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# ANALYSIS OF A CYLINDRICAL GUN FOUNDATION UNDER AXIAL LOADING

by



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## PERSONNEL

The analysis was developed by Edward Wenk, Jr., assisted by members of the Structural Mechanics Section of the David W. Taylor Model Basin. The report is the work of Edward Wenk, Jr.

## NOTATION

| | |
|---|---|
| $L$ | Axial load on trunnion, in pounds. |
| $M$ | Bending moment on any cross section of circular box girder, in inch-pounds. |
| $M_t$ | Twisting couple per unit length of centerline of box girder, in inch-pounds per inch. |
| $\theta$ | Angle of rotation of the entire cross section of the box girder, in radians. |
| $r$ | Radius of any point in the cross section of the box girder, in inches. |
| $y$ | Radial displacement of any point in the foundation, in inches. |
| $\epsilon$ | Unit elongation of any annular fiber, in inches per inch. |
| $E$ | Young's modulus for steel, in pounds per square inch. |
| $c$ | Radius* of the inner periphery of the circular box girder, in inches. |
| $d$ | Radius* of the outer periphery of the circular box girder, in inches. |
| $h$ | Height* of the box girder, in inches. |
| $a$ | Radius of the centerline of the circular box girder, in inches. |
| $M_0$ | Bending moment per unit length at the joint of cylinder and box girder, in inch-pounds per inch. |
| $P_0$ | Shearing force per unit length at the joint of cylinder and box girder, in pounds per inch. |
| $x$ | Axial displacement of any point in the foundation, in inches. |
| $e$ | Base of Napierian or natural logarithms. |
| $t_1$ | Thickness of cylinder wall, in inches. |
| $K$ | Modulus of elastic foundation, see text, page 5. |
| $\nu$ | Poisson's ratio (0.3 for steel): |
| $\beta$ | Arbitrary symbol for $\sqrt[4]{\frac{3(1-\nu^2)}{d^2 t_1^2}}$ |
| $A, B, C, D$ | Constants of integration. |
| $R$ | Axial force per unit length of the inner periphery of the box girder, in pounds per inch. |
| $\gamma$ | Angle of rotation of top and bottom of the box girder due to shear distortion, in radians. |
| $\tau$ | Shear stress in the web of the girder, in pounds per square inch. |
| $G$ | Shear modulus of the web, in pounds per square inch. |
| $t$ | Thickness of the web, in inches. |
| $\sigma$ | Stress in pounds per square inch. |

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* Subscripts $_o$ for $c$, $d$, and $h$ refer to outside dimensions of the hollow box. Subscripts $_i$ refer to inside dimensions of the hollow box.

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ABSTRACT

A new type of foundation for a 5-inch 38-caliber single-gun mount designed by the Bureau of Ships has been checked by tests of 1/5-scale steel and plastic models. A theoretical analysis to accompany the model tests of this structure is given in detail for the case of axial load. Predicted deformations and stresses based on this analysis are compared with data obtained from tests of the 1/5-scale steel model, and the two are found to be in good agreement.

INTRODUCTION

The 5-inch 38-caliber gun for which this foundation was designed is a double-purpose, high-angle gun in which the service loads may be horizontal or vertical or a combination of the two.

