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NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

AGREEMENT OF MODEL AND PROTOTYPE

RESPONSE AMPLITUDE OPERATORS AND WHIPPING RESPONSE

by

John N. Andrews and
Alfred L. Dinsenbacher

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STRUCTURAL MECHANICS LABORATORY

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Naval Ship Research and Development Center
Washington, D. C. 20007

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ABSTRACT

This report demonstrates the validity of employing structural segmented models to predict prototype response for use in ship design. By means of spectral analysis, bending moment and pitch angle response amplitude operators are obtained and compared for model and prototype. In addition, whipping bending moments resulting from bow-flare immersion for model and prototype are analyzed and compared. The results of these comparisons show good agreement between model and prototype.

ADMINISTRATIVE INFORMATION

To aid in design evaluation, the David Taylor Model Basin developed a model to simulate the structural response of a ship hull in a seaway. This model was built by the Davidson Laboratory of Stevens Institute of Technology, Newark, N.J., under Contract Nobs 78349 T/09. Test of this model and subsequent data analyses were performed at the Model Basin. The work was authorized by a Bureau of Ships letter F013 03 01, Ser 442-109 of 8 July 1963 and was done under Subproject S-F013 03 01 Task 1973.

INTRODUCTION

The need for displacement-type surface ships to operate at ever improving levels of performance in more severe seas is a constant necessity. These requirements have increased the complexity of the problem which the ship structural designer must face.

To provide rationally for the hull-girder strength, the designer must determine loads that reflect the greatest loads the ship may be expected to encounter in its service life. The efficiency of the design and the structural integrity of the ship hinge upon the extent to which these design loads adequately represent the corresponding true service loads experienced.

However, it is not certain whether methods currently employed to determine loads for hull-girder design are suitable for the design of new, high-performance ships. Existing design methods do not consider dynamic loads or the effect of high ship speed on the hull-girder loads. Sufficient full-scale trial data to substantiate current design methods are lacking and are also expensive and difficult to obtain.

With development of computers and advanced modeling facilities, the answer to the design problem appears to be model and/or computer techniques that will predict full-scale ship response. The Model Basin is currently engaged in such a program. The overall objective of this program is to correlate the results of full-scale, model, and computer programs, so that the designer can examine his new ideas before they are put into practice.

The broad program described above is divided into two phases. In the first phase, computer methods for obtaining vertical ship responses were developed. Heretofore, no definite correlation between computer and full-scale response (motions and bending moments) had been made; however, both analog and digital computers had been used in predicting ship response in head seas.[1,2] Results show good agreement with full-scale trial data. Although the analog development is complete, the method used for determining structural response on the digital computer requires foreknowledge of the ship motions. At present, the digital formulation is being expanded so that both the motions and structural response may be obtained simultaneously. Upon completion, these computer techniques can be further expanded to include athwartship response as well.

The second phase of the program is described in this report and is to develop and utilize model-testing techniques for obtaining ship response. Prior to this work no definite correlations of model with full-scale or model with computer results to obtain structural response (including whipping) has been made. To accomplish this second phase, a 6-foot segmented model of the ESSEX-Class aircraft carrier (scale 1:136) was constructed in accordance with Froude scaling. The model was towed in regular head waves of various lengths and heights at several speeds in the 140-foot basin. In addition, the model was tested at zero speed in a random head sea. Response information on motions (pitch angle and heave displacement) and structure (bending moment) was obtained. Model response-amplitude operators (RAO's) for vertical bending moments, and

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[1]References are listed on page 35.

motions derived from regular and random wave tests for zero model speed have been reported.[3] The results show good agreement. Model and computer results are compared also, and the agreement is good.

The objective of the program described in this report is to demonstrate that models can be employed to predict prototype response. In the future a report will be prepared in which RAO's for the model and prototype for various speeds in head seas will be compared. This will show the effect of ship speed on RAO's and establish whether model and prototype RAO's continue to show good agreement for nonzero speeds.

With confirmation of computer results, through comparison with model- and prototype-test results for similar conditions, costly and time-consuming prototype trials may be omitted. In fact, for new ship design, ship performance may be predicted far in advance of final construction.

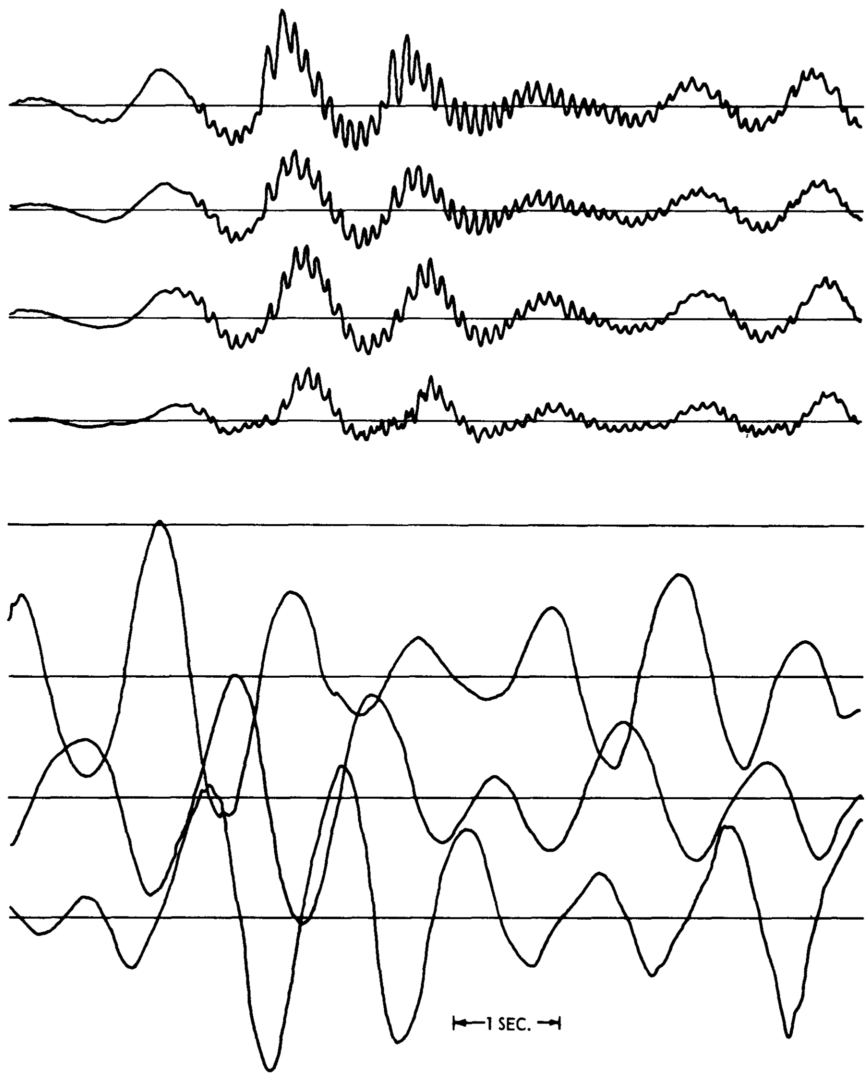
MODEL

The ESSEX-Class carrier was chosen for modeling because of the large amount of full-scale data available for comparison with model test results.[4] The model is 6 feet in length and consists of nine Fiberglas segments connected to a continuous magnesium alloy bar.

The scaling relationships applicable to this study are shown in Table 1. Model design, construction, and testing are presented in Reference 3. Table 2 shows the general characteristics for the model and prototype.

MODEL RANDOM WAVE TEST

The model was tested at zero speed in random waves in the 140-foot basin at the Model Basin. Freedom to heave and pitch was provided; but sway, roll, surge, and yaw motions were restrained. Tests were conducted in head waves only. A sample oscillogram of the waves and model response is shown in Figure 1.



QUANTITY MEASURED	SENSITIVITY	POLARITY
BENDING MOMENT STATION 6	$0.85 \times 10^6 \frac{\text{FT - TONS}}{\text{IN.}}$	SAG ↑
BENDING MOMENT STATION 10	$1.57 \times 10^6 \frac{\text{FT - TONS}}{\text{IN.}}$	SAG ↑
BENDING MOMENT STATION 15	$0.83 \times 10^6 \frac{\text{FT - TONS}}{\text{IN.}}$	SAG ↑
BENDING MOMENT STATION 17	$0.31 \times 10^6 \frac{\text{FT - TONS}}{\text{IN.}}$	SAG ↑
	1 IN.	
WAVE HEIGHT	$18.9 \frac{\text{FT}}{\text{IN.}}$	CREST ↑
HEAVE DISPLACEMENT	$11.3 \frac{\text{FT}}{\text{IN.}}$	UP ↑
PITCH ANGLE	$4 \frac{\text{DEG}}{\text{IN.}}$	BOW UP ↑

Figure 1 - Sample Oscillogram of Model Test in Random Waves

TABLE 1

Scaling Relationships

Quantity	Symbol: Model	Symbol: Full Scale	Relationship
Force	F_m	F_p	$F_p = \lambda^3 F_m$ *
Moment	M_m	M_p	$M_p = \lambda^4 M_m$
Stress	σ_m	σ_p	$\sigma_p = \frac{E_p}{E_m} \cdot \frac{C_p}{C_m} \cdot \frac{1}{\lambda} \sigma_m$ **
Pressure	P_m	P_p	$P_p = \lambda P_m$
Frequency	ω_m	ω_p	$\omega_p = \lambda^{-1/2} \omega_m$
Velocity	ν_m	ν_p	$\nu_p = \lambda^{1/2} \nu_m$
Time	t_m	t_p	$t_p = \lambda^{1/2} t_m$
Acceleration	a_m	a_p	$a_p = a_m$
Length	L_m	L_p	$L_p = \lambda L_m$
Bending Rigidity	$(EI)_m$	$(EI)_p$	$(EI)_p = \lambda^5 (EI)_m$
Mass	M_m	M_p	$M_p = \lambda^3 M_m$

*The scale for this model is $\lambda = 136$.

