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# EXPERIMENTAL METHOD FOR DETERMINING THE VIRTUAL MASS FOR OSCILLATIONS OF SHIPS

BY J.J. KOCH, DELFT

MAY 1949

TRANSLATION 225

# EXPERIMENTAL METHOD FOR DETERMINING THE VIRTUAL MASS FOR OSCILLATIONS OF SHIPS

(EINE EXPERIMENTELLE METHODE ZUR BESTIMMUNG DER REDUZIERTEN MASSE DES MITSCHWINGENDEN WASSERS BEI SCHIFFSSCHWINGUNGEN)

by

J.J. Koch, Delft

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Translated by Georg Weinblum

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## 1. DIFFERENTIAL EQUATION OF VIRTUAL MASS

A ship performing oscillations causes oscillatory motions of the surrounding water. The apparent mass of the ship is therefore greater than the ship's own mass, so that we have to consider the added (virtual) mass when calculating the frequency of free oscillations of the ship.

In the following we assume:

1. Ideal fluid,

2. The water particles are moving in planes normal to the longitudinal axis of the ship,

3. Potential flow.

The equations of motion of the water are given by [Euler's equations] (Figure 1)

$$-\frac{\partial p}{\partial x} = \varrho\left(v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + \frac{\partial v_x}{\partial t}\right) \qquad [1]$$

$$-\frac{\partial p}{\partial y} = \varrho\left(v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + \frac{\partial v_y}{\partial t}\right) \qquad [2]$$

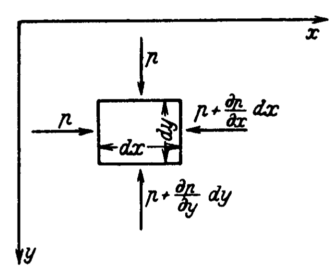


Figure 1

We have to add the equation of continuity

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \qquad [3]$$

and the condition of zero vorticity

$$\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} = 0 \qquad [4]$$

which expresses Condition 3. From Equations [1] through [4] follows as may be easily proved:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\{p + \tfrac{1}{2}\varrho\,(v_x^2 + v_y^2)\} = 0 \qquad [5]$$

Neglecting $\frac{1}{2}\varrho(v_x^2 + v_y^2)$ compared with p (which under normal conditions does not involve an error greater than about ±0.1%) and denoting $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ by $\Delta$ we get instead of [5]

$$\Delta p = 0 \qquad [6]$$

With the same degree of approximation, Equations [1] and [2] transform into

$$-\frac{\partial p}{\partial x} = \varrho \frac{\partial v_x}{\partial t} \qquad -\frac{\partial p}{\partial y} = \varrho \frac{\partial v_y}{\partial t} \tag{7}$$

We have suppressed the term $\rho g y$ expressing the static pressure as only those forces caused by the oscillation of the ship are to be considered.

## 2. BOUNDARY CONDITIONS

We assume the ship to be floating at the center of a rectangular channel so that by symmetry only one half of the ship AE has to be considered (Figure 2). Then the various boundary conditions are summarized as follows:

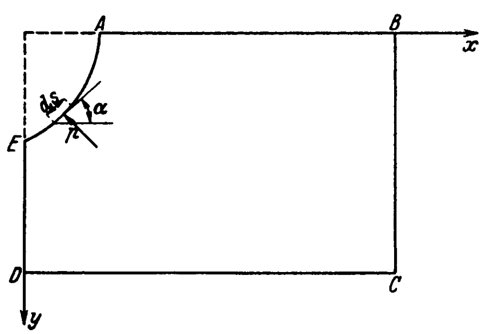


Figure 2

1. Along the free surface AB, p = 0,

2. On the wall BC of the channel $v_x = 0$, hence $\frac{\partial v_x}{\partial t} = 0$, $\frac{\partial p}{\partial x} = 0$,