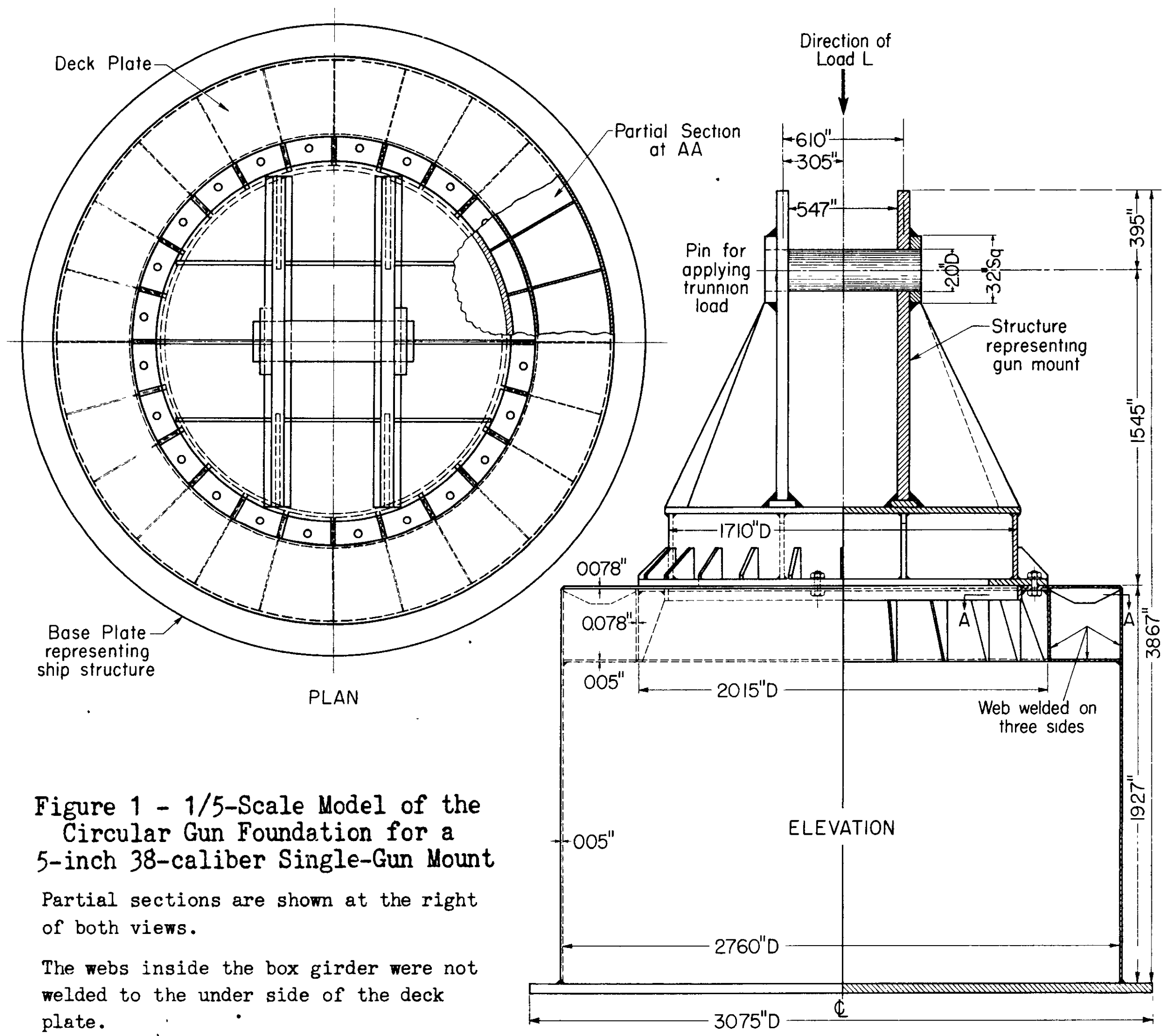


**Figure 1 - 1/5-Scale Model of the Circular Gun Foundation for a 5-inch 38-caliber Single-Gun Mount**

Partial sections are shown at the right of both views.

The webs inside the box girder were not welded to the under side of the deck plate.

The unstiffened cylindrical foundation or stool, which has proved so successful for turret guns in the United States Navy, was adopted because of its relative lightness. It was made large in diameter to accomodate special ammunition handling devices directly below the gun. As this diameter was considerably greater than that of the bolting circle of the gun mount, an inner cylindrical member was fitted at the top of the foundation just outside of the bolting circle, as shown in Figure 1. This inner member was connected to the main or outer stool by a series of 24 radial webs in a vertical plane, by a ring-shaped horizontal plate under these webs, and by the deck plate overhead. These members formed a ring-shaped hollow box girder of square section.

An extension to the deck plate inside the box, a series of 24 triangular bracket plates, and an inner stiffening ring served as a landing for the gun mount.

As the service load includes components both parallel and transverse to the axis of symmetry of the cylindrical foundation, it is necessary to investigate both components to determine the most severe case. The structure under transverse load, such as exists when the gun is firing horizontally, is not readily amenable to a theoretical analysis, but in the case of axial loading, the deformations are symmetrical so that a mathematical solution is feasible.

The analysis for axial loading is given here, in a form convenient for designers of similar structures in the future, and preliminary results of scale model tests of the cylindrical foundation are compared with the theoretical predictions.