**E is the modulus of elasticity in pounds per square inch, and C is the distance from neutral axis to extreme fiber.

TABLE 2

Model and Prototype Characteristics

	Model	Prototype
Length between Perpendiculars	72.375 inches	820 feet
Beam	9.10 inches	103 feet
Main Deck Height	4.85 inches	55 feet
Flight Deck Height	7.20 inches	81.5 feet
Weight	35.129 pounds	40,100 tons
Station Spacing	3.618 inches	41 feet

PROTOTYPE SEA TEST

From records obtained on ESSEX during sea trials, a record exhibiting operating conditions similar to the model, i.e., head seas and minimum speed, was selected. A sample of the record chosen is shown in Figure 2. The ship heading is about 22 1/2 degrees to the waves, and the speed is about 9 knots.

ANALYSIS

To derive response characteristics which may be used to compare model and prototype, a useful method is to employ spectral analysis.[5] A discussion of these techniques is given in Appendix A. Briefly, a spectrum of a random process is a frequency decomposition of the process and is a plot of the mean-squared value of the process per unit of frequency versus frequency.

Some of the important properties of a spectrum of a random process are given in Appendix A. One of the most important properties is that the area enclosed by the spectrum (E) may be employed to estimate maximum peak-to-peak variations of the process.* The most useful aspect of

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* 

$Y_{max} = 2\sqrt{\text{area under spectrum} \times \log_e N} = 2\sqrt{E \log_e N}$

where Y_{max} is the estimated peak-to-peak variation of the process, and N is the total number of peak-to-peak variations in the process.

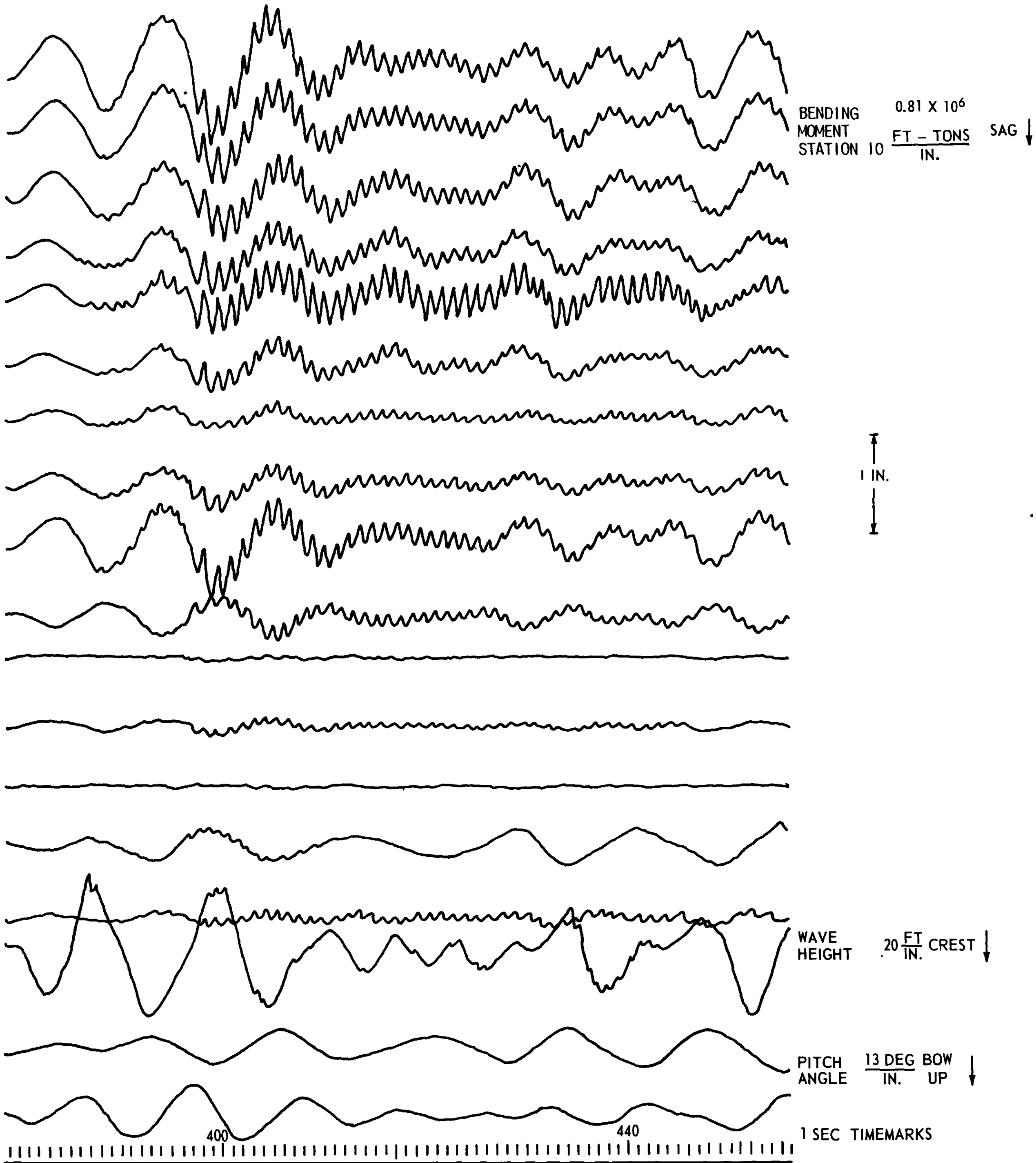

Figure 2 - Sample Oscillogram of Prototype Test in a Random Sea

spectral analysis is in determination of RAO's which characterize response from a knowledge of the excitation and response spectra. An RAO is a plot of the square of response per unit of excitation versus frequency. In the case of ships the random excitation is the waves, and the random response may be considered to be the motions and/or the structural loads (bending moment, shear-force, etc.).

To directly compare the model and prototype, it is advantageous to nondimensionalize the RAO's as described in Appendix A. Although RAO's are useful in defining the ship response characteristics, it should be emphasized here that the RAO's are valid only for the ordinary wave-induced responses, exclusive of vibratory (whipping) responses. In order to obtain the RAO for wave-induced response, the response records must be filtered so that the vibratory components are removed.

To determine whether or not whipping response (excitation of the fundamental mode of vibration) due to bow-flare immersion can be modeled, some method other than spectral analysis must be employed. To date, no statistical method for analyzing structural response to whipping has been formulated. To facilitate the establishment of the validity of models for predicting whipping response, the following analysis was performed.

Initial peak-to-peak variations of midship whipping response for each slam observed on the model and prototype records were read for the same number of wave-induced variations (this determined the length of record for analysis). The number of whipping variations for analysis were chosen to be one-tenth of the total number of wave-induced variations in each record. The variations were then tabulated in descending order, and a cummulative average of whipping response for model and prototype was obtained and normalized by dividing the averages by the root-mean-squared (rms) amplitudes ($\sqrt{\bar{E}}$) of the wave-induced midship moments derived from spectral analysis of the model and prototype records.

PRESENTATION AND DISCUSSION OF RESULTS

COMPARISON OF MODEL AND PROTOTYPE RESPONSE-AMPLITUDE OPERATORS

Figures 3 and 4 show the wave and response spectral densities as a function of wave frequency for model and prototype, respectively. The

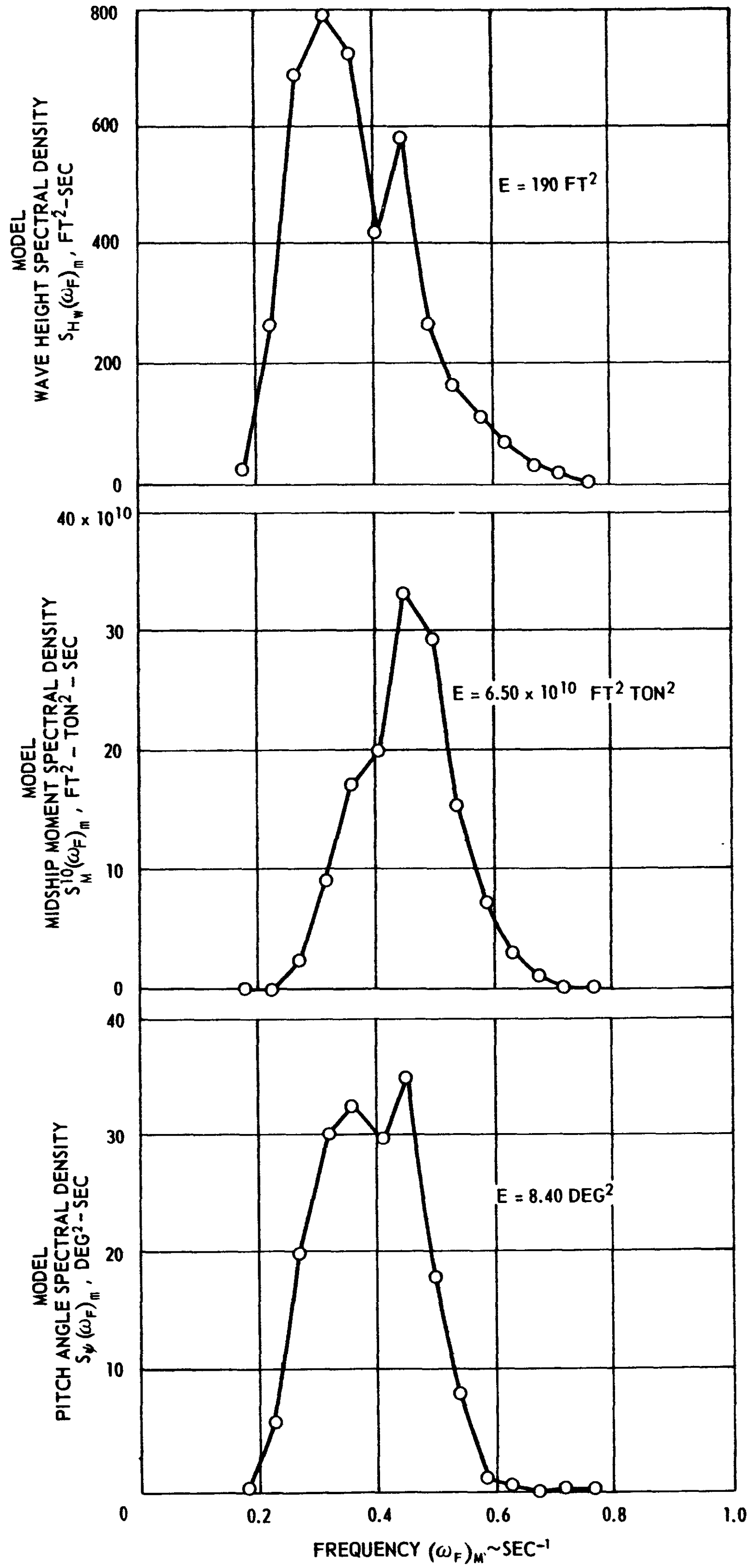

Figure 3 - Model Wave and Response Spectral Densities as a Function of Wave Frequency

