compute the decrease in current expected. Four or five voltages should be sufficient to establish an acceptable smooth curve, after which the product curve will give the power as a function of output voltage. Take the solar energy density to be 0.135 watts per cm^2 and calculate the theoretical efficiency.

C. Discuss the influences of such factors as the absorption of the conducting film that may be needed to reduce the IR drop through the cesium antimonide. Try to estimate very roughly the influence of IR drop

in case an additional conductor is not supplied. Discuss very briefly the extent to which your numerical calculations may be optimistic for any other reason.

Final Remarks

Even though this problem, to be carried through in all possible detail, would require much more time than is expected, it will be important to try to carry it through with more or less a minimum of individual effort so that it will not be too time consuming for any one person undertaking the analysis. The only requirement is that each student should write his own analysis.

A Quantitative Analysis of a Photoemissive Solar Energy Converter
Formulated by W. B. Nottingham

Introduction

In December 1960 the problem of the "Photoemissive Solar Energy Converter" was described in sufficient detail so that the students of the 8.21 class in Physical Electronics could analyze it. The detailed description has been made available to the students and for those reading this report not in class 8.21, the description is attached. On page 4 the first requirement is to give a word analysis of the problem indicating how it should be attacked. The first part of this report will therefore constitute an answer of the type expected under the heading A. Following this description, the method outlined will be applied quantitatively to answer the question under heading B. The final remarks will apply to heading C.

Analysis of the Problem

The analysis that is going to be given here will depend on "optimistic" assumptions. In the final analysis some discussion will be included to indicate roughly how very optimistic the numerical answer under Section B is likely to be. The solar radiation curve shown in Fig. 2 depends on the analysis of F.S. Johnson. This curve must be converted from one which gives quanta per square centimeter per second for a unit range in wavelength to another curve which gives quanta per square centimeter per second for a unit range in energy. This curve is shown in Fig. 3 and numerical values are given in Table 1. The optimistic analysis will first assume that all of this radiation is delivered in some manner or other to the immediate neighborhood of the photoemissive surface. This is optimistic for three reasons: (1) a certain amount of the radiation will be absorbed as it passes through the cesium antimonide semi-transparent surface; (2) if some kind of "transparent conductor, such as tin oxide, is between the cesium antimonide and its supporting surface, then the transmission of this material as a function of quantum energy must be considered; (3) Furthermore, if the feasibility test is to be made in a glass envelope, the transmission characteristic of the glass envelope must also be included.

It will be convenient in the analysis to follow to have some symbol which represents the "electron-volt-equivalent" of quantum energy. Equations 1 and 2 serve to illustrate the definition of the symbol $\bar{\nu}$.

$$h\nu = q\bar{\nu} \quad (1)$$

$$\bar{\nu} = \frac{h\nu}{q} \quad (2)$$

In these equations h is Planck's constant, ν is the frequency of light, q is the charge on an electron, and $\bar{\nu}$ is the voltage equivalent of the quantum characterized by the frequency ν . With this symbol in mind, the following function represents symbolically the solar quanta delivered to the effective emitting surface in quanta per square centimeter in a second for a unit range in energy.

$$\frac{\text{quanta}}{\text{cm}^2 \text{ sec-unit range in } \bar{\nu}} = T(\bar{\nu}) P(\bar{\nu}) \quad (3)$$

In this equation $P(\bar{\nu})$ is the solar energy distribution shown in Fig. 3, whereas $T(\bar{\nu})$ is the transmission coefficient as a function of quantum energy which in

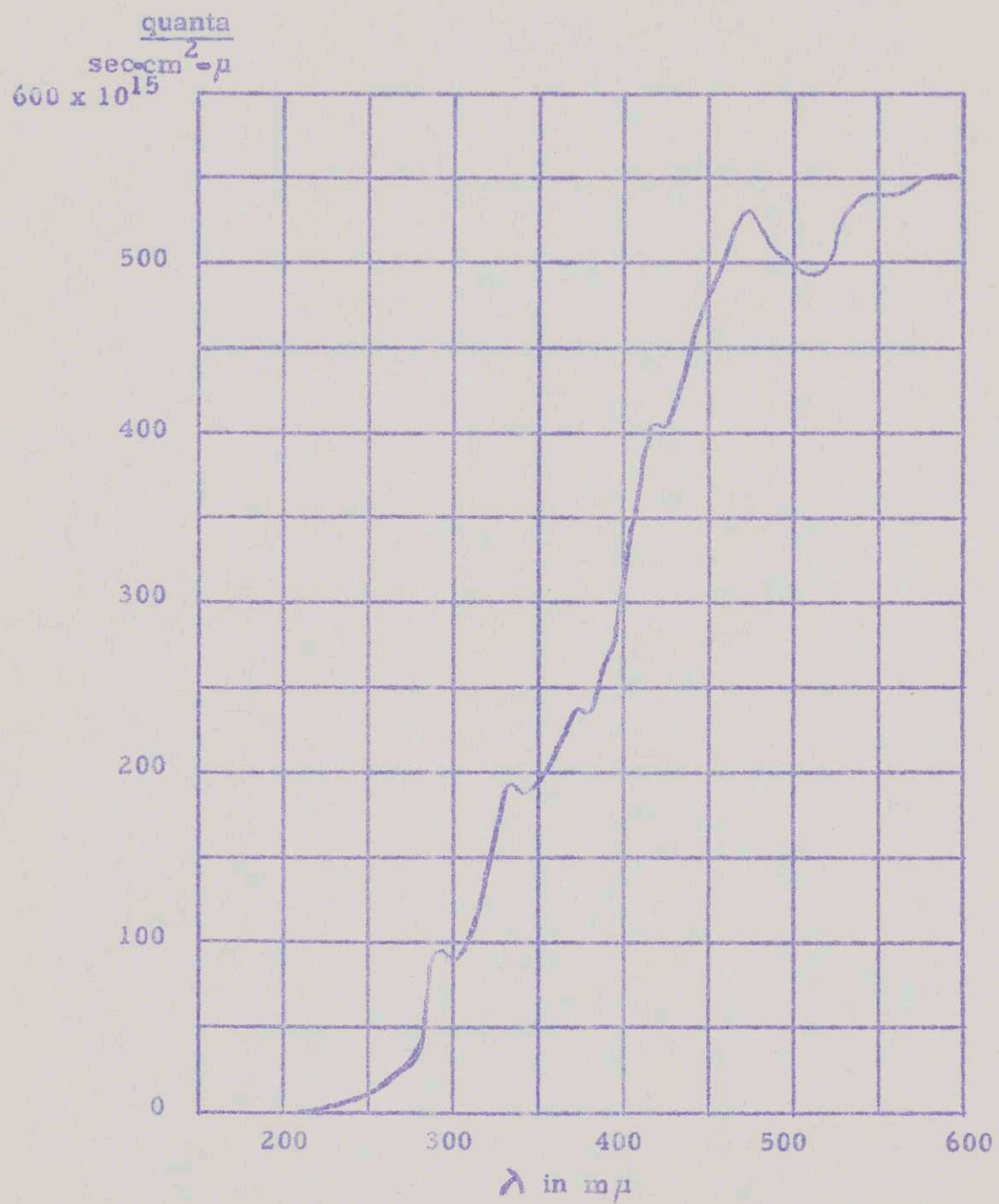
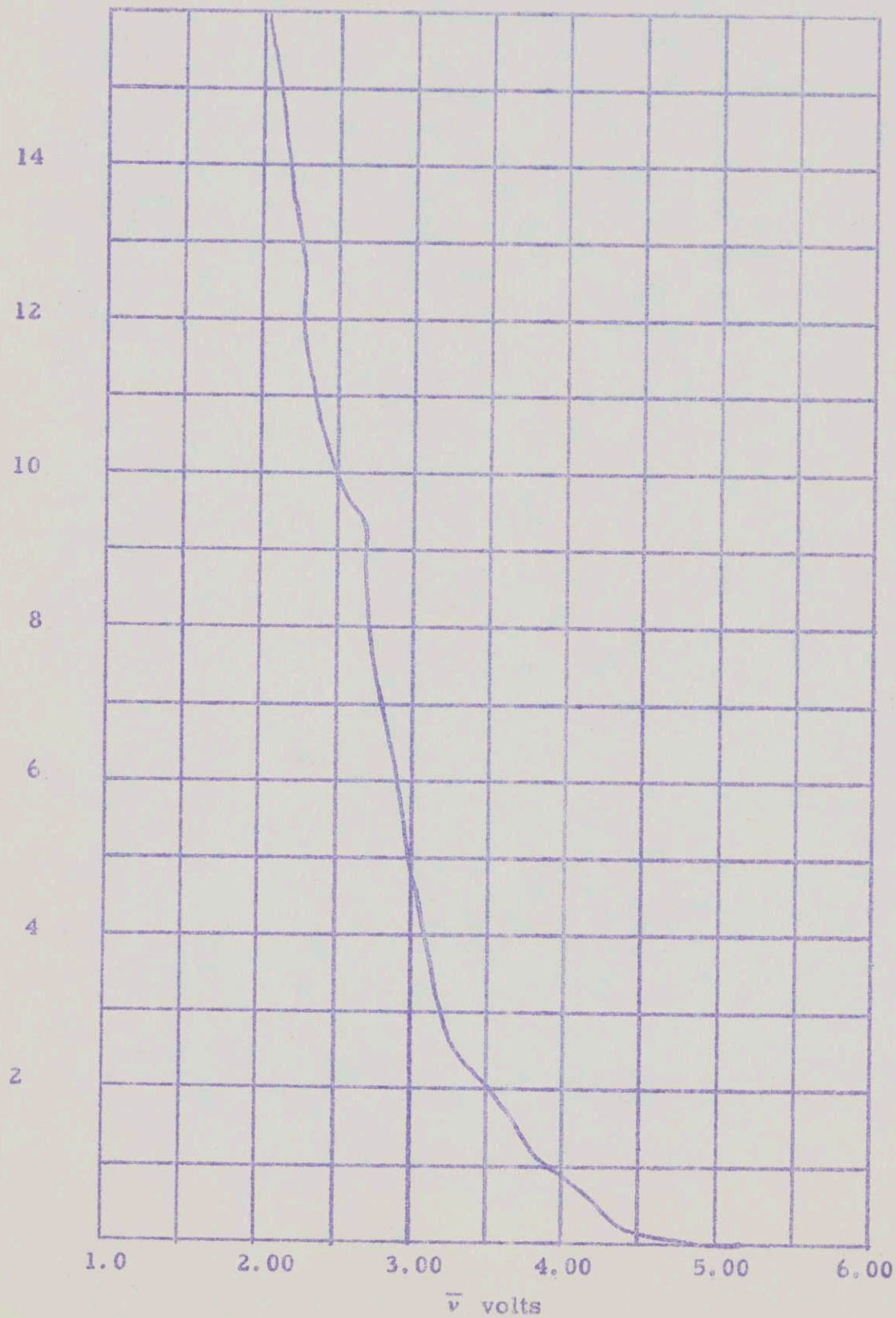