3. At the bottom CD $\frac{\partial p}{\partial y} = 0$,

4. At the section DE, by symmetry, $\frac{\partial p}{\partial x} = 0$,

5. On the surface EA the water velocity normal to this surface equals the velocity component of the ship's motion in the same direction; hence denoting by $v_0$ the vertical velocity of the ship (i.e., in the direction of ED) we get for any point on EA

$$v_x \sin\alpha + v_y \cos\alpha = v_0 \cos\alpha$$

or

$$v_x \tan\alpha + v_y = v_0 \tag{8}$$

Differentiation with respect to t yields

$$-\frac{\partial p}{\partial x}\tan\alpha - \frac{\partial p}{\partial y} = \varrho \frac{\partial v_0}{\partial t} \tag{9}$$

so that using [7], condition [8] may be written

$$\frac{\partial v_x}{\partial t}\tan\alpha + \frac{\partial v_y}{\partial t} = \frac{d v_0}{d t} \tag{10}$$

## 3. DEFINITION OF VIRTUAL MASS

Denoting the mass of a unit-length element of the ship by m and the difference of shearing forces acting at the limiting planes of this element by k we get the equation of motion in the vertical direction

$$m \frac{dv_0}{dt} = k - \int p\, ds \cos\alpha = k - \int p\, dx$$

or

$$\left( m + \frac{\int p\, dx}{\frac{dv_0}{dt}} \right) \frac{dv_0}{dt} = k \qquad [11]$$

From this it follows that the mass of the ship is increased by the virtual mass

$$m_r = \frac{\int p\, dx}{\frac{dv_0}{dt}}$$

For comparing different ship forms we define a factor $\varphi$ by

$$m_r = \varphi \varrho b^2 \qquad [12]$$

where b is half the beam; hence

$$\varphi = \frac{\int p\, ds}{\varrho b^2 \frac{dv_0}{dt}} \qquad [13]$$

Although by [13] we have solved in principle how to determine the added mass, the actual computation presents serious difficulties for arbitrary ship forms. For forms represented by Figure 3 the exact solution for added masses yields

$$m_r = \tfrac{1}{2} \pi \varrho b^2$$

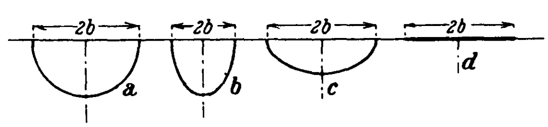


Figure 3

## 4. ELECTRICAL ANALOGY

We consider a two-dimensional electric current field in a homogenious plate of constant thickness; here the potential P is given by

$$\Delta P = 0 \qquad [14]$$

while the density of current in the direction of the x and y axis is given by

$$C_x = -\frac{1}{W}\frac{\partial P}{\partial x}, \qquad C_y = -\frac{1}{W}\frac{\partial P}{\partial y} \qquad [15]$$

Density of current means current referred to unit breadth of plate; W is the resistance of an element per unit of cross section.

Because of the complete analogy between Equations [6], [7], [14], and [15] the hydrodynamic problem can be solved by the electrical analogy provided the boundary conditions are applied in the proper way. We put

$$P = p, \qquad [16]$$

$$C_x = -\frac{1}{W}\frac{\partial P}{\partial x} = -\frac{1}{W}\frac{\partial p}{\partial x} = \frac{\varrho}{W}\frac{\partial v_x}{\partial t} \qquad [17]$$

$$C_y = -\frac{1}{W}\frac{\partial P}{\partial y} = -\frac{1}{W}\frac{\partial p}{\partial y} = \frac{\varrho}{W}\frac{\partial v_y}{\partial t} \qquad [18]$$

Thus the electrical boundary conditions are:

1. Along the surface AB p = 0, hence P = 0; AB must be soldered with a good conductor,
2. On the wall BC $\frac{\partial v_x}{\partial t} = 0$, $C_x = 0$; BC must be insulated,
3. At the bottom CD $\frac{\partial p}{\partial y} = 0$; DC must be insulated,
4. At the cross section DE $\frac{\partial p}{\partial x} = 0$; DE must be insulated,
5. Along EA we have

$$-\frac{\partial p}{\partial x}\tan\alpha - \frac{\partial p}{\partial y} = \varrho\frac{dv_x}{dt}$$

hence electrically

$$W C_x \tan\alpha + W C_y = \varrho\frac{dv_0}{dt}$$

or

$$C_x \tan\alpha + C_y = \frac{\varrho}{W}\frac{dv_0}{dt} \qquad [19]$$

The last condition is fulfilled by applying a current density a along EA.