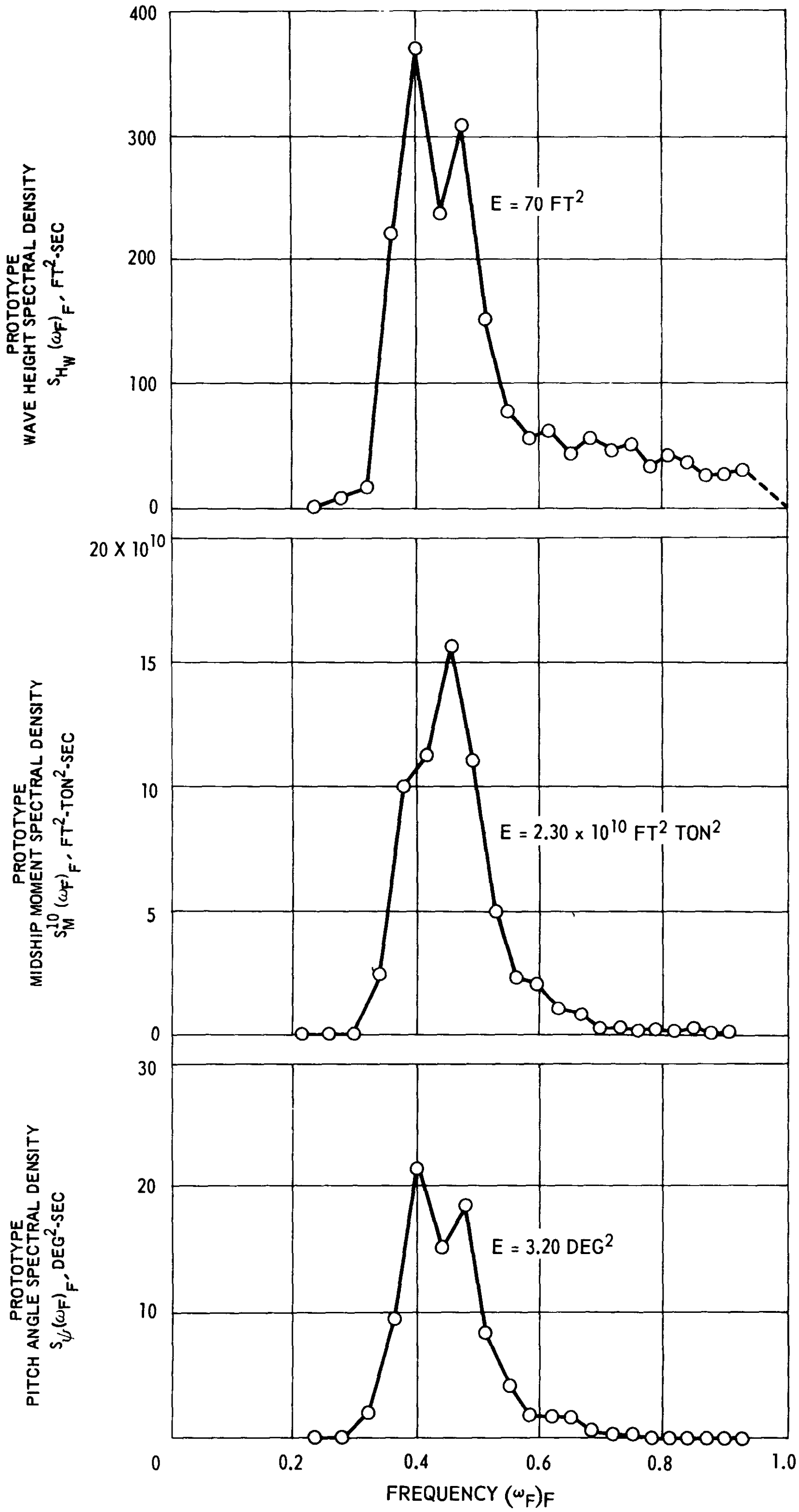

Figure 4 - Prototype Wave and Response Spectral Densities as a Function of Wave Frequency

model spectra are presented in terms of full-scale quantities. As previously stated, an effort was made to obtain similar environmental and operational conditions for direct comparison with prototype. Although the wave spectra for model and prototype apparently are somewhat dissimilar, both fall within the State 7 sea range (as defined by the scale of the U.S. Navy Oceanographic Office).

Since the prototype was operating in bow seas at a speed of approximately 8 to 10 knots, the wave response spectral densities and frequencies had to be corrected. The wave spectrum amplitudes were first corrected in order to compensate for the frequency characteristics of the Tucker wavemeter as described in Appendix B. Then the wave and response spectra were corrected to compensate for the forward motion and heading of the ship relative to the waves (see Appendix A). These corrections permit the spectral densities to be plotted in terms of wave frequency in lieu of encounter frequency, which facilitates direct comparison of model and prototype spectra.

Figure 5 shows the comparison between model and prototype RAO's of the midship-bending moment. Some of the differences in amplitude may be attributed to instrument errors. In particular, the accuracy of the Tucker meter is not precisely known, although a comparison of the Tucker meter with a wave buoy shows overall agreement of about 10 percent. In addition, it is not known to what extent the calculated and actual section modulus of prototype agree. Also as previously mentioned, model tests were conducted in head seas while the prototype test used for comparison was for a ship heading of 22 1/2 degrees relative to the waves. This would result in a reduction in the measured vertical bending moment. Furthermore, the errors are exaggerated in the figure since the ordinates are presented in terms of squared values of response per unit of wave height. The agreement in form is considered excellent, and the amplitude comparison is considered good.

Figure 6 shows the comparison of model and prototype midship-bending moment and pitch-angle response in nondimensional form. This form of presentation permits these curves to be compared readily with those obtained for other ships.

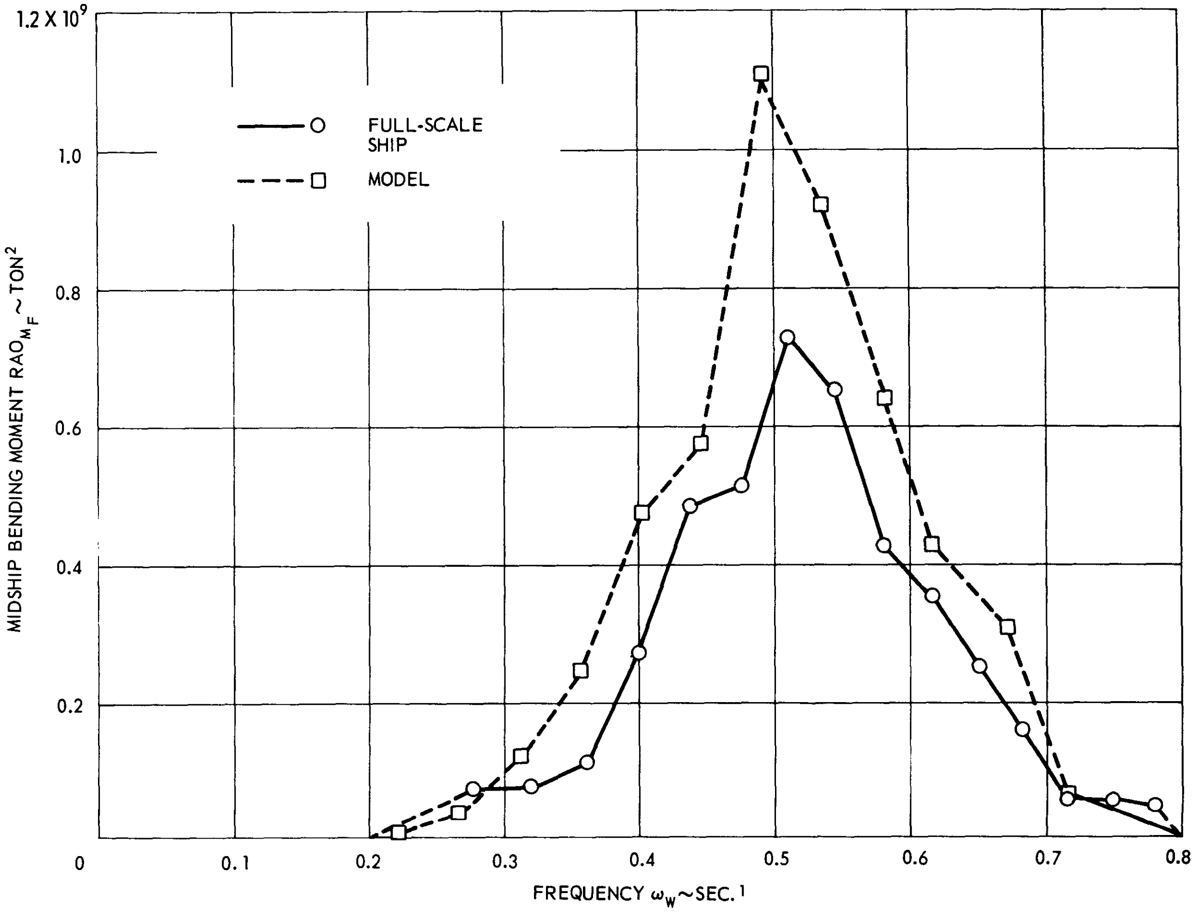

Figure 5 - Model and Prototype Response Amplitude Operators as a Function of Wave Frequency