## THEORETICAL ANALYSIS

The cylindrical gun foundation in its entirety is a somewhat complex indeterminate structure and does not lend itself readily to a complete stress analysis. However, for loading to be considered here, the structure is analogous to a flanged pipe under an axial load applied to the outer edge of the flange and distributed uniformly around it. The case of a solid flange loaded in this manner has been treated in the literature of Strength of Materials (1).* The gun foundation differs from this case in that the flange has the form of a hollow box and is located inside instead of outside the periphery of the pipe; see Figure 2.

The procedure in the calculations has been to determine the action of the axial load $L$ for a structure of the dimensions indicated in Figure 1, and the deformations in the circular box girder which result from the axial load.

In the calculations, the following assumptions concerning the structure have been made:

a. The gun mount above the circular foundation is rigid and distributes the axial load uniformly around the circumference as shown in Figure 2.

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* Numbers in parentheses indicate references on page 10 of this report.

b. The clearance of the bolt holes is sufficient so that the gun mount offers no restraint to motion of the box girder.

c. The structure under the cylinder is perfectly rigid.

Further necessary assumptions in the theoretical analysis are noted as they appear in the calculations.

Referring to Figure 3, it is evident that every radial section of the circular box girder is subjected to a torsional moment $M_t$ produced by the eccentricity of the load, and to a uniform shear $P_0$ and bending moment $M_0$ at the outside as a result of restraint to rotation offered by the cylindrical shell.

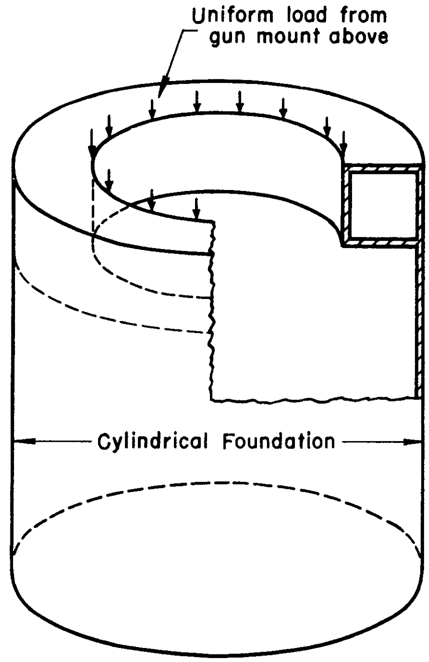


Figure 2 - Circular Box Girder analyzed as Hollow Flange of a Pipe

The axial load is $L$, as in Figure 1, and the unit axial load around the inner cylinder is $R$.

Consider first the isolated effect of the torsion $M_t$ on the box. This exists as a symmetrical and uniformly distributed action which, when separately applied, produces equilibrium in the ring considered as a free body. For convenience it is assumed that this moment is accompanied by no shearing action on the radial webs so that the girder retains its rectangular cross section under load. The effect of shear deflection of the webs is discussed separately on page 8. One-half of the box girder is shown as a free body in Figure 4. From the condition of equilibrium of moments about the diameter $oy$, there must be a bending moment $M$ on each cross section $m$ and $n$, and it can be demonstrated that

$$M = M_t a \qquad [1]$$

where $a$ is the radius of the center of the box section, and $M_t$ is the applied twisting couple per unit length of the circular centerline passing through the point $T$.

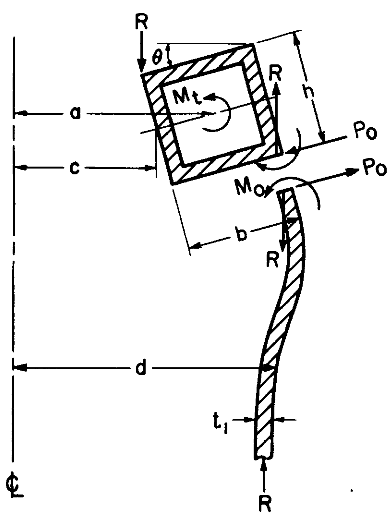


Figure 3 - Forces and Moments acting on Section of Circular Box Girder

From the condition of symmetry it can be concluded that under torsion every cross section rotates in its own plane through the same angle $\theta$. In Figure 4, let $T$ be the center of rotation, and $B$ any point in the cross section at a distance $\rho$ from $T$ and