Fig. 2

(See page 436 of F. S. Johnson, Jour. of Meteorology 11, 431 (1954) Dec.)

Fig. 3 $P(\bar{\nu}) \times$ Solar Radiation in quanta/cm²-sec. 16.00×10^{16} 

the optimistic analysis is taken to be unity but in fact will always be significantly less than unity.

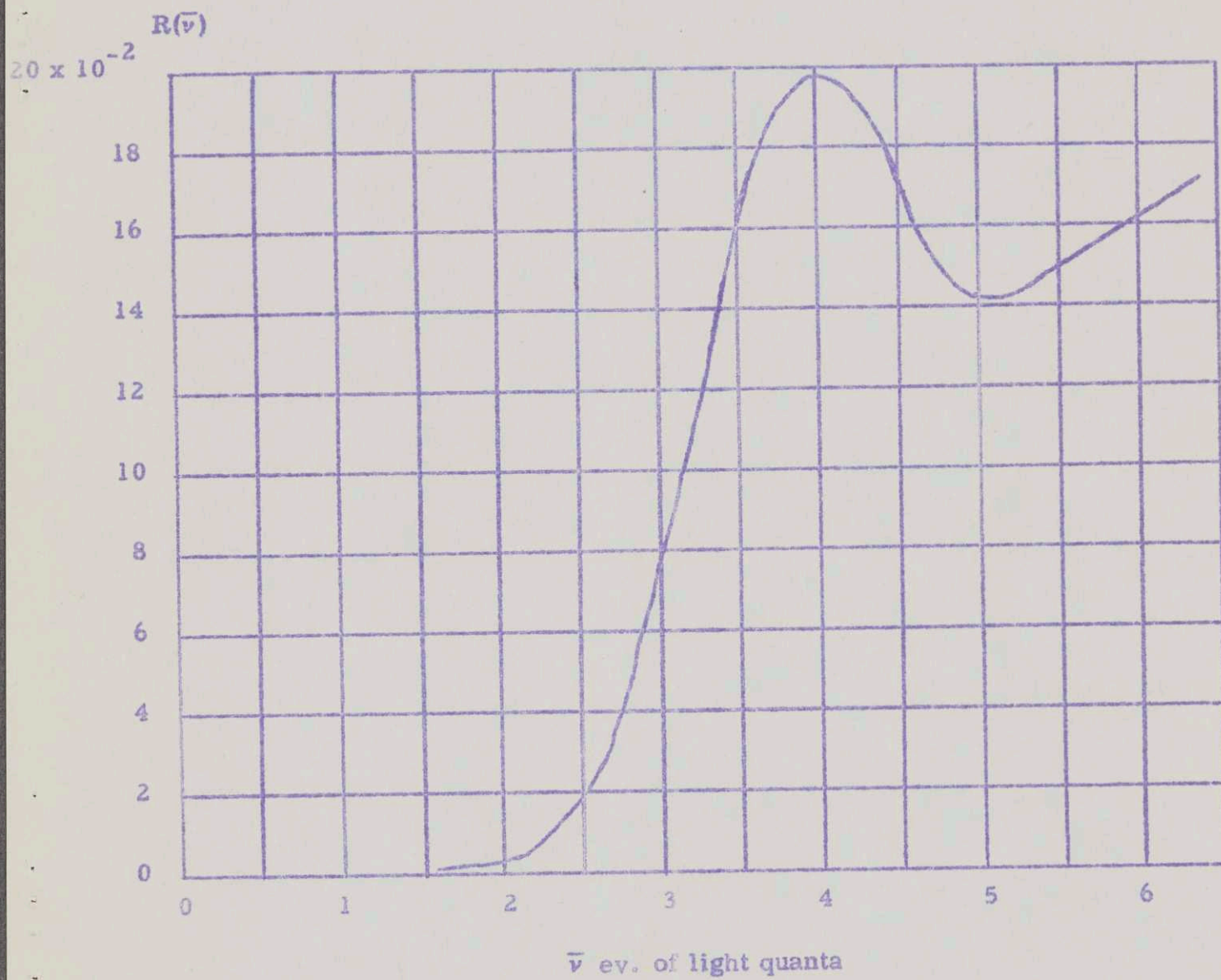
The second item of factual information needed is the quantum yield response characteristic of the photoemissive surface. There is no universally accepted curve that represents the quantum yield data for cesium antimonide. The curve which will be used for this analysis is from Apker, Taft, and Dickey referred to on page 1 of the problem proposal and shown by Weissler as his Fig. 40. This curve is reproduced in Fig. 4 and is represented by the generalized symbol $R(\bar{\nu})$. The reader should be reminded that this quantum yield represents the saturation emission from a photosurface as a function of the quantum energy and therefore does not represent the yield which can be obtained from the surface in the presence of a retarding field. In other words the energy distribution of the electrons as they are emitted from the surface must be taken into consideration in calculating the transmission of the photoelectrons over some barrier whether or not that is created by space charge or is the result of the application of retarding potentials as is the case for the photoemissive solar energy converter.

The curves shown by Weissler in his Fig. 50 are examples showing that the number of high energy electrons available from a cesium antimonide surface is in reality quite small. It will be an optimistic approach, therefore, if we assume for the purpose of this calculation that the energy distribution is uniform rather than rich in slow electrons.

Details of the band structure in the neighborhood of the surface of a cesium antimonide photoemissive unit are not well established. It is this writer's opinion that the manufacturing process carried through to optimize the photoelectric emission from such a surface leaves the interior material p-type in that there are a small number, perhaps not more than 10 or 20 per million, of the cesium lattice sites which are not occupied and therefore represent acceptor centers which create a "p" type of semiconductor. In the immediate neighborhood of the surface, presumably not more than 5 or 10 atom layers deep it is assumed that excess cesium converts this region into "n-type" material and that at the borderline these two types there is a "built-in" p-n junction which accelerates electrons generated within the junction by the absorption of light energy toward the surface and this accounts qualitatively for the high photoelectric yield of this compound. In spite of this complexity it is nevertheless the true work-function defined as the energy difference between the Fermi level and the surface potential that determines the distribution of the motive field in the space between the emitter and the collector electrode system. It is assumed for these calculations that this true work-function value will be 1.7 ev. This value cannot be determined in the conventional manner by assuming that it should be equal to the absolute minimum quantum limit sometimes expressed as the "long wavelength limit" of the surface. The precise choice of 1.7 is therefore arbitrary and could be in error by approximately ± 0.1 ev. This true work-function will be identified by the symbol ϕ_1 .