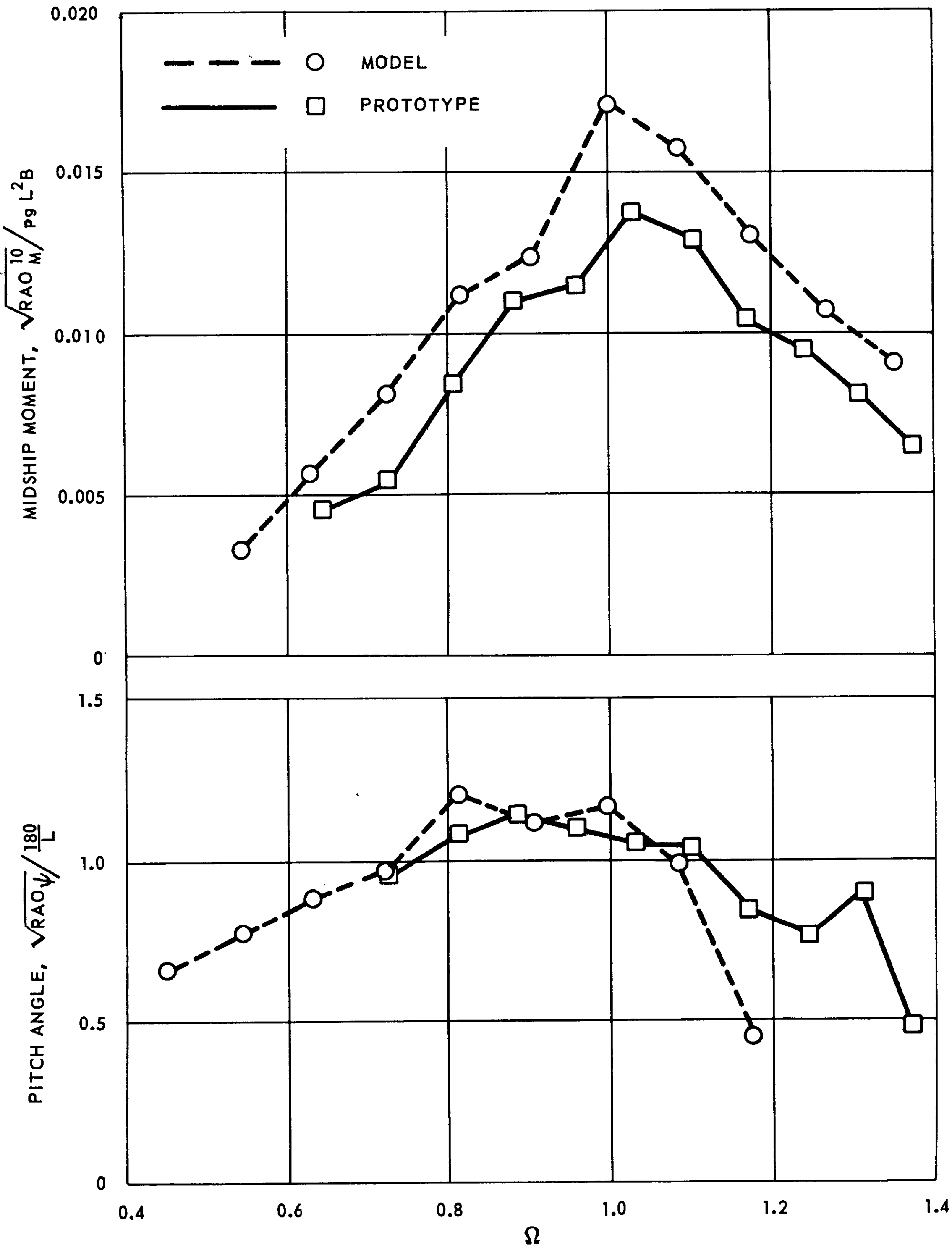

Figure 6 - Square Root of Nondimensional Response Amplitude Operators for Model and Prototype as a Function of Nondimensional Frequency

COMPARISON OF MODEL AND PROTOTYPE WHIPPING RESPONSE

In Figure 7, model and prototype whipping responses are compared. This figure is obtained from the data shown in Tables 3 and 4. As described in the analysis section, the highest 16 initial peak-to-peak whipping values for both model and prototype were chosen. The whipping moments were then arranged in descending order and the cummulative average values $\left(\text{i.e., } A_1, \frac{A_1 + A_2}{2}, \frac{A_1 + A_2 + A_3}{3}, \frac{A_1 + A_2 + A_3 + \ldots + A_n}{n}\right)$ were obtained and normalized by dividing each value by the rms amplitude for bending moment as obtained from spectral analysis. These values are shown plotted in Figure 7 as a function of n. The value of model and prototype for n = 1 represents the maximum whipping response for the records analyzed. The prototype values are slightly larger than model values which in part can be attributed to the prototype speed (about 9 knots). The agreement between model and prototype is considered good.

Thus, from the agreement of RAO's and whipping response for this model and prototype, model validity is established. Model techniques, as employed herein then, may be used to predict prototype response as described in the next section.

APPLICATION OF RESULTS IN SHIP DESIGN

The results described in the previous section can be used to predict the maximum structural response of a ship to any sea condition. To accomplish this, theoretical Neumann spectra of the sea for various wind velocities shown in Figure 8 (see Appendix C) are used together with the RAO obtained from model testing. The product of the Neumann spectra and the RAO results in response spectra. From the response and sea spectra, the $\sqrt{E}$ for the responses and the waves are obtained and plotted as shown in Figure 9. The figure shows two curves; data obtained from prototype testing is shown for comparison. Assymptotic values for model and prototype are given. These values are derived by assuming the wind velocity of the Neumann spectra to be infinite (see Appendix C). The solid points are the $\sqrt{E}$ values obtained from the actual model and prototype results. These points are included in the figure to show that Neumann spectra may be used to give realistic results.

