Report 2628
ACOUSTIC RADIATION FROM A DRIVEN, COATED INFINITE PLATE BACKED BY A PARALLEL INFINITE BAFFLE

NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Washington, D.C. 20007

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Naval Ship Research and Development Center
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ACOUSTIC RADIATION FROM A DRIVEN, COATED INFINITE PLATE BACKED BY A PARALLEL INFINITE BAFFLE

by

G. Maidanik

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Acoustic Radiation from a Driven, Coated Infinite Plate Backed by a Parallel Infinite Baffle

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The far-field acoustic pressure generated by an infinite plate driven by a point and a line force are computed. This paper constitutes an extension of previous computations [G. Maidanik, J. Acoust. Soc. Am 42, 27–31 (1967)]. The plate considered in this paper is covered uniformly and completely by a compliant coating which is applied in turn to the front and back of the plate. The formalism accounts for different fluids' occupying the space in front of the plate and in the cavity back of the plate.

INTRODUCTION

In a recent paper,[1] the far-field acoustic radiation from a driven infinite plate backed by a parallel infinite baffle was computed and discussed. In these computations, the influence of fluid loading and the presence of the baffle were of particular interest. The results indicated that under certain conditions fluid loading and the location of the baffle and its acoustic reflective properties could have considerable effect on the far-field acoustic pressure. In the present paper, we wish to extend these previous computations to take account of a situation where the plate is covered completely and uniformly by a compliant coating either on the front surface of the plate or the back surface of the plate so that the coating faces the baffle.

To avoid repetition, consistent with clarity, it is assumed that the reader is familiar with the formalism and discussions developed in Refs. 1 and 2. We concentrate primarily on those features of the computations that are directly associated with the coating. As in Refs. 1 and 2, we restrict consideration primarily to the frequency range below the critical frequency.[2] We further restrict consideration to coatings possessing negligible inertial impedance so that the spectral force fields that develop on its surfaces can be written

$$Z_c(\mathbf{k},\omega)[V_c(\mathbf{k},\omega)-V_p(\mathbf{k},\omega)] = P_{cp}(\mathbf{k},\omega) = -P_{pc}(\mathbf{k},\omega), \quad (1)$$

where $V_c(\mathbf{k},\omega)$ is the spectral velocity field on one of the coating surfaces; $V_p(\mathbf{k},\omega)$ is the spectral velocity field on the other coating surface; $Z_c(\mathbf{k},\omega)$, the spectral surface impedance of the coating; $P_{cp}(\mathbf{k},\omega)$, the spectral force field on the surface of the coating whose velocity field is $V_c(\mathbf{k},\omega)$ (FORCE FIELD≡PRESSURE); $P_{pc}(\mathbf{k},\omega)$, the spectral force field on the surface of the coating whose velocity field is $V_p(\mathbf{k},\omega)$; ω, the frequency variable in radians per unit time, the Fourier conjugate of the temporal variable t; and $\mathbf{k}$, the wave vector on the surface, the Fourier conjugate of the position vector $\mathbf{x}$ on the surface.

We assume that the coating is firmly attached to the plate so that the velocity field $V_p(\mathbf{k},\omega)$ on the plate is also the velocity field of the attached surface of the coating. We also assume that the coating and the plate are thin enough so that only normal (flexural)

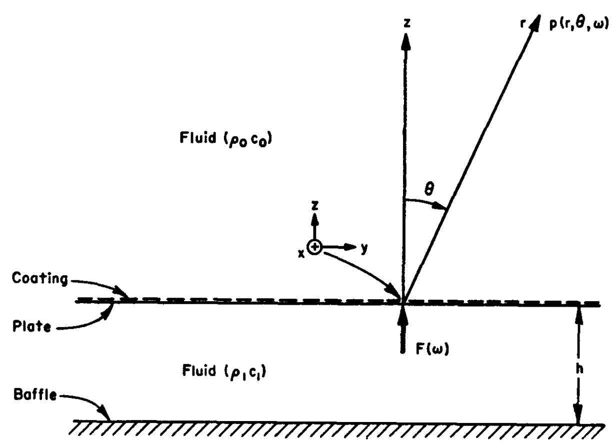

Fig. 1. Front surface-coated infinite plate backed by a baffle.