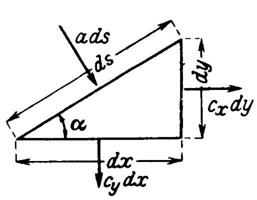


Figure 4

It follows from Figure 4 that this density a is determined by the condition that the total influx of current into the triangle with the sides, dx, dy, and ds is zero. Hence

$$a\,ds - C_x\,dy - C_y\,dx = 0 \qquad [20]$$

or with dx = ds cos $\alpha$, dy = ds sin $\alpha$

$$a = C_x \sin\alpha + C_y \cos\alpha \qquad [21]$$

Combining [19] and [21] we get finally

$$a = \frac{\varrho}{W} \frac{dv_0}{dt} \cos \alpha \qquad [22]$$

the current applied at every element of EA must be proportional to the projection of the element ds on the x-axis. The factor of proportionality is

$$\frac{\varrho}{W} \frac{dv_0}{dt}$$

The virtual mass is

$$m_r = \frac{\int p\,dx}{\frac{dv_0}{dt}} = \frac{\int P\,dx}{\frac{W}{\varrho} a} \qquad [23]$$

the factor $\varphi$ is

$$\varphi = \frac{\int P\,dx}{W b^2 a} = \frac{\int P\,dx}{W b\,(b\,a)} \qquad [24]$$

and ba is the total applied current.

## 5. EXPERIMENTAL DETERMINATION OF $m_r$ AND $\varphi$

Obviously it is impossible practically to get the right density of current at every point of the boundary curve EA. We therefore divide this curve into n parts each with equal horizontal projections on the x-axis. Each part carries a brass bar soldered to the manganine plate and connected with a wire through which flows a current $J = \frac{ba}{n}$. Formula 24 is approximated by

$$\overline{\varphi} = \frac{\frac{b}{n} \Sigma P}{W b n J} = \frac{\Sigma P}{W n^2 J}$$

The approximate virtual mass of the whole ship section is given by

$$m_r = 2\,b^2\,\overline{\varphi}$$

Figure 5 shows the system of wiring used. The manganine plate (5) denoted by ABCDEE' represents the section of the water; AB is the water surface, BC a wall, CD the bottom of the channel, DE the axis of symmetry. EE' represents the bottom of the ship and E'A its vertical side.

The storage battery (1) furnishes the current to the potentiometer (2). From this potentiometer the different currents feed the bars (4) soldered to the plate (5). They flow through the calibrated resistance (3). The loss of voltage due to the resistance is measured by the millivoltmeter (6) which is connected with both ends of each resistance (3) in turn by two switches (7) and (8). The millivoltmeter measures therefore the current

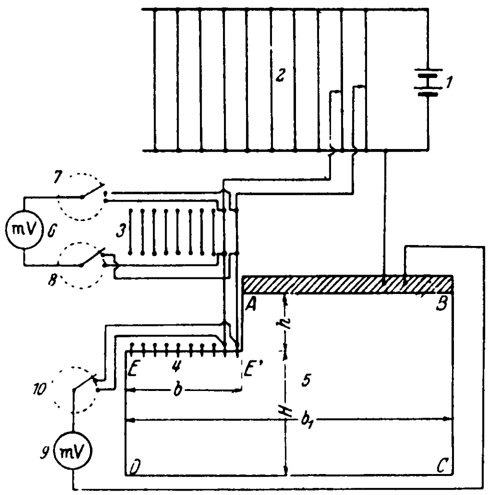


Figure 5

admitted to the bars (4); equal values of currents are reached by controlling the pertinent parts of the potentiometer.

The control is performed by trial and error, starting from currents which are approximately equal to each other. We needed from three up to seven variations till a state was reached where the currents differed not more than 1/2 % from each other; the necessary time varied between 10 minutes and 1 hour. After having reached final conditions the voltage at the bars (4) was measured by the millivoltmeter (9) which was connected in turn with them by the switch (10).

Measurements are performed for a rectangular ship section: four different drafts h(h = 0, $\frac{b}{2}$, b, $\frac{3}{2}$b) and different depths of Channel H + h (H is the distance between channel and ship bottom). The breadth of Channel $2b_1$ is so great compared with the beam of the ship 2b that it can be considered to be infinite ($b_1$ = 7b).