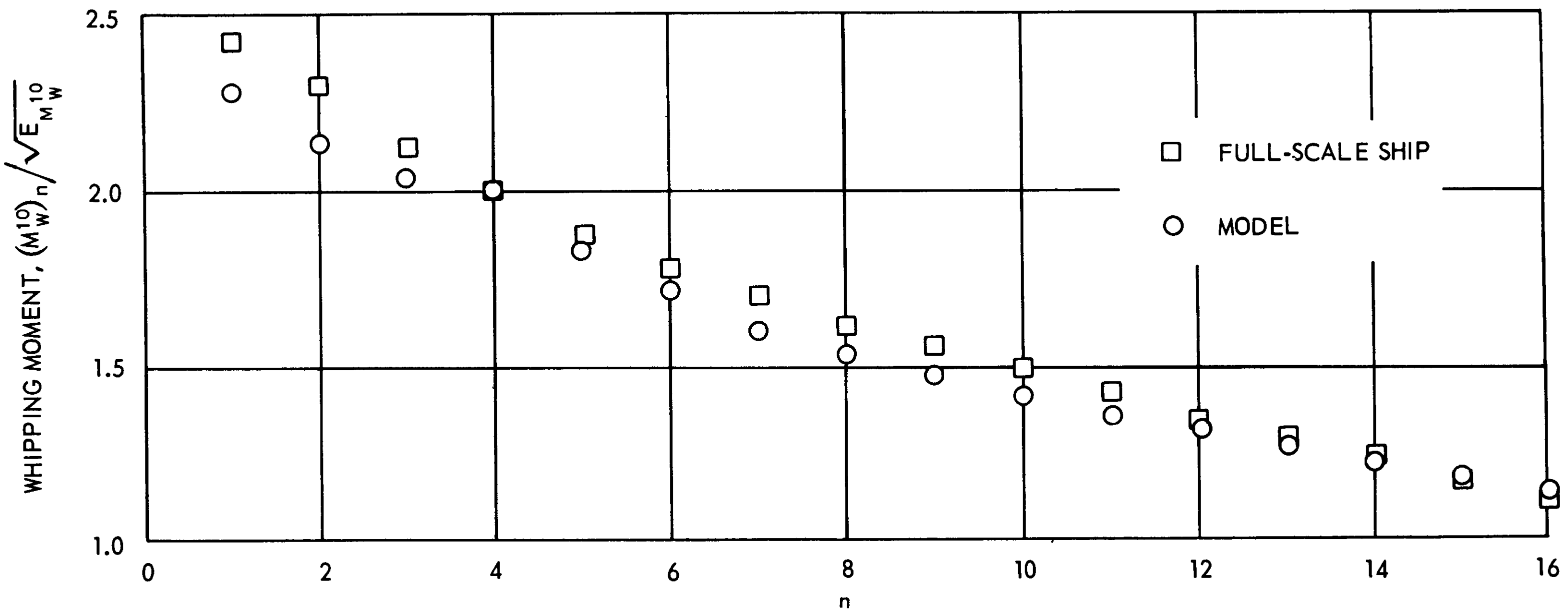

Figure 7 - Comparison of Model and Prototype Whipping Response

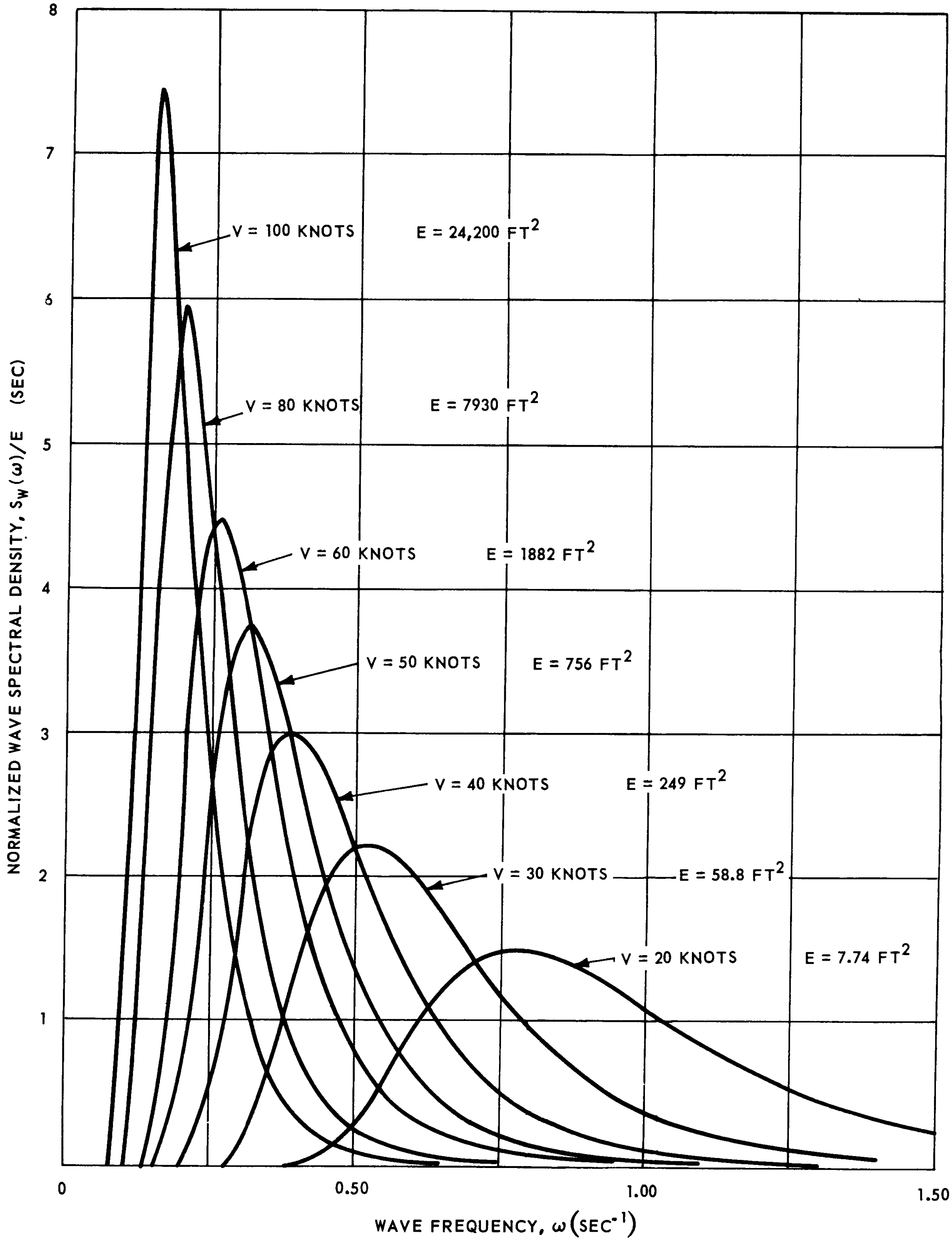

Figure 8 - Neumann Sea Spectra for Various Wind Velocities

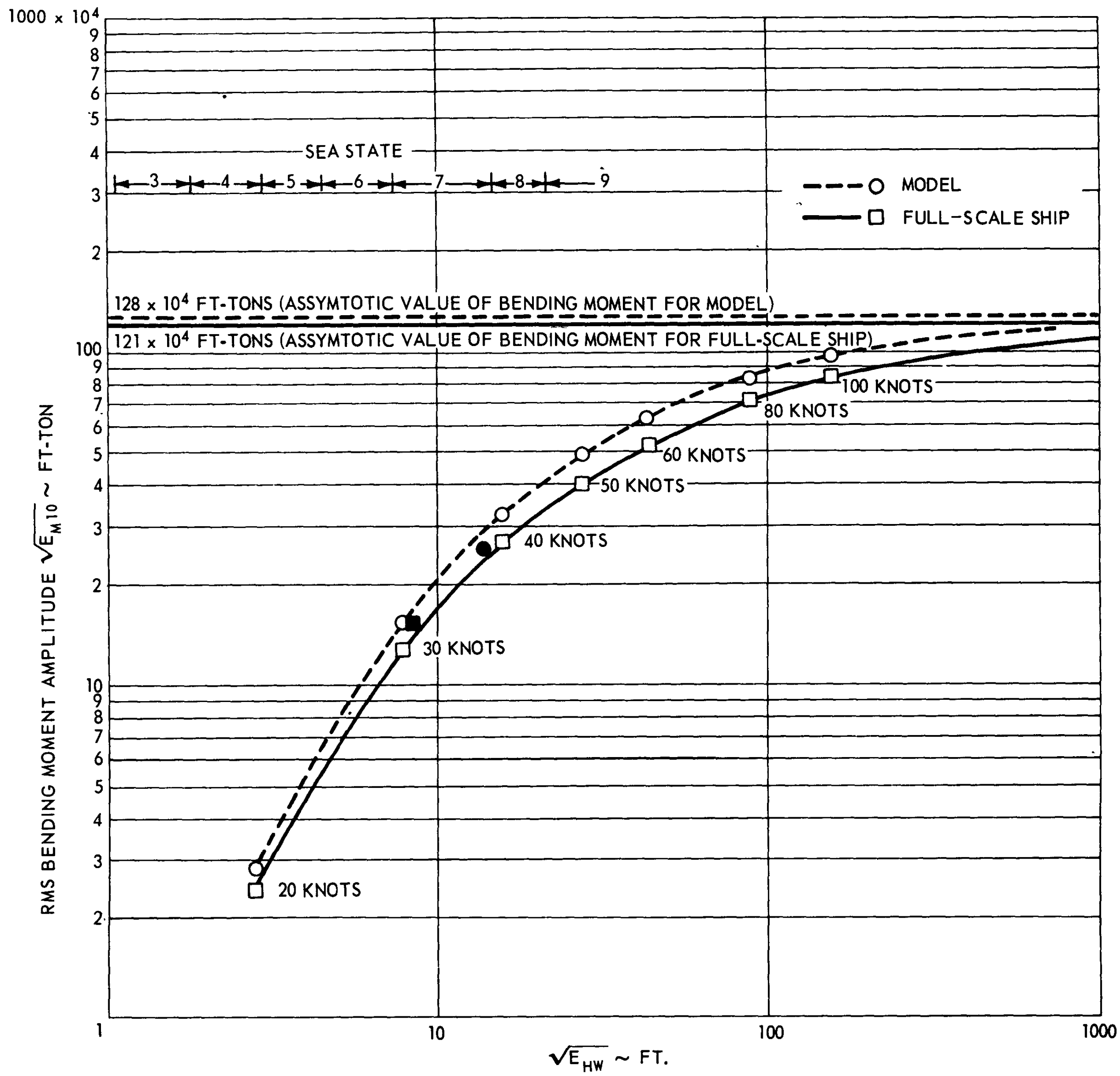

Figure 9 - RMS Bending-Moment Amplitude Amidship versus RMS Wave-Height Amplitude for Model and Prototype

TABLE 3

Model Whipping-Response Data

Data Points	Whipping-Moment M_W^{10} (ft-tons)	Cummulative Value (ft-tons)	Cummulative Average (ft-tons)	Ratio of Cummulative Average to RMS Value
1	580 X 10^3	580 X 10^3	580 X 10^3	2.27
2	500	1080	540	2.12
3	470	1550	517	2.03
4	470	2020	505	1.98
5	300	2320	464	1.82
6	280	2600	433	1.70
7	250	2850	407	1.60
8	250	3100	388	1.52
9	250	3350	372	1.42
10	240	3590	359	1.41
11	220	3810	346	1.36
12	190	4000	333	1.31
13	160	4160	320	1.25
14	160	4320	309	1.21
16	130	4590	287	1.13

TABLE 4

Prototype Whipping-Response Data

Data Points	Whipping-Moment M_W^{10} (ft-tons)	Cummulative Value (ft-tons)	Cummulative Average (ft-tons)	Ratio of Cummulative Average to RMS Value
1	370 X 10^3	370 X 10^3	370 X 10^3	2.43
2	330	700	350	2.30
3	280	980	323	2.12
4	240	1220	305	2.00
5	210	1430	286	1.88
6	200	1630	272	1.78
7	180	1810	259	1.70
8	160	1970	246	1.61
9	160	2130	237	1.55
10	140	2270	227	1.49
11	110	2380	216	1.42
12	80	2460	205	1.33
13	70	2530	195	1.28
14	70	2600	186	1.22
15	60	2660	177	1.16
16	60	2720	170	1.11

As previously stated in the analysis section (also see Appendix A), the estimated maximum peak-to-peak variation may be determined. If the area under the spectrum and the total number of variations are known. Figure 10 presents the estimated maximum values of response for model and prototype as a function of $\sqrt{E_{H_W}}$ as obtained from the Neumann spectra. The number of variations N employed are determined from the average period of response spectra (see Appendix A) and the duration of time the ship operates in a specific sea condition. The actual time chosen corresponds to that duration of time in which the model and prototype were tested. For the model this time in terms of prototype time is about 40 minutes and for the prototype 28 minutes. The average periods of response for model and prototype were computed to be 12.7 and 11.8 seconds, respectively. From these values and the time duration of the tests, predicted values of N = 194 and N = 173 were determined for model and prototype, respectively. These compare well with the actual values of 183 and 166. The actual maximum values for model and prototype as measured from the random records are shown in Figure 10 by the solid points. This agreement once again is considered good.

Figure 10 also shows the design value for midship-bending moment as obtained from the standard design calculation for an L/20 static wave. This value can occur in a State 9 sea as noted from Figure 10.