In the configuration described by Fig. 1, a retarding potential exists in front of the emitter which depends not only on the output voltage of the energy converter but also depends on the two coordinates x and y . The x coordinate represents the distance away from the emitter surface, and the y coordinate the distance parallel to the surface but perpendicular to the z direction which is taken to be along the axis of the collecting wires. Thus the potential function between the emitter and the collector depends on the three variables x , y , and V where V is the difference in potential between the Fermi level

Fig. 4 $R(\bar{\nu})$ = quantum yield response of Cs_3Sb in electrons/quanta.



of the emitter and the Fermi level of the collector.

In the neighborhood of the collecting wires there will be a line shown qualitatively in Fig. 5a which represents the maximum in the motive function for an electron as a function of x and y for a particular value of V which shows the location of the "ridge" of the motive function. On the left-hand side of this ridge line, the field acting on the electron is retarding, that is, it tends to send it back to the emitter. On the right-hand side of the ridge, the field accelerates the electron on its way. If an electron crosses the ridge line then the motive field there tends to carry it to one of the collector wires. Along the ridge line shown by the dots in Fig. 5a, the motive function has the minimum retarding value at the exact center line and has its highest retarding value as it intersects with the high work-function left-hand face of the collecting grid system shown here to have a value of $\phi_2 = 4.7$ ev. The backside of this surface is characterized by the optimistically low work-function for cesium silver oxide of 1.2 ev.

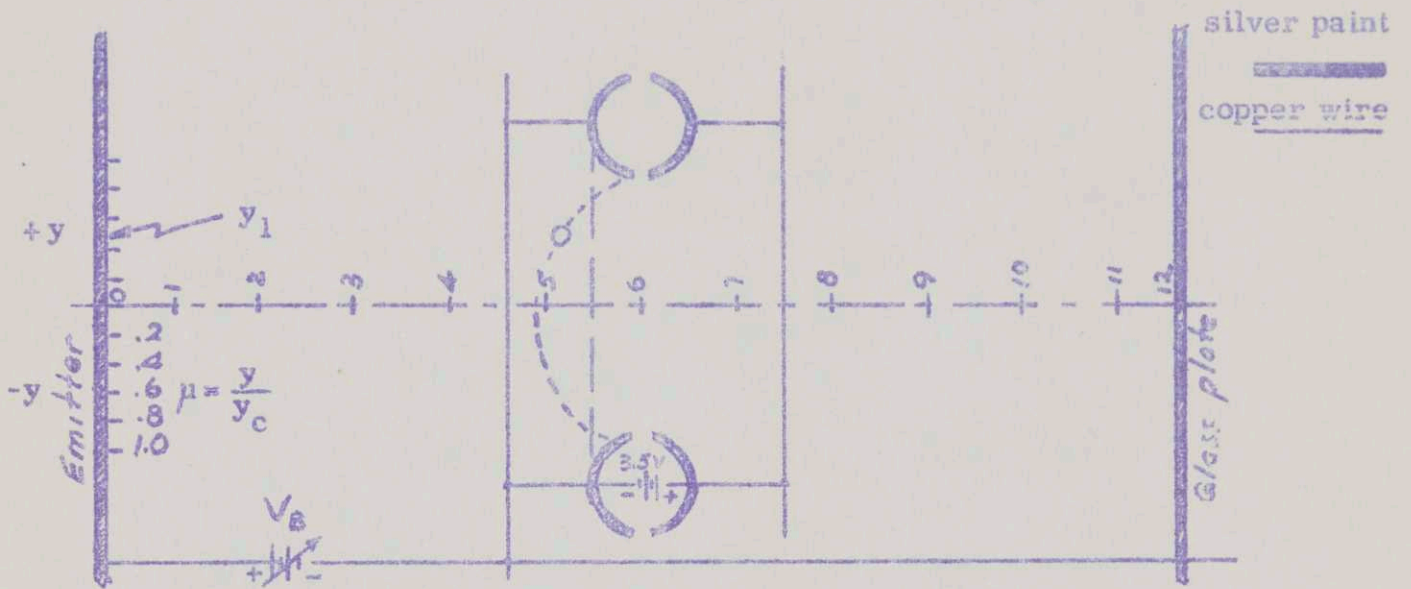
For all values of the output potential the motive function for an electron at the ridge is always lowest at $y = 0$, and rises symmetrically from this value in both the plus and the minus directions of y until it intersects the surface potential of the collector at the corresponding value of y identified here as y_c . A simplifying assumption which is to be used in the calculations is that an electron which originates at some point y_1 as shown in Fig. 5a will have to traverse the ridge potential at the corresponding point indicated in the diagram by the small circle on the dotted line. This assumption clearly ignores the statistical nature of the motions of electrons which leave the emitter with transverse components of momentum. It also ignores the fact that an electron which originates at y_1 will be acted upon by transverse fields. If this approximate analysis indicated that a photoemissive energy converter were at all likely to be a worthwhile device, then means are available for the more correct investigation of electron trajectories and this slightly erroneous assumption could be removed from the analysis. We will define a symbol V_y which is to represent the ridge potential for an electron at any distance y from the center line shown in Fig. 5a. This potential is relative to the surface potential of the emitter. The total range of interest is from $y = 0$ to $y = y_c$.

With this description of the symbols to be used the following expression gives the electron current per unit area available from a small element of the surface of unit length in the z direction and of width dy at y . For the quantum energy range between $\bar{\nu}$ and $\bar{\nu} + d(\bar{\nu})$. In this expression y_s is the half distance between the centers of the collector wires.

$$di = q T(\bar{\nu}) P(\bar{\nu}) R(\bar{\nu}) \frac{\bar{\nu} - \phi_1 - V_y}{\bar{\nu} - \phi_1} \frac{dy}{y_s} d\bar{\nu} \quad (4)$$

To obtain the total current density, Eq. 4 must be integrated.

$$I = \frac{q \int_{\bar{\nu}=\phi_1}^{\infty} \int_{y=0}^{y=y_c} T(\bar{\nu}) P(\bar{\nu}) R(\bar{\nu}) \frac{\bar{\nu} - \phi_1 - V_y}{\bar{\nu} - \phi_1} dy d\bar{\nu}}{y_s} \quad (5)$$



Actual Model Size Scale: 5.6 inches/30 mils
Assumed dimension of actual device: emitter-glass spacing 0.030 inch,
collecting grid wires spacing 0.010 inch, grid wire diameter 0.003 inch.