Testing smaller $b_1$ appeared useless as they practically do not occur. The depth of water H was changed by cutting down a strip of the manganine plate.

The specific resistance W of the plate was determined from a strip of the same material. Voltage and current were measured by the same instruments which were used for the main experiments. Thus the influence which may be caused by the resistance of the auxiliary devices was eliminated.

## 6. MODIFICATION OF WIRING DIAGRAM

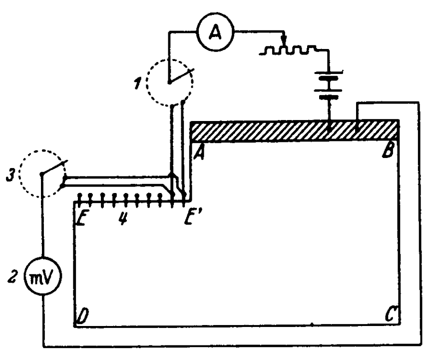


Figure 6 - Electrical Scheme for Vertical Oscillations (Simplified)

Figure 6 shows a simple wiring diagram which was used for some of the measurements. Here a fixed current is applied to one bar (4) only by switch (1) while the other bars do not get any current; in every case the voltage of all bars is measured by the millivoltmeter (2) and the switch (3). By superposition of the different results we get the voltages which would occur when all bars (4) are fed simultaneously by current.

Notwithstanding the simplicity of manipulation we have here the disadvantage that all $n^2$ voltages (n = the number of bars) must be added, which involves a loss of time when a great number of experiments are performed.

## 7. HORIZONTAL OSCILLATIONS

The virtual mass can also be determined for horizontal oscillations. For the straight lines AB, BC, CD (cf. Figure 7) the same reasoning applies as for the ship oscillating in a vertical direction. Because of the antisymmetry of the case, however, the potential will be constant along DE and equal to the potential on the straight line AB.

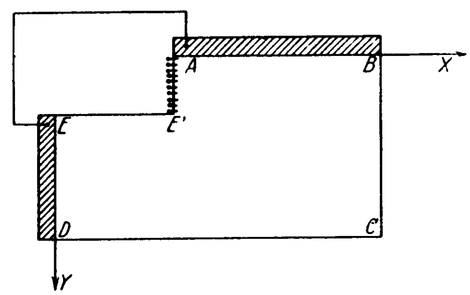


Figure 7 - Electrical Scheme for Horizontal Oscillations (Simplified)

We reproduce this fact by fitting a conductor along DE and connecting it with AB. For the line EA we get now the result, that the density of current per unit length of the vertical (y) projection must be constant.