To obtain the total maximum peak-to-peak variation in bending moment, whipping moment needs to be considered.[3] The maximum whipping moments can be obtained from model tests for specific sea conditions and operating speeds. The results obtained can then be added to the wave-induced results obtained from Figure 10.

This procedure gives the designer a more rational basis for ship design.

SUMMARY AND CONCLUSIONS

From Figures 6, 7, and 10 it may be concluded that RAO's, predicted wave-induced response, and whipping response obtained from model tests compare favorably with prototype test results. Figure 10 shows in

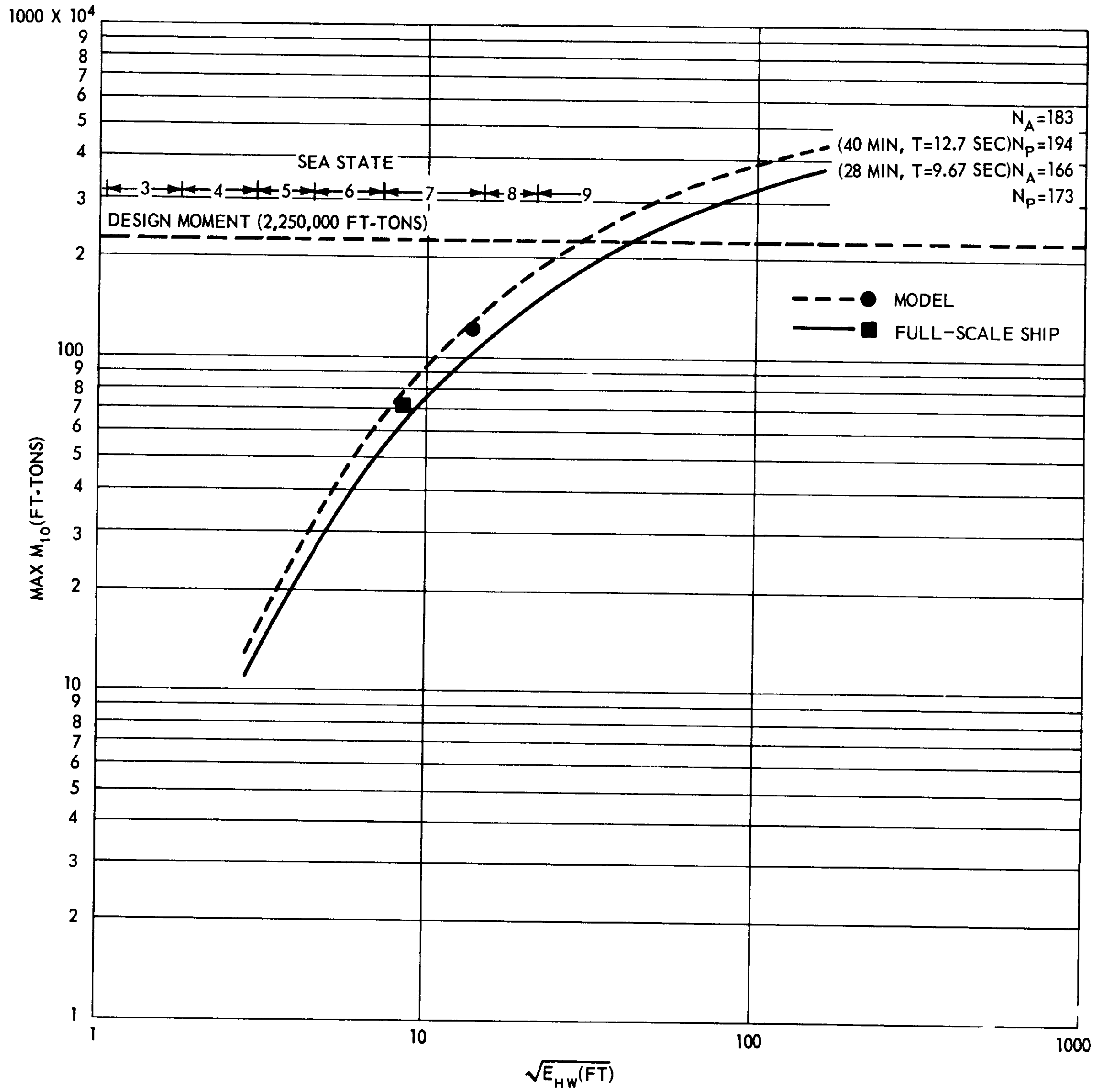

Figure 10 - Maximum Peak-to-Peak Bending Moment Amidship versus RMS Wave-Height Amplitude for Model and Prototype

particular that the employment of Neumann sea spectra gives realistic results. It also can be concluded that the model techniques described herein are valid in predicting prototype response, including whipping, and thus can be used in ship design.

ACKNOWLEDGMENTS

The authors wish to thank Dr. N. Jasper, Technical Director of the Mine Defense Laboratory, for his help and suggestions in the earlier development of the program. Acknowledgments are also made to Mr. D. Pincus for his assistance in the reduction of portions of the data.

APPENDIX A

RESPONSE OPERATORS AND SPECTRAL ANALYSES

A useful method of presenting the response characteristics of a ship is in the form of response-amplitude operators, which can be used to predict response to random seas, and which are given as the square of response per unit of wave height versus wave frequency. That is

$$\mathrm{RAO}\ (\omega) = \left[\frac{R_o(\omega)}{H_w(\omega)}\right]^2 \qquad [A.1]$$

where H_w (ω) = the height (crest to trough) of the waves of frequency ω.

RAO (ω) = response-amplitude operator as a function of the wave frequency ω.*

R_o (ω) = peak-to-peak or total variation of response when the ship is subjected to waves of frequency ω.

The curves defining these RAO's may easily be found from regular wave tests by conducting several tests, each with the model running through waves of a particular length; obtaining the ratio of response variation to wave height; and plotting the squares of these ratios versus the wave frequency.

Obtaining response-amplitude operators from random wave tests cannot be performed by such a simple method. In the random wave tests, the peak-to-peak response variations and the wave heights are not constant as in the regular wave tests (see Figures A-1 and A-2). The ratios of individual response variations to corresponding individual wave heights are not constant either, and it often becomes difficult to ascertain the particular variation of response that corresponds with a particular variation of wave elevation.

The reader may have wondered about the use, or even the meaning, of a response-amplitude operator in connection with random wave tests. An

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*The circular frequency associated with a wave of length L_w is given as $\omega = \sqrt{\frac{2\pi g}{L_w}}$.

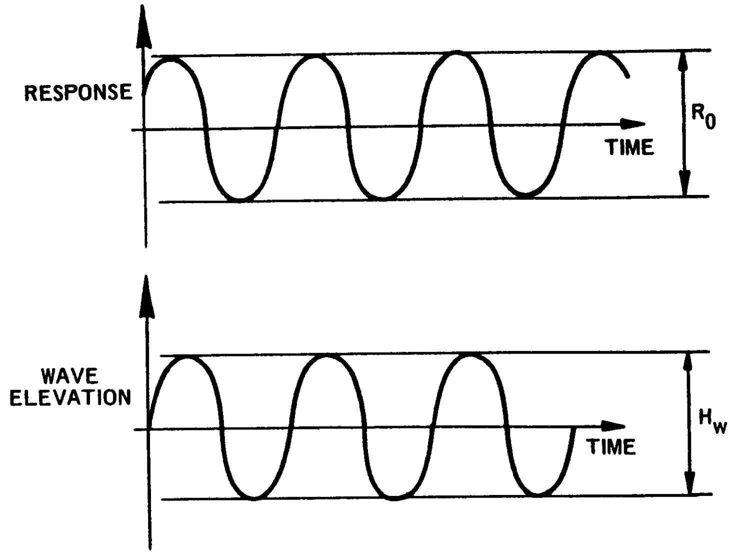

Figure A-1 - Response and Wave Height--Regular Wave Test

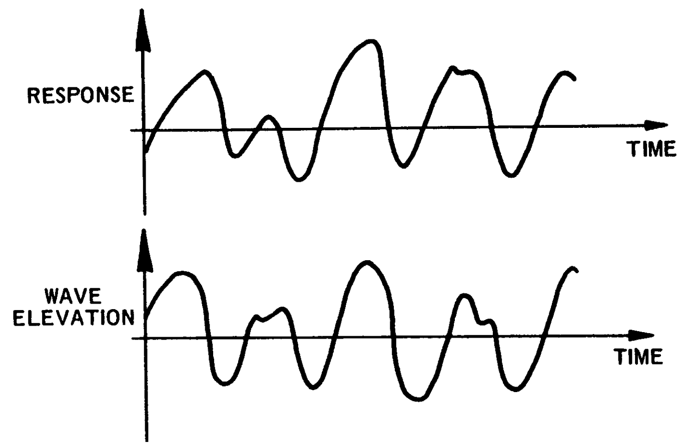

Figure A-2 - Response and Wave Height--Random Wave Test

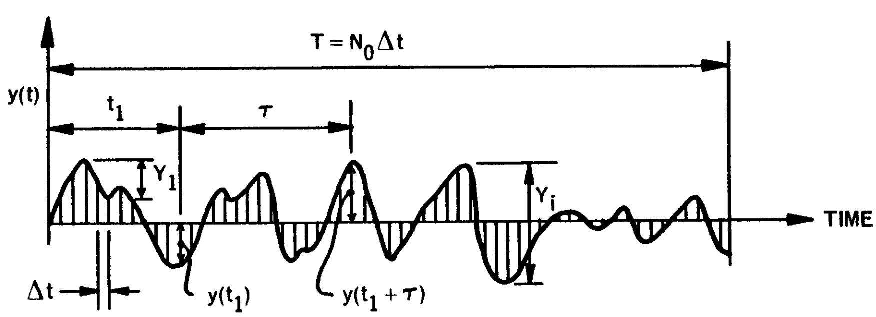

Figure A-3 - Illustration of a Random Process intuitive appreciation of the rationality of response-amplitude operators and spectral analysis may be derived from the following discussion. Let us suppose that an unidirectional random sea is composed of a superposition of an infinite number of sinusoidal wavelets of different frequencies and amplitudes, all moving in the same direction. A plot of the square of the heights of these wavelets versus their frequency would be a curve not unlike an actual amplitude spectrum of the sea. Let us also suppose that each wavelet acts upon the ship independently so that we can determine the response of the ship to each of these wavelets separately by simply multiplying the square of its height by the square of the ratio of response per unit wave height (the RAO). The product is the square of the response caused by that wavelet, and the frequency of the response is equal to the frequency of the wavelet. Now, if we plot the square of the response versus frequency, we have another curve which is indeed similar to the response spectrum.