Fig. 5a

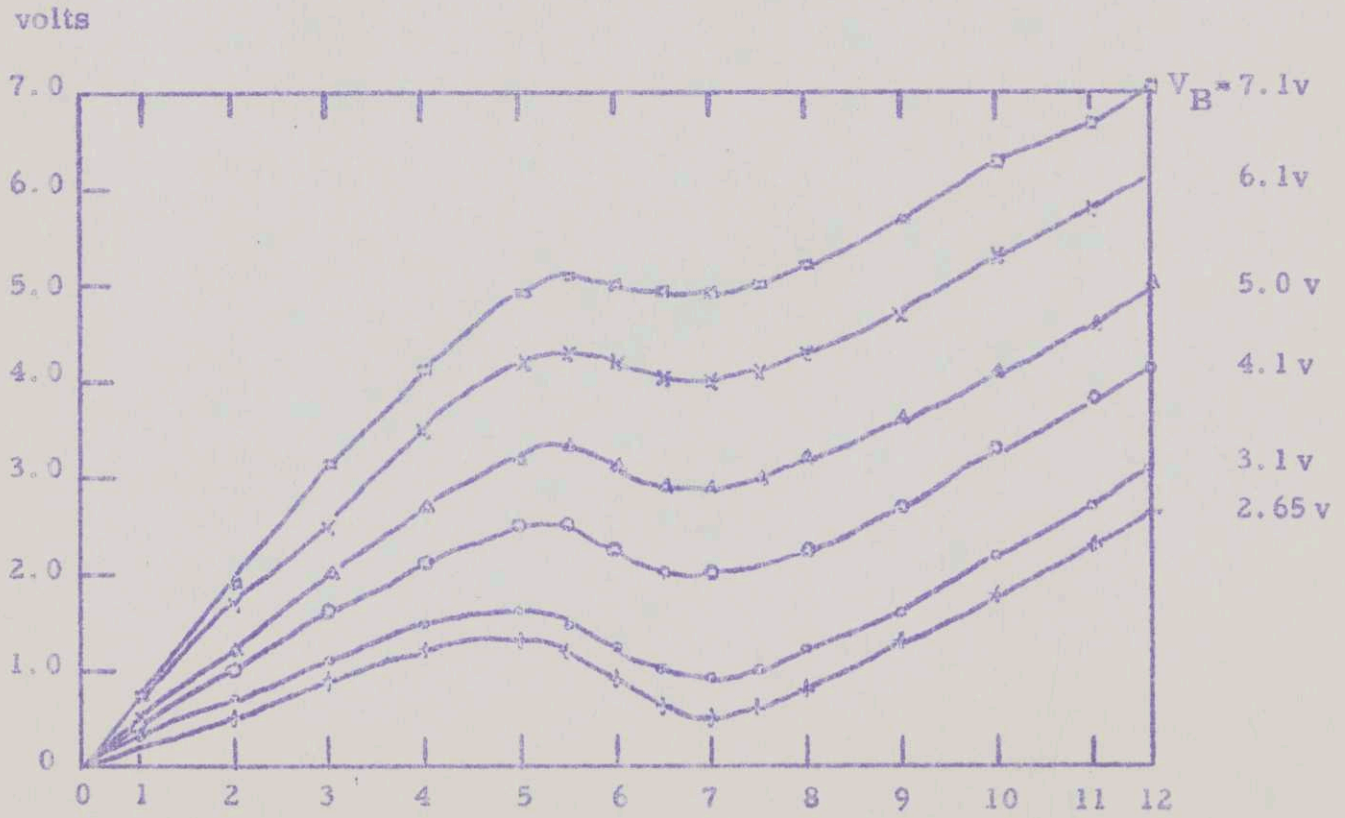
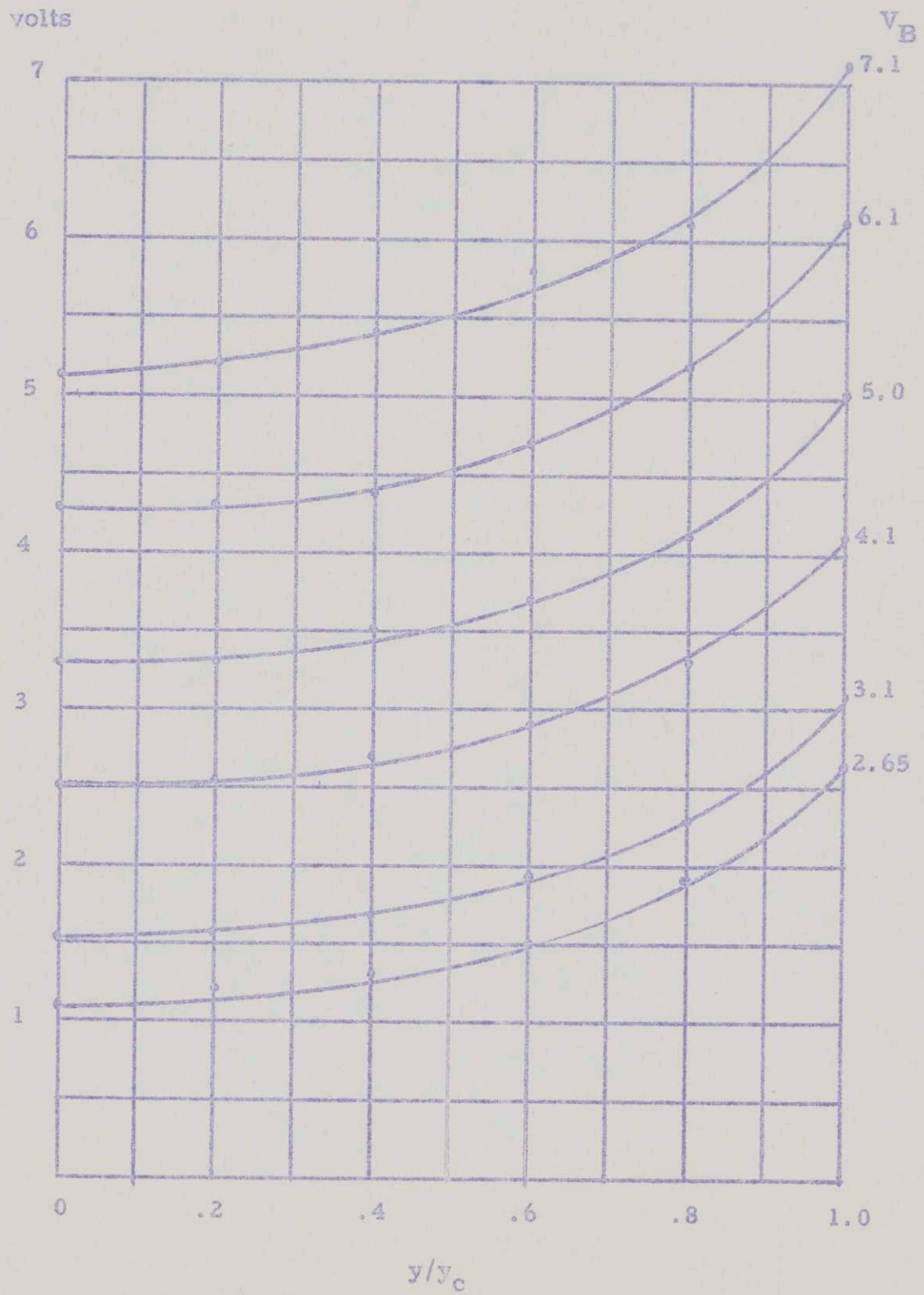


Fig. 5b

Fig. 5c Measurement of potential along the dashed line of Fig. 5a.



Part of the complexity of Eq. 5 comes from the fact that V_y is a function of the applied potential; the difference in work-function between the illuminated and the dark side of the collector wire; the surface potential of the glass wall at the right of the structure; and the coordinate y . Not only does this quantity enter in the integrand but it also enters into the limit of integration as is indicated. The form of the expression has incorporated the concept that the energy distribution of the emitted electrons is uniform. As mentioned before, this is not borne out by experiment but is clearly an optimistic position.

In order to work through to obtain numerical values it will be necessary to discover suitable means for carrying through graphically or otherwise the integration of Eq. 5 for various applied potentials between the Fermi level of the emitter and the Fermi level of the collector. The product of the current-voltage curve thus obtained will represent the maximum possible power obtainable as a function of the output voltage. Clearly this power curve will have a maximum and a comparison can be made between the maximum power density available as indicated by these calculations and the solar power input. The result of this calculation will give an optimistic value for the ultimate efficiency of such a device. The final discussion of this report will indicate that realizable efficiencies will be lower than this optimistic one by a factor probably not less than 5, and not more than 15.

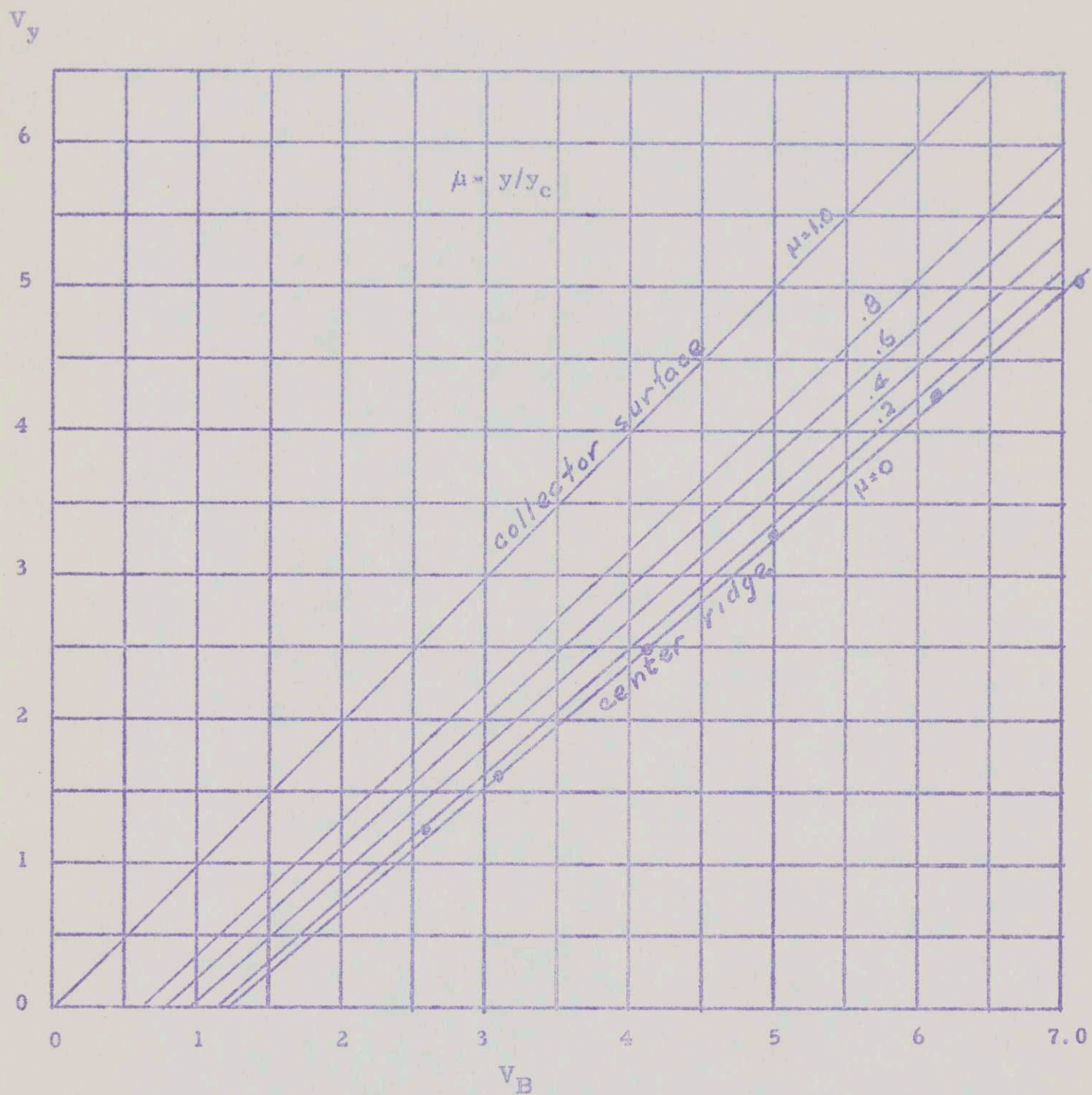
Although the functions $P(\bar{v})$ and $R(\bar{v})$ have already been described in this report and shown in Figs. 3 and 4, the potential function at the ridge of the motive diagram will have to be developed. Fortunately, two of the students, Mr. Keung Luke and Mr. Toby Norris undertook an experiment using "Telladeltos paper"^Δ to map out the approximate distribution of potential in the neighborhood of the ridge. Since this experiment was done in just a few hours, they did not actually plot the ridge values as a function of their applied potentials. Nevertheless, they did obtain a set of potential curves which permit one to make a very good estimate of the ridge values. Figures 5b and 5c represent the data actually obtained. In the next section, the procedure by which I applied these data will be discussed in more detail.