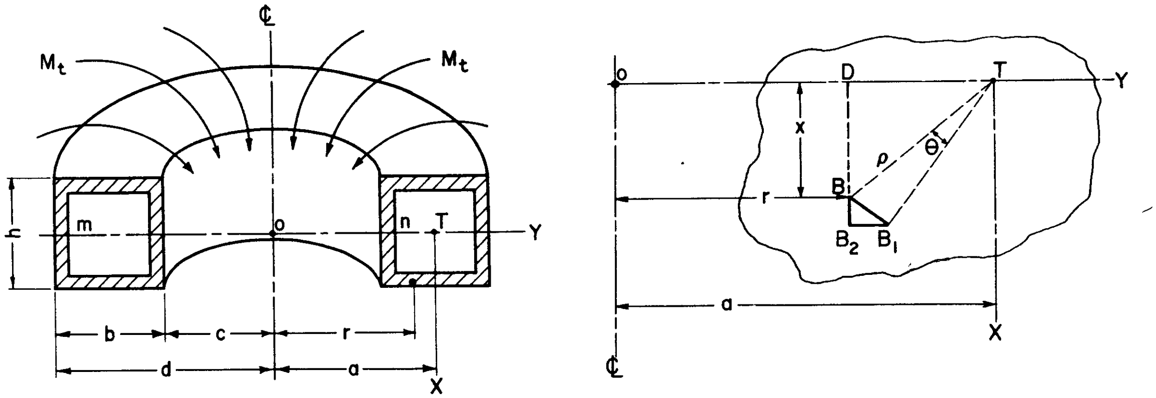


Figure 4 - Half of Circular Box Girder analyzed as a Free Body under Uniform Torsion

a radius $r$ from the center of the ring. Owing to rotation of the cross section, point $B$ describes a small arc $\overline{BB_1} = \rho\theta$. Then, owing to this displacement, the radius of the annular fiber of the circular box girder which is perpendicular to the section at point $B$ is increased from $r$ to $r + B_1B_2$. For small angles of $\theta$, $BB_1$ is almost perpendicular to $BT$ and triangles with perpendicular sides are similar. From similarity of the triangles $BB_1B_2$ and $BDT$ we have

$$\overline{B_1B_2} = \overline{BB_1}\,\frac{DB}{BT} = \rho\theta\,\frac{x}{\rho} = \theta x \qquad [2]$$

Thus the unit elongation $\epsilon$ of any circumferential fiber at radius $r$ in the box girder is

$$\epsilon = \frac{\theta x}{r} \qquad [3]$$

where $x$ is the distance of the fiber from the horizontal plane through the center of rotation. The stress $\sigma$ at any fiber in the section may be determined from Equation [3] by application of Hooke's law.

In order to relate the rotation $\theta$ to the trunnion load $L$, it is first necessary to calculate the moment $M$ about the neutral axis of the normal forces acting on the section. If $drdx$ denotes an elemental area of the cross section, the following expression is obtained

$$\iint \frac{E\theta x^2}{r}\,drdx = M \qquad [4]$$

To perform the integration on the hollow section, it is necessary to use the difference between two solid sections, one having the outside dimensions of the circular box girder and the other having the inside dimensions. Denoting the outer dimensions by the subscript $o$ and the inner by $i$,

$$M = \int_{-\frac{h_o}{2}}^{\frac{h_o}{2}}\int_{c_o}^{d_o} \frac{E\theta x^2}{r}\,drdx - \int_{-\frac{h_i}{2}}^{\frac{h_i}{2}}\int_{c_i}^{d_i} \frac{E\theta x^2}{r}\,drdx \qquad [5]$$

By integration

$$M = \frac{E\theta h_0^3}{12} \log_e \frac{d_0}{c_0} - \frac{E\theta h_i^3}{12} \log_e \frac{d_i}{c_i} \qquad [6]$$

Substituting for the values of $c$, $d$, and $h$ the dimensions of the model

$$M = 1.2296\ E\theta - 1.0647\ E\theta = 0.1649\ E\theta \qquad [7]$$

This expression thus relates the moment $M$ at every cross section directly to the angle of rotation $\theta$.