[1] G. Maidanik, J. Acoust. Soc. Am. 42, 27–31 (1967).
[2] G. Maidanik and E. M. Kerwin, Jr., J. Acoust. Soc. Am. 40, 1034–1038 (1966).

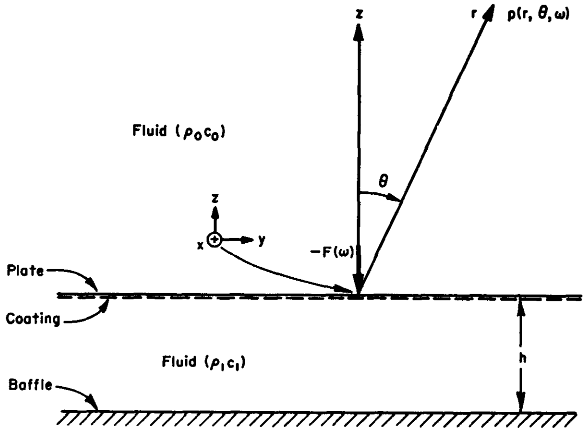

FIG. 2. Back surface-coated infinite plate backed by a baffle.

velocities are of concern. Note also that the coating impedance chosen is of a form such that it does not scatter waves, and thus, it is unable to generate spectral components in the force field that do not exist in the velocity field V_p or V_c; e.g., if the coating were finite or nonuniform, this condition on the impedance could no longer be made.

If we denote by $V_s(\mathbf{k},\omega)$ the spectral velocity field on the surface bounding the semi-infinite space $z>0$ (see Figs. 1 and 2), which is filled with a fluid medium designated by $\rho_0 c_0$ (ρ_0 being the density and c_0 the speed of sound), the frequency spectral acoustic pressure at the field position $\{\mathbf{x},z\}$ in this semi-infinite space is given by[1-3]

$$p(\mathbf{x},z,\omega)=(2\pi)^{-1}\rho_0 c_0 \iint_{-\infty}^{\infty} d\mathbf{k} \times\left\{\left(\frac{k_{z0}}{k_0}\right)^{-1}\exp[-i(\mathbf{x}\cdot\mathbf{k}+zk_{z0})]\right\}V_s(\mathbf{k},\omega), \quad (2)$$

where

$$\mathbf{x}=\{x,y\};\quad \mathbf{k}=\{k_x,k_y\};\quad d\mathbf{k}=dk_x dk_y;\quad |\mathbf{k}|=k,$$

$$k_0=\omega/c_0, \quad (3)$$

$$k_{z0}=(k_0^2-k^2)^{\frac{1}{2}}. \quad (4)$$

The purpose here is to ascertain $p(\mathbf{x},z,\omega)$ in terms of the external force field and the mechanical properties of the plate, the coating, the baffle, and the fluid media (we allow the fluid medium between the plate and the baffle to be different from that occupying the space above the plate, $z>0$). We have thus to establish the relationship in spectral form between $V_s(\mathbf{k},\omega)$ and $P(\mathbf{k},\omega)$ in terms of these mechanical properties; $P(\mathbf{k},\omega)$ is the external force field in spectral form. We assume throughout that the external force field is acting directly only on the plate. (The formalism can, however, be readily generalized to account for force fields that act on the free surface of the coating.)

In Sec. I, we consider the coating to be placed on the front side of the plate so that the semi-infinite space ($z>0$) is bounded by the free surface of the coating. In Sec. II, we consider the coating to be placed on the back side of the plate so that the free surface of the coating faces the baffle.