In summary of this section, it is Eq. 5 which must be integrated by some satisfactory approximate method in order to obtain the voltage-current curve needed to calculate the maximum power available from this device. The next section will deal with the actual procedure used by me in preparation for the answering of the proposition given in B of the "expected results."

Computational Procedure to Obtain Numerical Results

The first step in this procedure involves the interpretation of the Luke-Norris data. Inspection of the curves allows one to make a plot of the observed potential at the ridge line as a function of the applied potential used in their experiment and designated as V_B . These data are plotted as shown in Fig. 6. The straight line that represents these data is identified as "center ridge." A second straight line on this plot is that identified as "collector surface." It is now assumed that as a function of the variable y the other ridge values will lie also on straight lines that fall between these two limits. The straight lines chosen for equal increments in y are also plotted in Fig. 6. A reanalysis,

^Δ Telladeltos paper. Manufactured and distributed by Western Union for use in their Telefax machines for facsimile reproduction and also available for purchase in rolls and sheets of various sizes. Western Union, Government and Contract Service Division, Rm. 1725, 60 Hudson St., New York, N. Y.



V_y is the potential relative to emitter surface at saddle point.

Fig. 6

experimentally, could be made to establish the correctness of my choice of additional ridge points. Any error introduced here as a result of the arbitrariness of my relations will undoubtedly have a negligible influence on the final calculation since other approximations are less likely to be strictly valid. In a generalized way, the equation form that represents all of these data follows:

$$V_y = \alpha_y (V_B - V_{by}) \quad (6)$$

The two new symbols here are α_y which is the slope of each of the lines shown in Fig. 6 and is a function of y and also V_{by} which is the intercept of each of the lines on the abscissa and again is a function only of y . Figures 7a and 7b show the abscissa in terms of the dimensionless parameter (y/y_c) and the corresponding values of α_y and V_{by} .

There is a direct relation between the applied potential V ; the work-function of the emitter surface; the high work-function value on the collector; and the potential V_B . This relation is given by:

$$V + \phi_2 = V_B + \phi_1 \quad (7)$$

Equations 6 and 7 may be combined together to yield

$$V_y = \alpha_y (V + \phi_2 - \phi_1 - V_{by}) \quad (8)$$

The reader may be reminded now that α_y and V_{by} are functions of y only, and therefore the expression in Eq. 8 gives the difference in potential between the surface of the emitter and the ridge point located at a specific value of y for any chosen value of applied potential V . The fact that the two quantities α_y and V_{by} are well enough represented by a quadratic function of y is purely an empirical result which assists in the integration to be worked out later. Graphically the functional dependence is shown in Figs. 7a and 7b.

Inspection of Eq. 5 shows that one of the steps in the procedure is to form the product curve which is $P(\bar{v})R(\bar{v})$ and this is shown in Fig. 8. Along the right-hand side of the actual curve there is a straight line which, as is evident, is a reasonable and yet slightly optimistic representation of the actual curve between the limits of 3.2 volts and 4.7. The use of this linear representation instead of the actual curve is confined between these two energy limits. Obviously, this line must be used within this range only and no extrapolation of any kind outside of the range will have any meaning. The equation for this line follows:

For range $3.2 \leq \bar{v} \leq 4.7$ only

$$Y = P(\bar{v})R(\bar{v}) = 2.7 \times 10^{15} (4.7 - \bar{v}) \quad (9)$$

Since Eqs. 4 and 5 both require multiplication by the charge of the electron to convert them to current density, this will be done at once to give the numerical expression as:

$$qP(\bar{v})R(\bar{v}) = 4.3 \times 10^{-4} (4.7 - \bar{v}) \quad (10)$$

Fig. 7a Slope as a function of distance.