Consider now the effect on the rotation of the box that results from shear and from bending moments at the joint of the cylinder and the box. The section rotates through the angle $\theta$ and the cylinder wall bends as shown in Figure 3. $M_0$ and $P_0$ are the bending moment and shearing force at the joint per unit length of the outer circumference of the girder, and the magnitude of these quantities can be found from the condition of continuity at the junction of the cylindrical shell and the girder. The circular box girder is very rigid in the plane perpendicular to the axis of the cylinder so that radial displacements produced by forces $P_0$, if separately applied, are negligible. Consequently the horizontal deflection at the edge of the cylinder can be considered zero, i.e., in this case, the horizontal deflection in addition to that accompanying rotation of the box, Figure 4. The angle of rotation of the edge of the cylinder is equal to $\theta$, the angle of rotation of sections of the box.

Considering the cylinder as a free body, the applied loads consist of the moments and shear at the top. These are resisted mainly by circumferential tension in the cylinder. An analogous case is discussed on page 166 of Reference (1).

Taking a differential strip parallel to the cylinder axis, Figure 5, these reactive forces are distributed along the strip in proportion to the lateral deflections $y$ and the strip is in the same condition with respect to bending as a beam on an elastic foundation whose spring constant* $K = \frac{Et_1}{d^2}$. The differential equation associated with that case is therefore applicable to this; the equation of the deflection curve of the strip is thus

$$\frac{d^4y}{dx^4} = -\frac{3(1-\nu^2)}{d^2t_1^2}\, y \qquad [8]$$

Introducing the symbol

$$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{d^2t_1^2}} \qquad [9]$$

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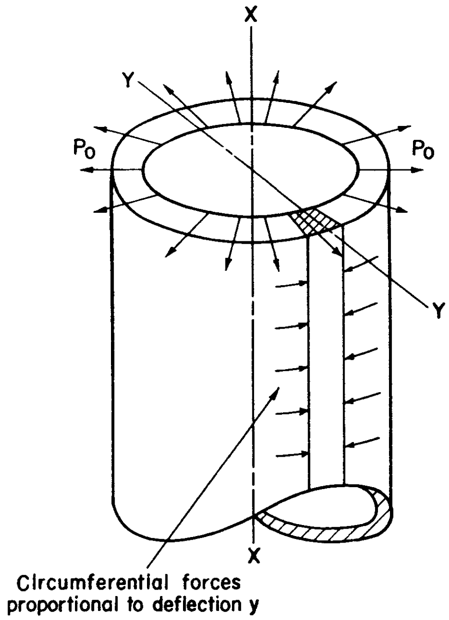
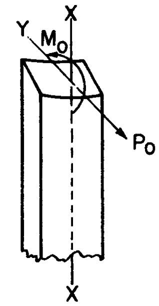
* Here $K$ is the spring constant per unit width. The familiar expression for rigidity in the theory of flexure of flat plates is given by $D = \frac{Et_1^3}{12(1-\nu^2)}$. This is related to $k$ by the equation

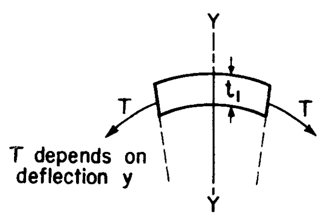
$$k = \frac{12(1-\nu^2)D}{d^2t_1^2}$$



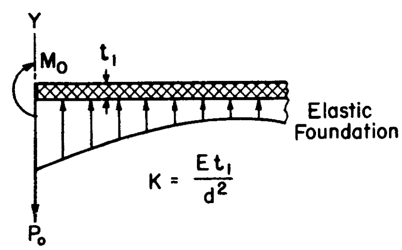

(a) Analysis of Longitudinal Strip of Cylindrical Shell




(b) Forces and Moments acting on Upper Edge of Longitudinal Strip




(c) Circumferential Tensile Forces on Strip Proportional to Lateral Deflection




(d) Longitudinal Strip Analyzed as a Beam on an Elastic Foundation

Figure 5 - Analysis of Forces and Moments in Cylindrical Shell

the general solution of Equation [8] can be represented as follows:

$$y = e^{\beta x}\,(A \cos \beta x + B \sin \beta x) + e^{-\beta x}\,(C \cos \beta x + D \sin \beta x) \qquad [10]$$

The arbitrary constants $A$, $B$, $C$, and $D$ can be determined from known conditions at certain points of the strip. Since at points infinitely distant from the force $P_0$ the deflection and curvature are equal to zero, $A$ and $B$ must be taken equal to zero. The constants of integration $C$ and $D$ can now be determined from the conditions at the origin, where the transverse load $P_0$ and the moment $M_0$ have known values.* The constants