We thus see that the use of a sea spectrum and the RAO can be useful for determining the response of the ship to a random sea. Conversely, the RAO may be determined through knowledge of the sea and the response spectra.

The problem is in actually obtaining these spectra from a random sea test and then deriving the RAO from the spectra. A method for doing this will now be outlined.

The autocorrelation function of a random process y (t) satisfying certain conditions may be defined as[5,6]

$$C(\tau) = \lim_{T\to\infty} \frac{2}{T} \int_0^T y(t)\, y(t+\tau)\, dt \qquad [A.2]$$

where τ is called the autocorrelation lag.

The spectral density may then essentially be defined as the Fourier cosine transform of $C(\tau)$

$$S(\omega) = \frac{2}{\pi} \int_0^\infty C(\tau) \cos \omega\tau \, d\tau \qquad [A.3]$$

Numerical integration may be employed to obtain the autocorrelation function and spectral density for the waves or responses from the test records. The record of time duration T is subdivided into N_o equal increments of time Δt (Figure A-3). The incremental time spacing Δt is usually taken as not greater than one-fourth of the period of the variation having the highest frequency of interest in the record.

Readings of y(t) are taken from the record at these equally spaced increments. The autocorrelation function is then obtained by using a discrete approximation of the form[5,6]

$$C_n = \frac{2}{N_o - n} \sum_{q=1}^{N_o - n} y_q \, y_{q+n} \quad (n=0,1,2, \ldots, m) \qquad [A.4]$$

where

C_n is the autocorrelation estimate for lag $\tau = n\Delta t$,

Δt is the time interval between values of y(t) read from original record,

y_q is the value of y(t) at time $q\Delta t$,

N_o is the number of data points in record,

n is the number of intervals defining lag $\tau = n \, \Delta \, t$, and

m is the maximum number of lags, usually taken as not more than $N_o/4$.

A discrete approximation to the spectral density is then obtained as

$$S_k = \frac{\Delta t}{\pi} \left[C_o + 2 \sum_{n=1}^{m-1} C_n \cos \frac{nk\pi}{m} + C_m \cos k\pi \right] \qquad [A.5]$$

$$(k = 0, 1, 2 \ldots, m)$$

where S_k is the estimate of the spectral density at the frequency

$$\omega = \frac{k \, \pi}{m\Delta t} = k \, \Delta\omega \qquad [A.6]$$

The computations for obtaining the C_n and S_k may be made by using a digital computer program available at the TMB Applied Mathematics Laboratory (AML). Each response-amplitude operator may then be found for a test run by dividing the ordinates of the response spectrum by the ordinates of the wave spectrum at their corresponding frequencies; that is,

$$RAO\ (\omega) = S_R\ (\omega) / S_{H_W}\ (\omega) \qquad [A.7]$$

where $S_R(\omega)$ is the spectral density of the response at frequency ω, and $S_{H_W}(\omega)$ is the spectral density of the waves at frequency ω.

Actually, since the ship (or model) advances through the waves at a certain speed V_s and wave to course angle θ, the frequencies of the recorded waves and responses will reflect this forward motion. The recorded or "encounter" frequency is related to the wave frequency, ship speed, and wave to course angle as follows:

$$T_e = L_W \left| V_s \cos \theta + V_W \right| \qquad [A.8]$$

so that

$$\omega_e = \left| \frac{\omega^2}{g} V_s \cos \theta + \omega \right| \qquad [A.9]$$

where T_e is the period of encounter,

L_W is the wavelength, crest to crest,

V_W is the speed of advance of wave crest,

ω_e is the frequency of encounter,

ω is the wave frequency,

θ is the wave to course angle, and

V_s is the ship speed.

If it is desired to predict the number of peak-to-peak variations (N) from a response spectrum in terms of wave frequency for given ship speed and heading, for a given time (T) then one may proceed as follows:

The average encounter frequency may be determined from

$$\bar{\omega}_e = \sqrt{\frac{\int_o^\infty \omega_e^2\, S(\omega_e) d\, \omega_e}{\int_o^\infty S(\omega_e) d\, \omega_e}} \qquad [A.10]$$

Upon substituting Equation [A.9] into Equation [A.10], ω_e is determined. With $\bar{\omega}_e$ known, N may be determined by

$$N = \frac{T \bar{\omega}_e}{2 \pi} \qquad [A.11]$$

The analyses described previously for the wave and response spectra thus result in spectral densities as a function of encounter rather than wave frequency, unless the ship (or model) is operating at zero speed. The spectral density as a function of encounter frequency can be related to the spectral density as a function of wave frequency as follows. The total area under the spectrum must be the same whether the spectral density is given as a function of wave or as encounter frequency so that

$$\int_0^\infty S(\omega) d\omega = \int_0^\infty \overline{S(\omega_e)} \, d\omega_e \qquad [A.12]$$

where $S(\omega)$ is the spectral density as a function of wave frequency, and

$\overline{S(\omega_e)}$ is the spectral density as a function of encounter frequency

But from the relation between ω and ω_e

$$d\omega_e = \left|1 + \frac{2\omega}{g} V_s \cos\theta\right| d\omega \qquad [A.13]$$

so that

$$\int_0^\infty S(\omega) d\omega = \int_0^\infty \overline{S(\omega_e)} \left|1 + \frac{2\omega}{g} V_s \cos\theta\right| d\omega \qquad [A.14]$$

or

$$\overline{S(\omega_e)} = \frac{S(\omega)}{\left|1 + \frac{2\omega}{g} V_s \cos\theta\right|} \qquad [A.15]$$

Furthermore, the response amplitude operator may be found by using the sea and response spectra in terms of either wave or encounter frequencies:

$$\overline{RAO(\omega_e)} = \overline{S_R(\omega_e)} \Big/ \overline{S_{H_W}(\omega_e)} = \frac{S_R(\omega)}{S_{H_W}(\omega)} = RAO(\omega) \qquad [A.16]$$

where

$\overline{RAO(\omega_e)}$ is the response-amplitude operator as a function of encounter frequency,

$\overline{S_R(\omega_e)}$ is the response spectral density as a function of encounter frequency,

$\overline{S_{H_w}(\omega_e)}$ is the wave spectral density as a function of encounter frequency,

$S_R(\omega)$ is the response spectral density as a function of wave frequency,

$S_{H_w}(\omega)$ is the wave spectral density as a function of wave frequency, and

$RAO(\omega)$ is the response amplitude operator as a function of wave frequency.

An important property of the spectrum is that the area under the spectrum curve (mean-squared amplitude E) is equal to twice the mean-squared value of the function (autocorrelation function for zero lag).

$$E = \int_o^\infty S(\omega)d\omega = \frac{2}{T}\int_o^t \{y(t)\}^2\, dt = C(o) \qquad [A.17]$$

Also, under the condition that the function y(t) has zero mean, C(o) is also equal to twice the variance.[5] It is important to note, too, that for a time-stationary process having a narrow band spectrum (such as a short-term sea or response spectrum) and zero mean the area under the spectrum is equal to one-fourth the mean-squared value of the peak-to-peak variations;[5] so that

$$E = \text{area under spectrum} = \frac{1}{4}\sum_{i=1}^{N}\frac{Y_i^2}{N} = \frac{E_p}{4} \qquad [A.18]$$

where

Y_i is i^{th} peak-to-peak variation (see Figure A-3),

N is the number of variations in the sample record, and

E_p is the mean-squared value of the peak-to-peak variations.

Furthermore, for a time-stationary random process having a narrow band spectrum, such as for a ship maintaining a particular speed and heading for a short time in a particular seaway, the probability density function for the peak-to-peak variations is characterized by the mean-squared value of the variations and is given by[5,7]

$$P(Y) = \frac{2Y}{E_p} e^{-Y^2/E_p} \qquad [A.19]$$

This distribution is known as a Rayleigh distribution.

In addition, if the sample contains N peak-to-peak variations (with $N \geq 100$) the probable maximum variation is given as[5,7]

$$Y_{max} = \sqrt{E_p \log_e N} \qquad [A.20]$$

Other relations which apply in this case are[5,7,8]

$0.707\sqrt{E_p}$ is the average of most frequent peak-to-peak variation,

$0.886\sqrt{E_p}$ is the average value of all peak-to-peak variations in the sample,

$1.416\sqrt{E_p}$ is the average of the one third highest variations, and

$1.8000\sqrt{E_p}$ is the average of the one tenth highest variations.

APPENDIX B

CORRECTION FOR TUCKER WAVEMETER

During full-scale trials conducted aboard USS ESSEX a Tucker wavemeter was employed to obtain data on wave height. When employing the wavemeter, an amplitude correction due to the electronics is necessary. This correction is dependent upon the encounter period between the ship and wave and the location of the wavemeter-pressure transducers below the stillwater line.

This correction factor is given by

$$C.F. = \frac{1}{1.2rf} = \frac{(H_w)_A}{(H_w)_m} \qquad [B.1]$$

where $(H_W)_A$ is the actual wave height, $(H_W)_m$ is the measured wave height, f is an attenuation coefficient dependent upon the wave encounter period as shown in Figure B-1, and r is given by

$$r = e^{-4\pi^2 h/gT_e^2} \qquad [B.2]$$

where h is the location of the pressure transducers below the still waterline in feet, and T_e is the encounter period of the waves in seconds. Plots of the correction factor in Equation [B.1] for various depths of the pressure transducers are shown in Figure B-2. For the ESSEX test h = 15 feet.