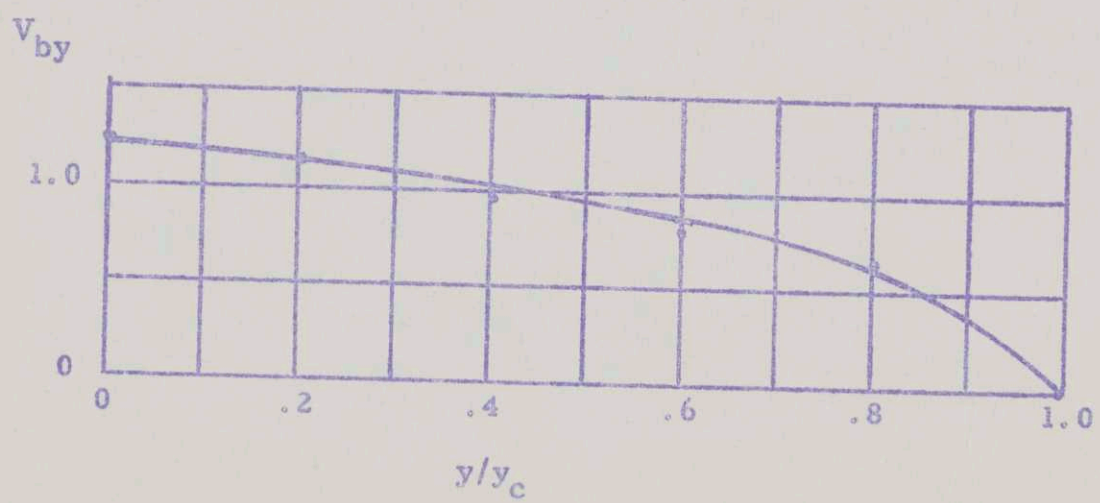
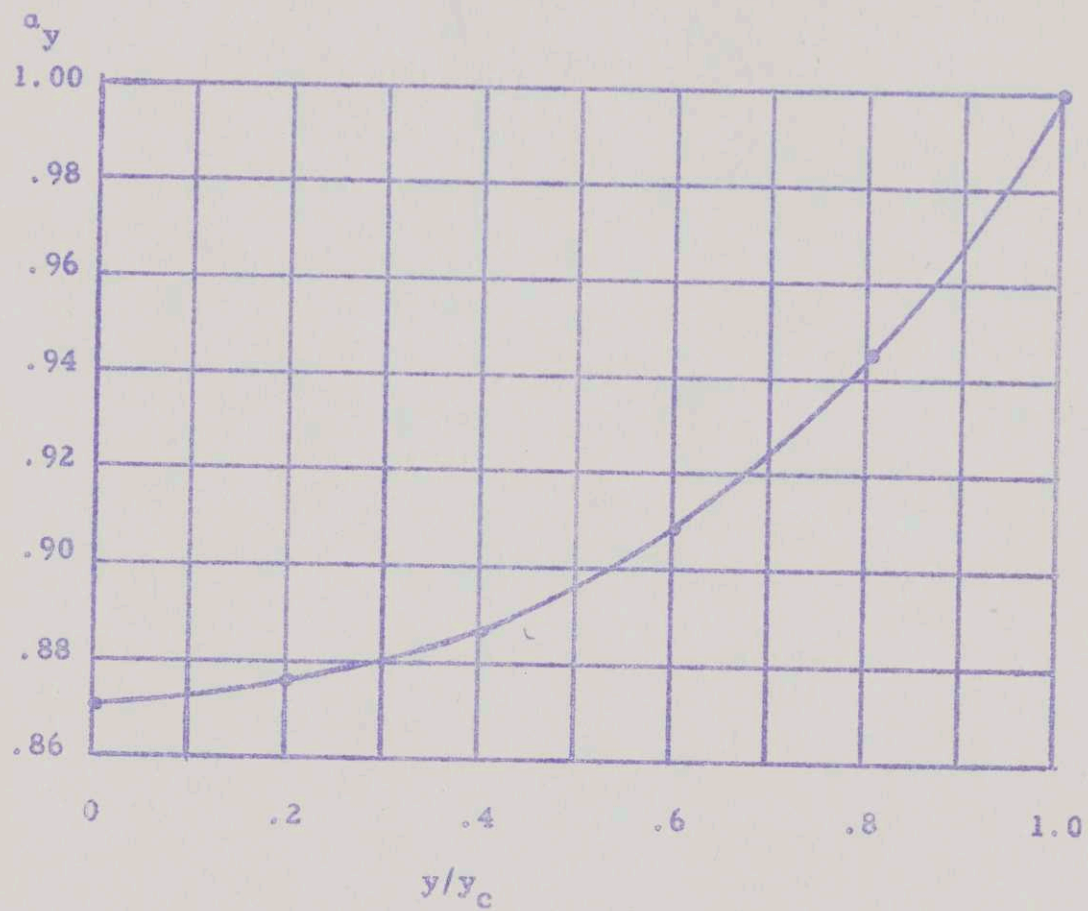
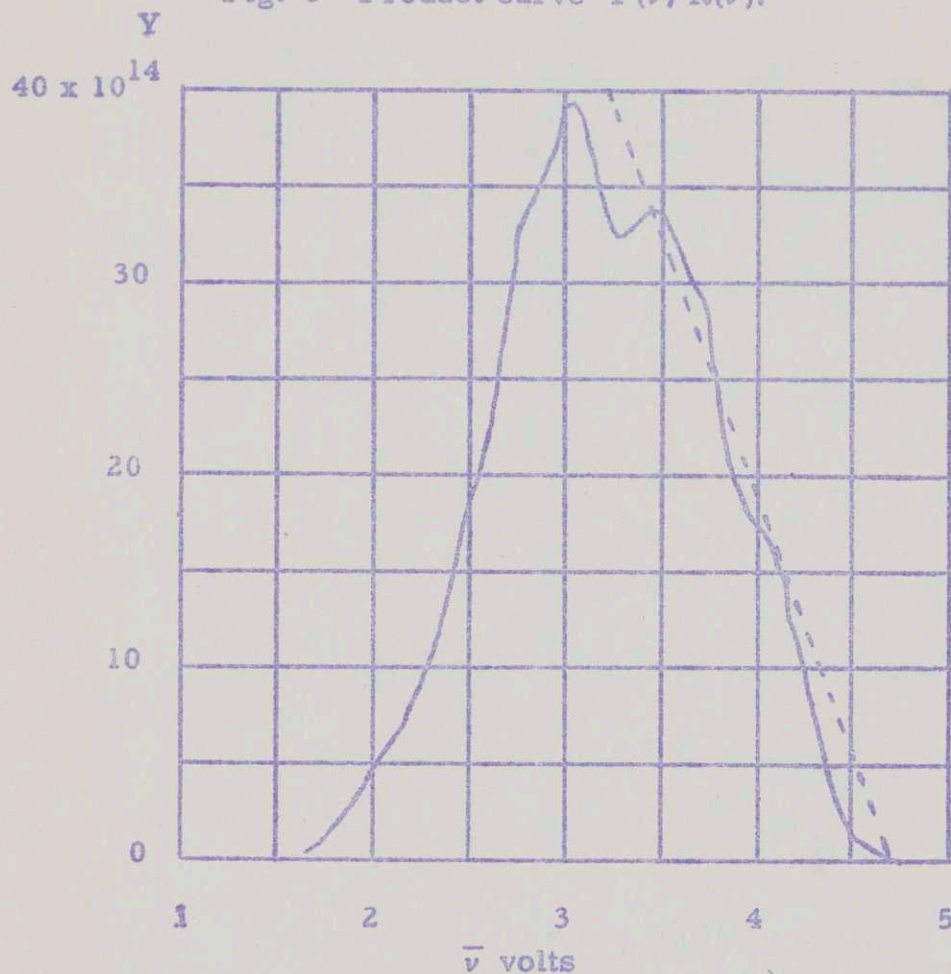


Fig. 7b

Fig. 8 Product curve $P(\bar{v}) R(\bar{v})$.

The quadratic functions referred to above as representations of Figs. 7a and 7b are:

$$a_y = 0.87 + 0.13 \left(\frac{y}{y_c}\right)^2 \quad (11)$$

$$V_{by} = 1.2 - 1.2 \left(\frac{y}{y_c}\right)^2 \quad (12)$$

For the purpose of this calculation it has been assumed that ϕ_1 is equal to 1.7, and ϕ_2 is equal to 4.7. After these values have been introduced into Eq. 8, it may be combined with Eqs. 11 and 12 to give the final relation between the applied voltage V and the ridge point potential V_y as evaluated at any distance y between the center line and the surface potential of the collector located at y_c

$$V_y = (1.566 + 0.87 V) + (1.278 + 0.13V) \mu^2 + 0.156 \mu^4 \quad (13)$$

As explained above, the approximation represented by Eqs. 9 or 10 that the maximum value of \bar{v} is 4.7 and that the effective work-function ϕ_1 is 1.7 volts, means that the maximum value of V_y of Eq. 13 is 3 volts. It follows, therefore, that for any choice of (y/y_c) there is a corresponding maximum value of V which will satisfy Eq. 13 when the maximum value of V_y is 3 volts. The equation form which satisfies this relation is the following:

$$V_{\max} = \frac{1.434 - 1.278 \mu_{\max}^2 - 0.156 \mu_{\max}^4}{0.87 + 0.13 \mu_{\max}^2} \quad (14)$$

The symbol μ_{\max} is defined by

$$\mu = \frac{y}{y_c} \quad \text{and} \quad \mu_{\max} = \frac{y_{\max}}{y_c} \quad (15)$$

The use that one makes of the solution to this equation will follow the pattern that for any arbitrary choice of voltage output V for the converter between the range of zero and 1.65 there is a corresponding limit to the range in y that can be used before the value y_{\max} is reached. The curve in Fig. 9 shows this relation to indicate that for an output voltage of 0.6 volt, the maximum value of μ is 0.795. At this value of μ and the chosen value of $V = 0.6$, Eq. 13 gives the limiting value of V_y of 3 volts.

It is now possible to introduce Eq. 10 into Eq. 5 and set in the proper limits of integration so as to give an expression for the photocurrent which can be collected on the electron collector of the photoemissive solar energy converter. This equation is:

$$I = 4.3 \times 10^{-4} \left(\frac{y_c}{y_s} \right) \int_0^{\mu_{\max}} \int_{\bar{v}=1.7+V_y}^{\bar{v}=4.7} (4.7 - \bar{v}) \left(1 - \frac{V_y}{\bar{v} - 1.7} \right) d\bar{v} d\mu \quad (16)$$

This equation is readily integrated with respect to \bar{v} and after the limits of integration have been inserted, it takes the form:

$$I = 3 \times 10^{-4} \int_0^{\mu_{\max}} (3 V_y \ln V_y + 4.5 - 3.3 V_y - 0.5 V_y^2) d\mu \quad (17)$$

Since V_y is a function only of the output voltage V and the parameter μ as given in Eq. 13, it is possible to integrate Eq. 17 for arbitrarily chosen specific values of the output voltage V . The first term of Eq. 17 offers difficulty in terms of its direct explicit integration. The other terms can be integrated directly. On this basis, it would be possible to integrate the first