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* The values of $M_0$ and $P_0$ at $x = 0$ are:

$$D\left(\frac{d^2y}{dx^2}\right)_{x=0} = -M_0 \qquad D\left(\frac{d^3y}{dx^3}\right)_{x=0} = P_0$$

These are reduced to terms of $k$ by the relationship $k = 4D\beta^4$.

are found to be

$$C = \frac{2\beta}{k} (P_0 - \beta M_0) \qquad [11]$$

$$D = \frac{2\beta^2 M_0}{k} \qquad [12]$$

Substituting in Equation [10], the deflection $y$ becomes:

$$y = \frac{2\beta}{k} e^{-\beta x} [P_0 \cos \beta x - \beta M_0 (\cos \beta x - \sin \beta x)] \qquad [13]$$

Returning to the assumption that displacements produced by $P_0$ are negligible

$$(y)_{x=o} = 0 = \frac{2\beta}{k} (P_0 - \beta M_0) \qquad [14]$$

and

$$\left(\frac{dy}{dx}\right)_{x=o} = \theta = -\frac{2\beta^2}{k} (P_0 - 2\beta M_0) \qquad [15]$$

From the first of these equations

$$P_0 = \beta M_0 \qquad [16]$$

Then

$$M_0 = \frac{k}{2\beta^3} \theta \qquad [17]$$

Taking $R$ as the axial force per unit length of the inner periphery of the girder, the torque per unit length of the centerline of the box girder produced by the forces as shown in Figure 3 acting along the outer edge is:

$$M_t = \frac{d}{a} \left[R(d - c) - M_0 - P_0 \frac{h}{2}\right] = \frac{d}{a} \left[R(d - c) - M_0 - M_0 \frac{h}{2} \beta\right] \qquad [18]$$

Here, as in Equation [5], it is assumed that the center of rotation is the geometric center of the section. As indicated by Equations [1] and [7],

$$M_t a = M = 0.1649\ \theta E$$

Substituting in Equation [18] and solving for $\theta$

$$\theta = \frac{d}{0.1649E} \left[R(d - c) - M_0 - M_0 \frac{h}{2} \beta\right] \qquad [19]$$

Elimination of $M_0$ from Equations [17] and [19] gives

$$\theta = \frac{\frac{d}{0.1649E} R(d - c)}{1 + \frac{k}{2\beta^3} \left(1 + \beta \frac{h}{2}\right) \frac{d}{0.1649E}} \qquad [20]$$

Using Poisson's ratio $\nu$ as 0.3 and $E = 30 \times 10^6$ pounds per square inch and substituting the various dimensions of the model in the expressions for $\beta$ and $k$

$$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{d^2 t_1^2}} = 1.49 \qquad [21]$$

$$k = \frac{E t_1}{d^2} = 0.00026E$$

Substituting the correct values in Equation [20] and solving for $\theta$

$$\theta = 1.00 \cdot 10^{-5} R \qquad [22]$$

The angle of rotation $\theta$ of the box girder of given dimensions is thus a direct function of the unit axial load $R$ and can be easily computed for various loads.

The values of $M_0$ and $P_0$ can now be obtained if desired from Equations [16] and [17], but they will be found to be small when compared to the external forces acting on the circular box girder.

Returning to the cross section of the circular box girder, it can be seen from Figure 3, page 3, that transmission of the load from the inner periphery of the box girder to the cylindrical shell is strongly assisted by the webs acting in shear. Distortion of the box girder as indicated in the broken lines of Figure 6 may occur without giving rise to a moment $M$ as in Equation [4], and it actually does occur to the extent that the webs permit it. As a result the top and bottom members of the girder rotate through the angle $\gamma$ in addition to the rotation $\theta$ of the section as a whole. $\gamma$ may be determined from the expression

$$\gamma = \frac{\tau}{G} \qquad [23]$$

where $\tau$ is shearing stress in the web, and $G$ is the shear modulus of the material. Assuming that the applied load is divided equally among the 24 webs, $\tau$ becomes $\frac{2\pi c R}{24} \div h_i t$ and substituting the proper dimensions of the model and $G$ = 11,500,000 pounds per square inch,

$$\gamma = 0.133 \cdot 10^{-5} R \qquad [24]$$

Thus, the shear distortion is proportional to the load $R$ and can be determined for any of its values. The total rotation of the top of the box girder is the sum of the two angles $\gamma$ and $\theta$.