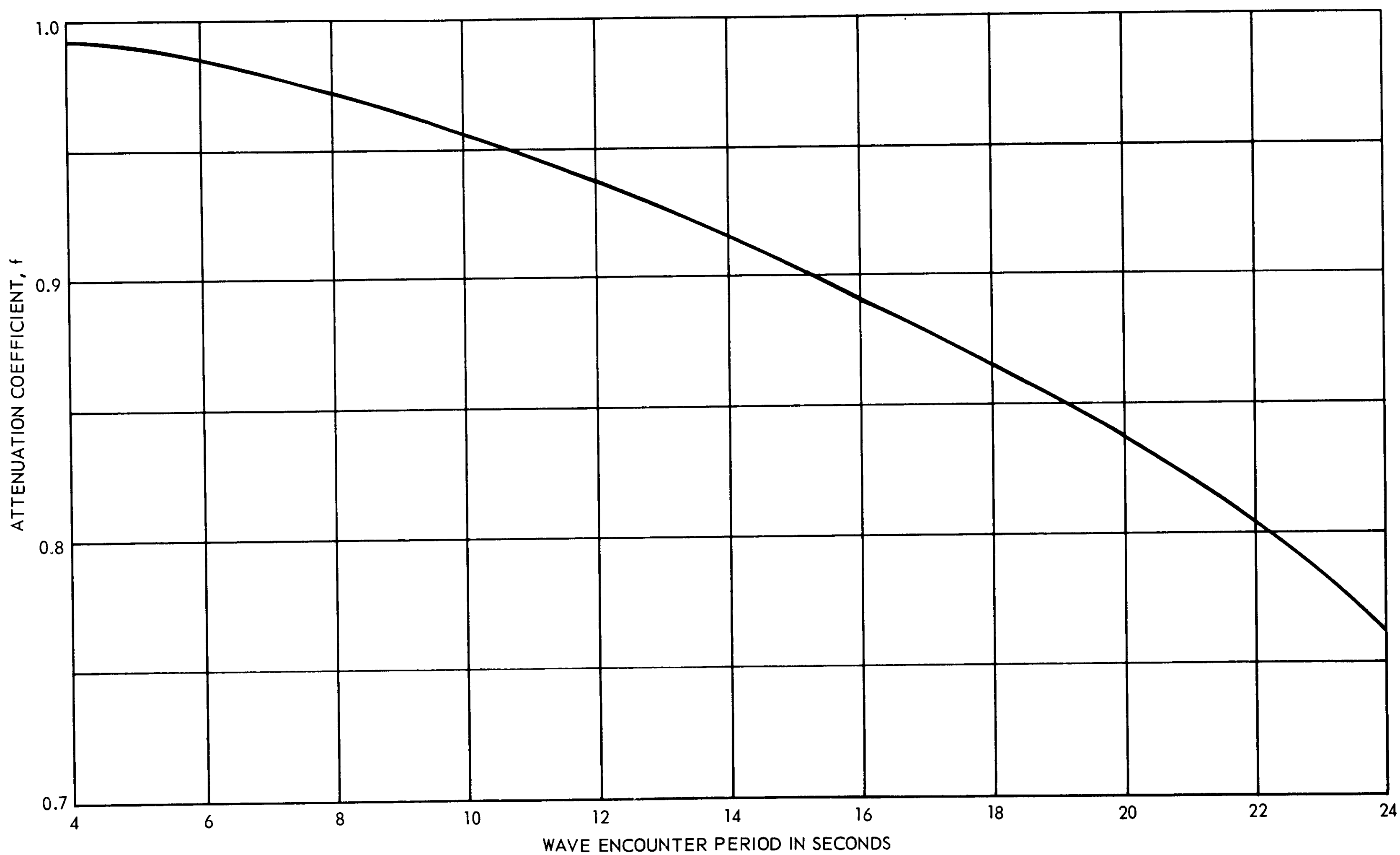

Figure B-1 - Attenuation Coefficient Curve for Wave-Height Meter

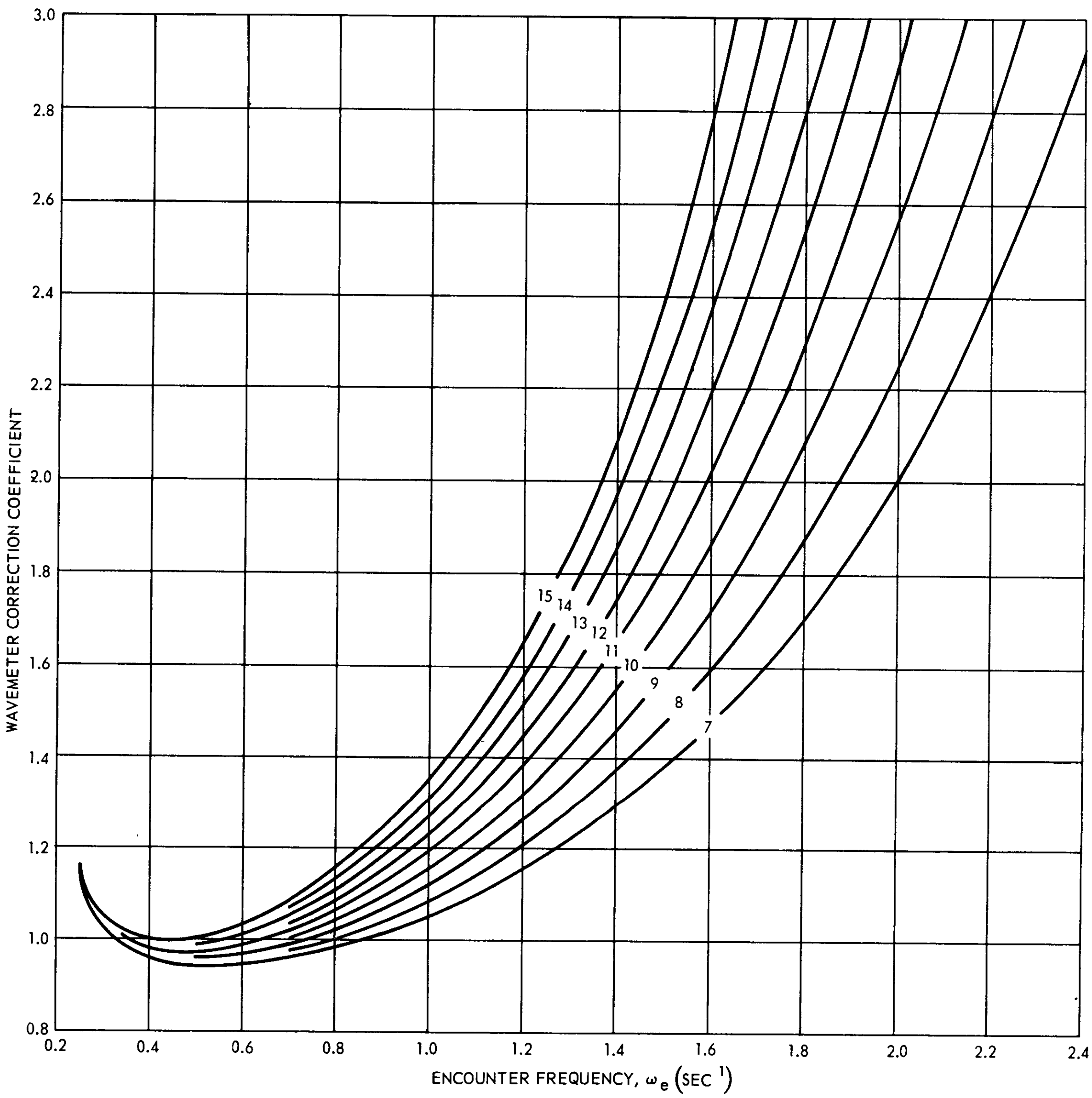

Figure B-2 - Wave-Height Meter Corrections for Various Depths

APPENDIX C

NEUMANN SEA SPECTRA FOR VARIOUS WIND VELOCITIES

The Neumann sea spectral density function for various wind velocities is defined by the following relationship

$$S(\omega) = \frac{\pi c'}{2\omega^6} e^{-2g^2/\omega^2 V^2} \qquad [C.1]$$

where g is acceleration due to gravity in ft/sec^2,

ω is circular frequency in sec^{-1},

V is wind velocity in ft/sec,

c' is 32.9 in ft^2-sec^{-5}, and

$S(\omega)$ is the spectral density function in ft^2-sec.

Upon integration of [C.1] there results

$$E = \int_0^\infty S(\omega)d\omega = \frac{3}{16} c' \left(\frac{\pi^3}{32}\right)^{1/2} \left(\frac{V}{g}\right)^5 \qquad [C.2]$$

where E is the area under the spectrum.

Upon dividing the spectral density function given in Equation [C.1] by E given in Equation [C.2], the normalized sea spectra shown in Figure 8 are obtained.

By differentiating Equation [C.1] with respect to the frequency and setting the result equal to zero, the frequency at which the peak value of the spectrum occurs may be found from the following relationship.

$$\omega_{max} = \frac{g}{V}\sqrt{\frac{2}{3}} \qquad [C.3]$$

Upon substituting Equation [C.3] for Equation [C.1] the peak value of the spectrum is found to be

$$S(\omega)_{max} = \frac{27}{16} \pi c' \left(\frac{V}{g}\right)^6 e^{-3} \qquad [C.4]$$

The spectral density function for an infinite wind speed is given as

$$S(\omega)_{\infty} = \frac{\pi c'}{2\omega^6} \qquad [C.5]$$

This is obtained from Equation [C.1] when the wind velocity is assumed to be infinite.

The ratio of the maximum peak spectral density to the area under the spectrum may be found from the ratio of Equation [C.4] to Equation [C.2] as

$$\frac{S(\omega)_{max}}{E} = 36 \sqrt{\frac{2}{\pi}} \left(\frac{V}{g}\right) e^{-3} \qquad [C.6]$$

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