μ_{\max}

$\mu_{\max} = f(v)$

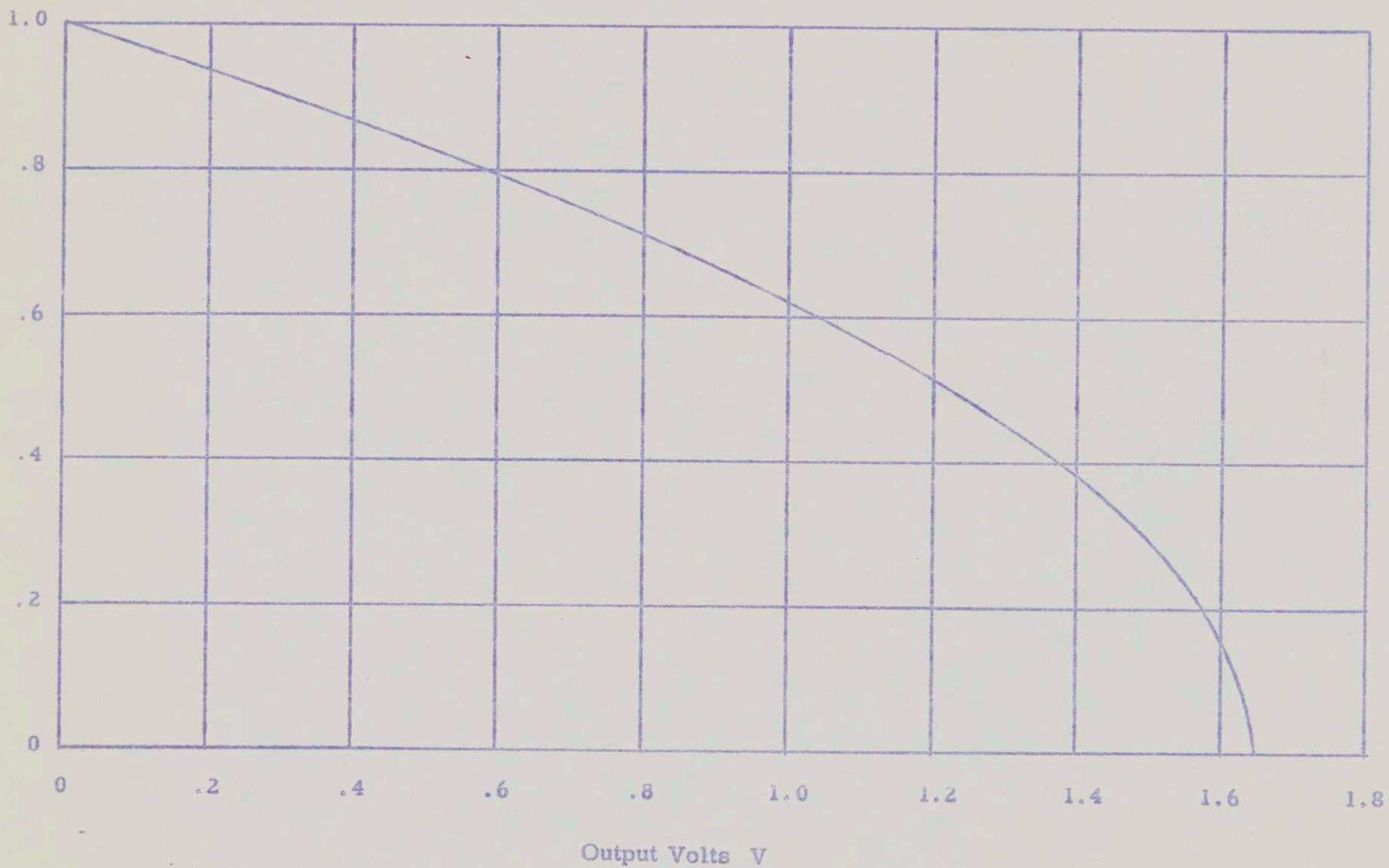


Fig. 9

term by graphical means and the other terms directly. This method leads to the difficulty that the integral of the positive term is only slightly greater than the integral of the negative terms. Since the integrand itself can be expressed to the necessary accuracy at specific values of μ it is more convenient to plot the total integrand as a function of μ and then integrate this curve graphically. The graph produced in this manner applicable to an output voltage of zero is shown in Fig. 10. Even though the curve shown in Fig. 9 indicates that the legitimate range of integration extends to $\mu = 1$, the value of the integral beyond $\mu = 0.8$ is zero. The total value of the integral is 9.34×10^{-2} which when combined as shown in Eq. 17 indicates a current density of 28.0×10^{-6} amp/cm² as the optimistic yield of photocurrent for the device with an output voltage of zero.

Also shown on Fig. 10 is the curve that gives the current as a function of μ with an output voltage of 0.3 volt. This curve is shown because upon integration, it shows that the current density is 12.4×10^{-6} amp/cm² and therefore corresponds to a power available of 3.7 microwatts/cm². Thus the efficiency at the maximum is 0.0027 per cent.

The above procedure was followed for a sufficient number of calculated output voltages so that the current-voltage curve could be computed as shown in Fig. 11 by the solid line. Since the current and voltage are both known, it is therefore possible to calculate the power available as a function of the voltage and it is seen that the maximum comes at 0.3 volt.

General Discussion

Although it is thought that the above calculations are quite realistic, there are a number of points that need additional explanation. Some of these will be discussed under the specific subheadings below.

Space Charge

Since the minimum value of \bar{v} that can deliver electrons over the ridge line, even with an output voltage of zero, is 3.3 volts, solar radiation in the range 1.7 to 3.3 can inject electrons into the space outside of the emitter, but all of these will be returned to the emitter. This wavelength range is from 380 to 740 millimicrons. This is a range rich in quanta from a solar source which is not only useless to the energy converter but also definitely detrimental because of the space-charge cloud which it can generate. Such a cloud will let the ridge line rise to slightly higher values than those calculated when space charge is neglected. Statements have been made that filtering can minimize this effect but it would be difficult to obtain a filter which would pass all wavelengths shorter than 370 millimicrons effectively and at the same time cut out the longer wavelength range.

Conductivity of the Emitter Surface

One of the descriptions of this type of solar energy converter indicated that a grid system of conducting material would be put onto the emitter support in order to reduce the power lost there. Calculations show that considerable power would be lost unless some additional conducting surface were placed under the photoemitter. Such a surface would have to have a high transparency in the ultraviolet and yet from the limited information available it seems as though a tin oxide surface as suggested would absorb a considerable number of light quanta in this useful ultraviolet range. The neglect of this factor in the calculations also results in a more optimistic evaluation of the converter than can be realized in practice.