I. FRONT SURFACE OF THE PLATE COATED

This system is illustrated in Fig. 1. Employing a procedure similar to that by which Eq. 3 of Ref. 1 was derived, we obtain the following expression for the spectral velocity field $V_c(\mathbf{k},\omega)$ on the free surface of the coating in terms of the external spectral force field $P(\mathbf{k},\omega)$:

$$V_c(\mathbf{k},\omega)=P(\mathbf{k},\omega)[\{(Z_p+Z_b)(Z_c+Z_0)/Z_c\}+Z_0]^{-1}, \quad (5)$$

where

$$Z_b=\rho_1 c_1(k_{z1}/k_1)^{-1}[1+R\exp(-2ihk_{z1})] \times[1-R\exp(-2ihk_{z1})]^{-1}, \quad (6)$$

$$Z_0=\rho_0 c_0(k_{z0}/k_0)^{-1}, \quad (7)$$

$$k_{z1}=(k_1^2-k^2)^{\frac{1}{2}}, \quad (8)$$

$$k_1=\omega/c_1. \quad (9)$$

ρ_1 and c_1 are, respectively, the density and speed of sound of the fluid medium occupying the space between the plate and the baffle, R is the acoustic reflection coefficient of the baffle,[1] and h is the distance between the baffle and the plate. The coating and the plate are assumed thin enough that regardless of whether h is measured from the plate or the coating surfaces, negligible variation is introduced in the value of h.

Substituting $V_c(\mathbf{k},\omega)$ for $V_s(\mathbf{k},\omega)$ in Eq. 2 and carrying out the integration under the condition that $(x^2+y^2+z^2)^{\frac{1}{2}}k_0\gg 1$, the frequency-spectral far-field acoustic pressure generated from the free surface of the coating is obtained. The expression for this far-field acoustic pressure is[4]

$$p(r,\theta,\omega)=[k_0\beta_p F(\omega)/2\pi][r^{-1}\exp(-ik_0 r)]D(\theta), \quad (10)$$

[3] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Co., New York, 1953), Chap. 11.

[4] In order to obtain the far-field acoustic pressure from $p(r,\theta,\omega)$, it is necessary to calculate the Fourier transform

$$p(r,\theta,t)=(2\pi)^{-\frac{1}{2}}\int_{-\infty}^{\infty} p(r,\theta,\omega)\exp(i\omega t)d\omega.$$

In the discussion, both in Ref. 1 and the present paper, we are chiefly examining the behavior of the frequency-spectral far-field acoustic pressure $p(r,\theta,\omega)$. These discussions would be wholly inclusive if we assume the external force fields to be pure tone defined by the frequency ω and thus set $F(\omega)=F_0\exp(i\omega t)$, $F_e(\omega)=F_{e0}\exp(i\omega t)$, and $p(r,\theta,\omega)=p(r,\theta,t)$ throughout the papers; where F_0 and F_{e0} are constants.

for a point-force drive,[1] and

$$p(r,\theta,\omega) = -[k_0^{\frac{1}{2}}\beta_p F_e(\omega)/2] \times [r^{-\frac{1}{2}} \exp(-ik_0 r + i\pi/4)] D(\theta), \quad (11)$$

for a line-force drive.[1] In Eq. 10 r and θ are defined in a spherical coordinate system, and in Eq. 11, in a cylindrical coordinate system.[1,2] $F(\omega)$ and $F_e(\omega)$ are the frequency-spectral components of the point force and the line force, respectively.[1,2] The directivity function $D(\theta)$ is given by

$$D(\theta) = \cos(\theta)[\cos(\theta)(1+\beta_p\nu)(1+\beta_c/\cos\theta)+\beta_p]^{-1}, \quad (12)$$

where[5]

$$\beta_p = \rho_0 c_0 / i\omega m, \qquad \beta_c = \rho_0 c_0 / Z_c(k_0 \sin\theta,\omega), \quad (13)$$

$$\nu = \frac{\rho_1 c_1}{\rho_0 c_0 \cos\theta_1} \cdot \frac{1+R(\cos\theta_1,\omega)\exp(-i2hk_1\cos\theta_1)}{1-R(\cos\theta_1,\omega)\exp(-i2hk_1\cos\theta_1)}, \quad (14)$$