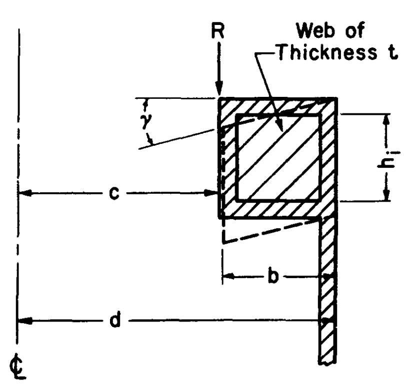


Figure 6 - Shear Distortion of Section of the Circular Box Girder

Calling $L$ the axial component of the load acting upon the trunnion of the gun mount, the circumferential load $R$ is related to $L$ by

$$R = \frac{L}{2\pi c} \qquad [25]$$

Then the expressions used to determine rotations $\theta$ and $\gamma$ of the box girder in terms of trunnion load are

$$\gamma = 0.21 \cdot 10^{-7} L \qquad [26]$$

$$\theta = 1.56 \cdot 10^{-7} L \qquad [27]$$

As mentioned in the foregoing, the stress $\sigma$ at any point in the section may be found from Equation [3] by application of Hooke's law. Thus

$$\sigma = \epsilon E = \frac{\theta x}{r} E \qquad [28]$$

Using the expression of $\theta$ in terms of load, Equation [27], the stress of any fiber at radius $r$ and distance $x$ from the axis of rotation of the section may be determined for any given load $L$.

$$\sigma = 4.68 \frac{x}{r} L \qquad [29]$$

COMPARISON WITH TEST RESULTS

A 1/5-scale steel model of the circular gun foundation shown in Figure 1 has been constructed and tested under axial loading at the David W. Taylor Model Basin (2). The test procedure and complete results will be given in a subsequent report.

An excerpt from the experimental results is given in Table 1. Observed deformations are given for an axial load of 9240 pounds, which corresponds to 3 times the brake recoil load, and these are compared with the predicted deformations. Measurements are expressed as rotations of the top and side of the circular box girder at two diametrically opposite stations.

TABLE 1

Comparison of Theoretical and Experimental Deformations for an Axial Load of 9240 pounds

A load of 9240 pounds corresponds to three times the brake recoil load of a 5-inch 38-caliber gun. Rotations are given in radians.

| Gage Station | Outer Side of Box Girder | | Top of Box Girder | |
|---|---|---|---|---|
| | Observed Rotations | Predicted Rotations $\theta$ | Observed Rotations | Predicted Rotations $\theta + \gamma$ |
| I | $10.9 \times 10^{-4}$ | $14.4 \times 10^{-4}$ | $14.8 \times 10^{-4}$ | $16.3 \times 10^{-4}$ |
| II | $11.7 \times 10^{-4}$ | | $15.7 \times 10^{-4}$ | |

Observed rotations are in every case smaller than the theoretical ones. This may be due to the restraint offered by the rigid gun mount but which was neglected in the calculations. Nevertheless, the small difference between observed and calculated values indicates that the theory is reasonably satisfactory and can be used as a guide in design calculations.

Model test data indicated that the center of rotation of the girder section was above the theoretical center of rotation, probably because of restraint introduced by the gun mount. As indicated in Equation [29], the stress in any fiber of the girder section depends on its distance from this center of rotation so that stresses in the girder may differ somewhat from calculated values. The most highly stressed fiber in the box girder was located at the inner corner, where $r$ = 10.08 inches and $x$ = 1.8 inch. It was calculated that this fiber would yield at a load of 36,000 pounds. Model tests indicated that yield occurred in the box girder at a load of 30,000 pounds. This is in good agreement with the calculated values.

A similar analysis and experimental comparison for the case of transverse loading on the gun foundation will be described in a subsequent report.

## CONCLUSION

Deformations of the circular gun foundation predicted by the theoretical method here evolved are in good agreement with experimental results.

## REFERENCES

(1) Strength of Materials, Part II, by S. Timoshenko, D. Van Nostrand Company, New York, Second Edition, 1941, pages 177-183.

(2) BuShips letter C-S74-1(341) of 2 October 1942 to TMB.