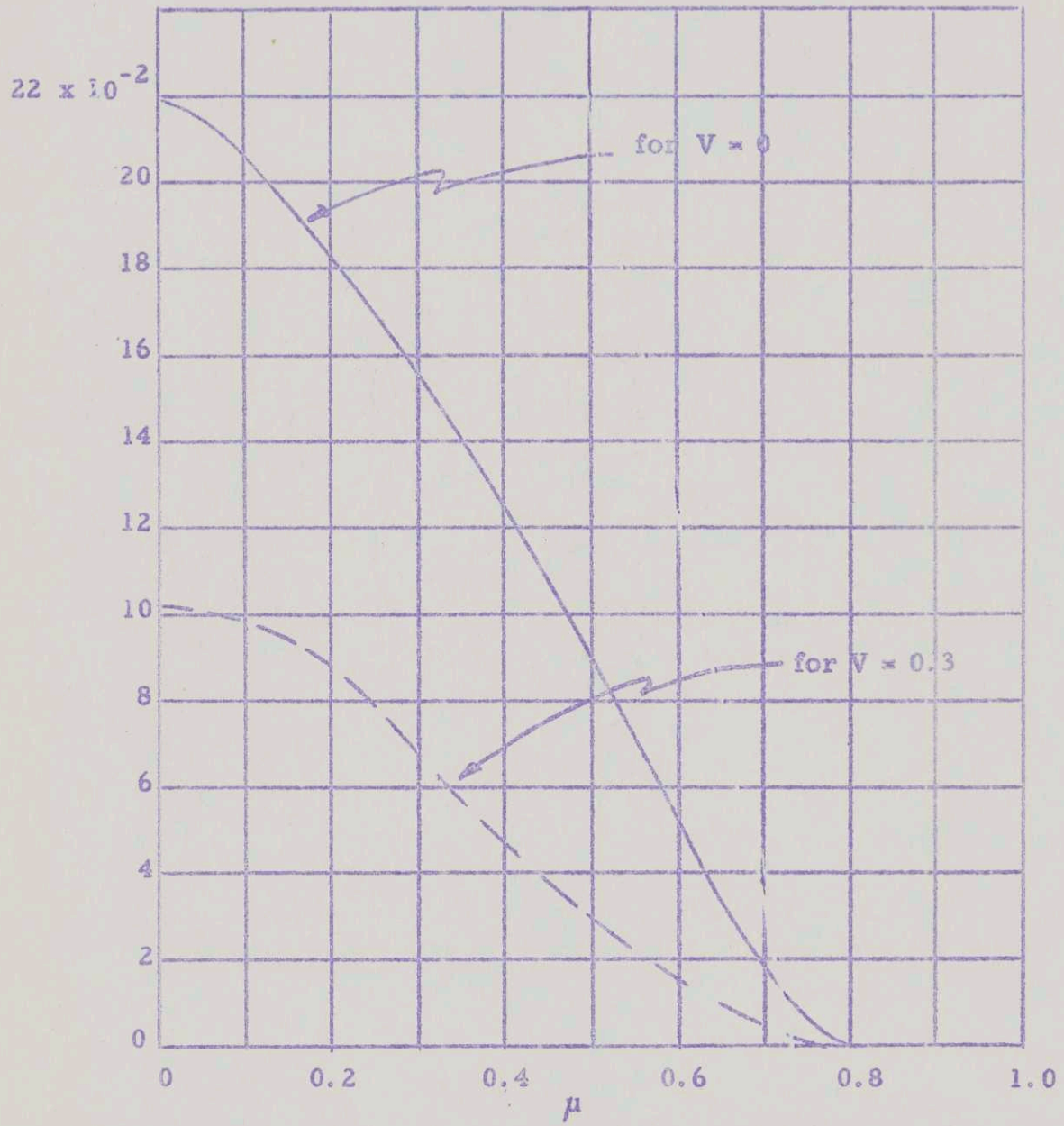
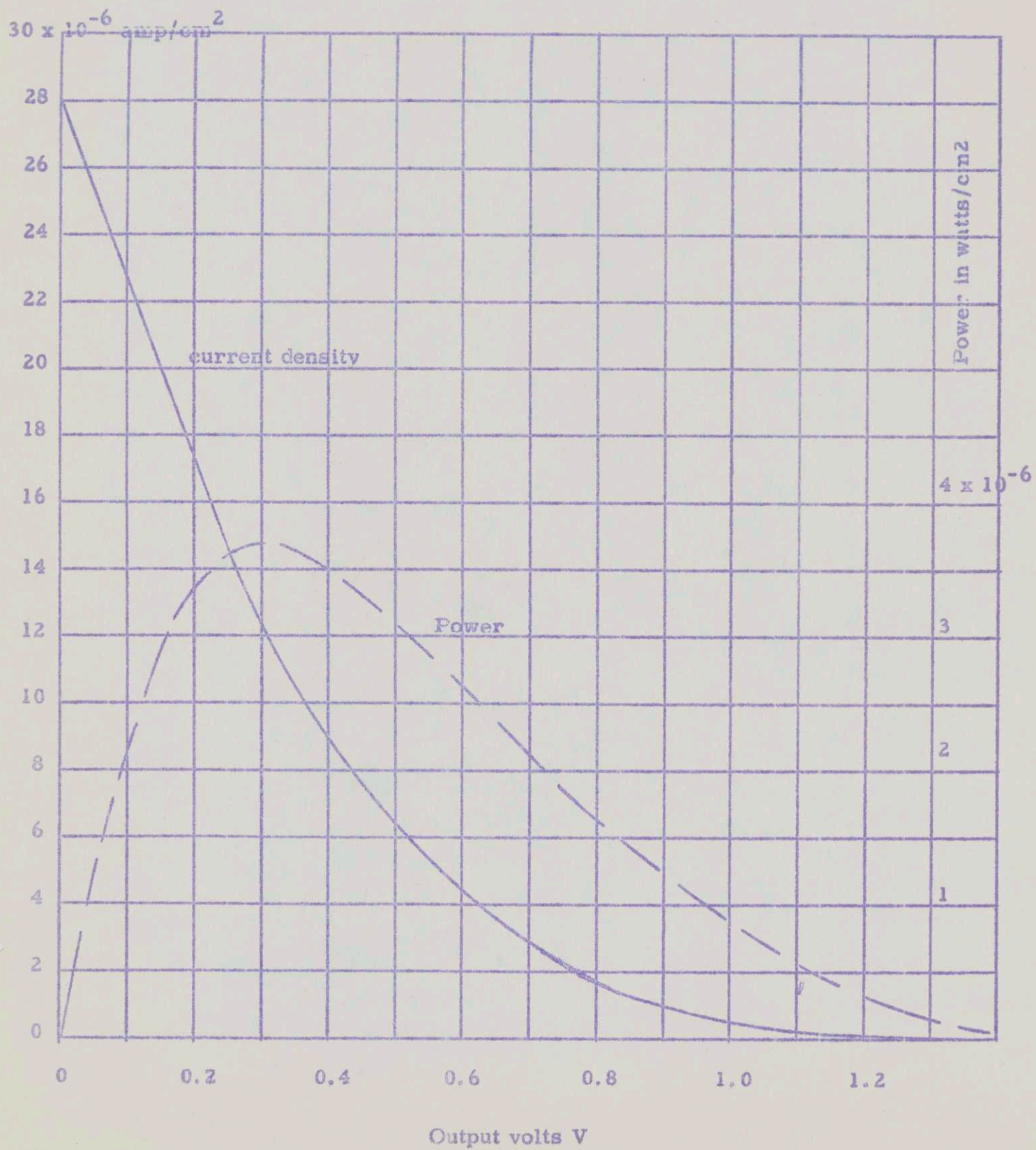


Fig. 10

Fig. 11



Electron trajectories

Even with the simplifying assumptions made, the calculation of a reasonably accurate current-voltage curve has been difficult. The approximation made was that transverse motions were neglected. Whether or not this assumption makes for a more or less optimistic result is difficult to state with certainty. If a device of this general type held more promise than would be indicated by these calculations, means are available to evaluate the influence of this simplifying approximation. It is doubtful that the error introduced as a result of it is more than a factor of 2 either way.

Photoemissive Surface

The most obvious area for exploitation to make a photoemissive converter more efficient lies in the area of the photoemitter itself. The requirement here is principally that the very high quantum yield associated with cesium antimonide or related compounds should be maintained. Some lowering of the work-function might be helpful. The likelihood of a major advance in this area of research that would increase the efficiency even as much as a factor of 10 is so small that effort directed here as it applies specifically to a solar energy converter is hardly worthwhile. At most it might bring the efficiency up to 0.03 per cent.

Effort expended in this direction purely from the point of view of a better understanding of the electron physics of this complex type of photoemissive surface could very well be worthwhile.

Work-Function of Photoelectron Receiver

The lower the work-function of the dark side of the electron receiver, the stronger its influence will be to encourage electrons to come between the collector wires and become collected. Because this surface is certain to receive some light, it is important that it should be low work-function, low photoelectric-yield material. For the calculation, it has been assumed that a work-function of 1.2 volts is possible. Such a low value is indeed optimistic. In the calculation it was assumed that no light would fall on the receiving surface. This favorable situation would also be impossible to realize. The work-function of the illuminated side of the collector was taken as 4.7 volts which would be close to that of a clean surface even though a lower value than this would be more favorable. It would be very difficult to maintain the necessary high work-function in the presence of cesium-covered surfaces for long periods of time. Even though a test specimen might be produced with a work-function close to 4 volts, it could not be expected to maintain this value indefinitely. As the work-function becomes reduced, its photoelectric yield will increase and the back current will bring about a decrease in overall efficiency.

Summary of 8.21 Class Results

This problem was assigned to the group of fourteen students interested in the physics of electronics. They were instructed to use approximate methods to solve the problem "optimistically." In general, the approximations used were more optimistic and perhaps less exact than the solution outlined in this report. Of the fourteen final results, seven showed efficiencies of 0.006 per cent and less. Seven others were slightly above this value but not significantly. The output voltage obtained in their calculations averaged close to 0.45 volt. These values may be compared with mine of 0.3 for output voltage and 0.0027 per cent for efficiency.

CONCLUSIONS

My present calculations are described in considerable detail to serve two purposes. The first has been to make available to the students of the 8.21 class a detailed solution to the problem which they, themselves, had the opportunity to evaluate. The second purpose of this report is to make it available to interested parties outside of the 8.21 class who have had occasion to study the feasibility of a photoemissive solar energy converter. Clearly, the answers given here are unfavorable, first from the point of view of its overall efficiency, and second, we must consider that the construction of a device of this kind capable of producing more than milliwatts of power would offer practically insurmountable constructional difficulties.

Since the above analysis does not claim to be more than a thoughtful study of the particular model of a photoemissive solar energy converter, correspondingly critical comments concerning the basic assumptions and methods of calculation will be welcome by the writer.

Table 1

Solar Quanta/cm²-sec-Volt as a Function of Quantum Energy $\bar{v} = h\nu/q$

\bar{v} volts	P(\bar{v})	\bar{v} volts	P(\bar{v})
1.5	2440 x 10 ¹⁴	0.6	185
1.6	2270	0.7	155
1.7	2110	0.8	130
1.8	1965	0.9	110
1.9	1835	4.0	85
2.0	1680	4.1	70
2.1	1550	4.2	55
2.2	1380	4.3	35
2.3	1225	4.4	23
2.4	1065	4.5	19
2.5	1010	4.6	13.5
2.6	960	4.7	10.5
2.7	860	4.8	8.5
2.8	705	4.9	6.7
2.9	620	5.0	5.0
3.0	535	5.1	4.0
0.1 3.1	400	5.2	3.1
0.2 3.2	315	5.3	2.4
0.3 3.3	270	5.4	2.0
0.4 3.4	235	5.5	1.6
0.5 3.5	205	5.6	1.4

To Be Added To: "A Quantitative Analysis of A Photoemissive Solar Energy Converter" by Professor Wayne B. Nottingham.