$$\theta_1 = \cos^{-1}\{1-[c_1/c_0)\sin\theta]^2\}^{\frac{1}{2}}. \quad (15)$$

The presence of the coating is manifested in the term in Eq. 12 that involves the impedance ratio β_c. Two limiting cases are immediately apparent. If $\beta_c \to 0$, the coating is considered to possess negligible compliance, and thus it becomes essentially indistinguishable from the plate; the directivity function defined in Eq. 12 reduces to the one defined in Eq. 8 of Ref. 1, with $g(\theta)$ as given in Eq. 17 of this reference. Thus, we recover appropriately the previous results in this limiting case.[1] Note that the low compliance required to approximate this limiting case becomes more stringent for those waves in the plate-coating system that radiate into grazing angles. In fact, this criterion is applicable to an uncoated plate; in this case β_c is simply the impedance ratio associated with the compressibility of the plate material and its thickness. It is customary to assume that in plates, $\beta_c = 0$ without further examination. Provided that the plates are thin enough and of material of low enough compressibility, this assumption is adequate for most practical purposes.

The second limiting case is brought about if $\beta_c \to \infty$. In this case, the coating has substantial compressibility. Under this condition, the coating is incapable of transmitting the velocity of the plate to the fluid; and therefore, the system cannot induce an acoustic field in the fluid medium. Indeed, if $\beta_c \to \infty$, the directivity function defined in Eq. 12 becomes substantially zero as expected.

[5] The far-field acoustic pressure may be evaluated for the frequency range above the critical frequency. Gutin [L. Ya. Gutin, Soviet Phys.—Acoust. 4, 369–371 (1965)] has shown that such an evaluation would lead to a result where β in Eq. 7 assumes the expression $\rho_0 c_0 / Z_p(k_0 \sin\theta,\omega)$ instead of $\rho_0 c_0 / i\omega m$. Unfortunately, Gutin has not discussed the condition for the validity of this expression. It appears that the condition for the far field may have to be replaced by $\gamma^2(x^2+y^2+z^2)k_0^2 \gg 1$.

We turn to consider briefly a few additional limiting cases of interest. However, it is not intended to exhaust all such cases in this paper.

A. Back Space a Vacuum

In this case, $\rho_1 = 0$, the directivity function $D(\theta)$ reduces to

$$D(\theta) = \cos(\theta)[\cos(\theta)+\beta_c+\beta_p]^{-1}. \quad (16)$$

This result has been known for sometime; it was computed previously by Maidanik and Kerwin.[6] Of particular interest is the resonance condition in Eq. (16). Below the critical frequency β_p is the fluid-loading parameter associated with the surface-mass impedance of the plate. The parameter β_c is the fluid-loading parameter associated with the surface impedance of the coating; this impedance we assumed to be substantially stiffness in character. (This statement holds provided that we assume the coating to possess a loss factor[2] that is small as compared with unity.) Thus, β_p and β_c are of opposite sign; and if their magnitudes are substantially equal, the fluid-loading terms vanish in Eq. 16. Physically, that means that the mechanical terms, including the fluid, combine so that the velocity field on the free surface of the coating is equal to the velocity field that the plate would have had if the coating and the fluid loading were removed. Further, if β_c is substantially independent of θ in the range of frequency below the critical frequency, the material discussed in Ref. 2 is applicable in whole to the present case, provided that the fluid-loading parameter β in the directivity function in Ref. 2 is made equal to $\beta_c + \beta_p$.

B. Rigid and Soft Baffles

The results in Ref. 1 concerning the behavior with respect to rigid and soft baffles are applicable to the present situation; e.g., if $\theta_1 = \theta_{1n} = \cos^{-1}(\pi n/hk_1)$, n being an integer (including zero), the directivity function is substantially zero and essentially no far-field acoustic pressure is generated into the angular directions $\theta_n = \sin^{-1}[(c_0/c_1)\sin\theta_{1n}]$ at the frequency defining $k_1 = \omega/c_1$ in the angular conditions just described.

II. BACK SURFACE OF THE PLATE COATED

This system is illustrated in Fig. 2. Again employing a procedure similar to that by which Eq. 3 of Ref. 1 derived, we obtain the following expression for the spectral velocity field $V_p(\mathbf{k},\omega)$ on the plate surface in terms of the external spectral force field $P(\mathbf{k},\omega)$:

$$V_p(\mathbf{k},\omega) = P(\mathbf{k},\omega)[Z_p + Z_c Z_b (Z_c + Z_b)^{-1} + Z_0]^{-1}, \quad (17)$$

[6] G. Maidanik and E. M. Kerwin, Jr., "The Influence of Fluid Loading on the Radiation from Infinite Plates Below the Critical Frequency," 1 December 1965, unpublished notes.

where the quantities were previously defined in the Introduction and Sec. 1.

The expression for the frequency-spectral far-field acoustic pressure generated from the front surface of the plate is obtained by substituting V_p of Eq. 17 for V_s into Equation 2 and carrying out the integrations involved. The expressions for the frequency-spectral far-field acoustic pressures when the plate is driven by a point force and a line force are given in Eqs. 10 and 11, respectively, except that now the directivity function assumes the form

$$D(\theta)=\cos(\theta)[\cos(\theta)\{1+\beta_p\nu(1+\beta_c)^{-1}\}+\beta_p]^{-1}, \quad (18)$$

where, again, the quantities were previously defined in the Introduction and Sec. I. Here, as in Sec. I, we consider first the two limiting cases where $\beta_c \to 0$ and $\beta_c \to \infty$. In the former case, the compliance of the coating is negligible, so that the coating is essentially part of the plate. Under this condition, Eq. 18, as did Eq. 12, reduces to Eq. 8 of Ref. 1, with $g(\theta)$ as given in Eq. 17 of this reference. Thus, we recover appropriately the previous results[1] in this limiting case. The latter case, $\beta_c \to \infty$, is the situation when the compliance of the coating is substantial. The coating is then incapable of transmitting the velocity field of the plate to the back cavity, and consequently, the back cavity is not aware of the plate vibrational state; the coating decouples the plate from the back cavity. Indeed, under this condition the directivity function becomes

$$D(\theta)=\cos(\theta)[\cos(\theta)+\beta_p]^{-1}, \quad (19)$$

which is precisely the directivity function one obtains if the back space is a vacuum (cf. Ref. 2 and the discussion of this limiting case in Section I).

We turn to consider briefly a few additional cases of interest. Again, we do not exhaust all such cases in this paper.

A. Rigid and Soft Baffles

In Ref. 1, we showed that for waves on the radiating surface, which radiates into some well-defined angles, the impedance presented to the surface facing the baffle becomes infinite If these conditions are imposed on the present system, the radiation from these waves does not vanish. Although these waves on the free surface of the coating have vanishing amplitudes because they are inhibited by the presence of the baffle, the corresponding waves on the plate need not have vanishing amplitudes. The force field can generate such waves on the plate because the coating is compressible; and consequently, it decouples the plate somewhat from the back cavity so that the back cavity cannot completely inhibit these waves. For the plate waves just described, $\nu \to \infty$, and the directivity function takes the form

$$D(\theta)=\cos(\theta)[\cos(\theta)\{1+(\beta_p/\beta_c)\}+\beta_p]^{-1}, \quad (20)$$

provided that $\beta_c \neq 0$.

For those waves on the plate that radiate into angles for which $\nu \to 0$, the usual results are obtained, whereby these radiating waves are substantially decoupled from the back cavity. This can be readily verified from Eq. 18.

A final remark is in order regarding special, yet practical, cases of coated plates. If the fluid medium is a liquid, the coating can take the form of a thin layer of gas. Thus, for example, the behavior of the case just presented is applicable to a situation where the back cavity contains a thin layer of air adjacent to the plate. The case treated in Sec. I can be found, in practice, when a thin layer of bubble-infested water forms on the front side of the plate.

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13. ABSTRACT

The far-field acoustic pressure generated by an infinite plate driven by a point and a line force are computed. This paper constitutes an extension of previous computations [G. Maidanik, J. Acoust. Soc. Am. 42, 27–31 (1967)]. The plate considered in this paper is covered uniformly and completely by a compliant coating which is applied in turn to the front and back of the plate. The formalism accounts for different fluids' occupying the space in front of the plate and in the cavity back of the plate.

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