We refer now the virtual mass to the draft of the ship h

$$m_r = 2\,\overline{\psi} h^2$$

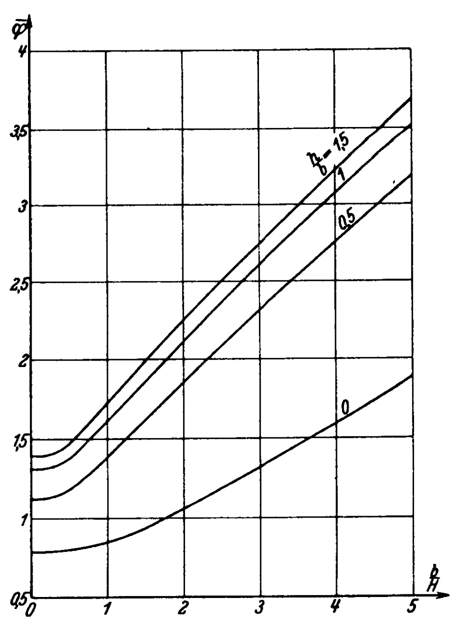

Figure 8 - Added Mass Factor for Vertical Oscillations As a Function of b/H

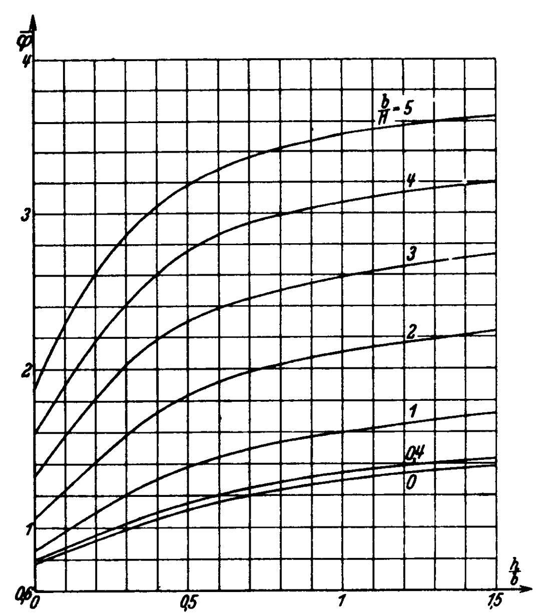

Figure 9 - Added Mass Factor for Vertical Oscillations As a Function of h/b

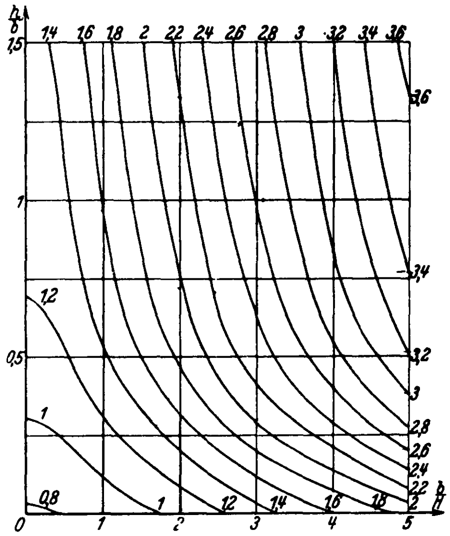

Figure 10 - Constant Added Mass Factors for Vertical Oscillations

## 8. RESULTS

For convenience we reproduce the results graphically.

All facts pertaining to vertical oscillations are represented by Figures 8, 9, and 10; to horizontal oscillations by Figures 11, 12, and 13.

Figure 8 represents $\overline{\varphi}$ as function of $\frac{b}{H}$ for $\frac{h}{b}$ = 0; 0.5; 1; 1.5.

Figure 9 represents $\overline{\varphi}$ as function of $\frac{h}{b}$ for $\frac{b}{H}$ = 0; 0.4; 1; 2; 3; 4; 5.

Figure 10 shows the curves $\overline{\varphi}$ = const for axes of reference $\frac{b}{H}$; $\frac{h}{b}$.

Figures 11 to 13 are analogous diagrams for $\overline{\psi}$ horizontal motion.

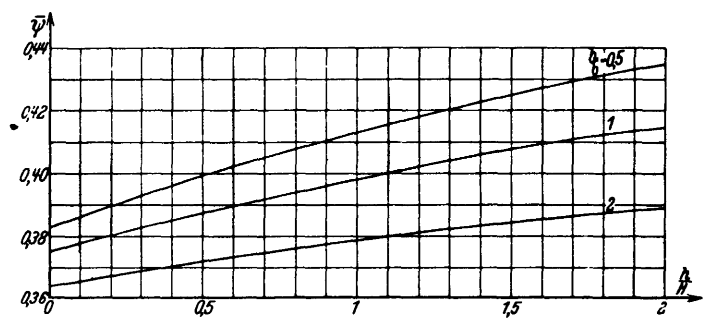

Figure 11 - Added Mass Factors for Horizontal Oscillations As a Function of h/H

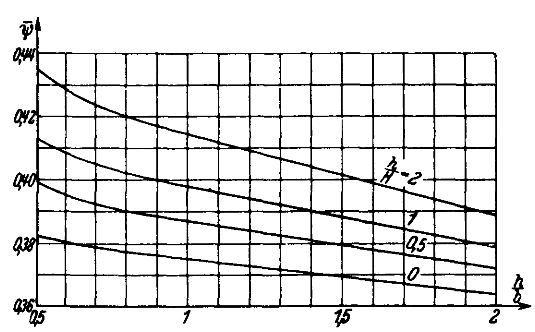

Figure 12 - Added Mass Factors for Horizontal Oscillations As a Function of h/b

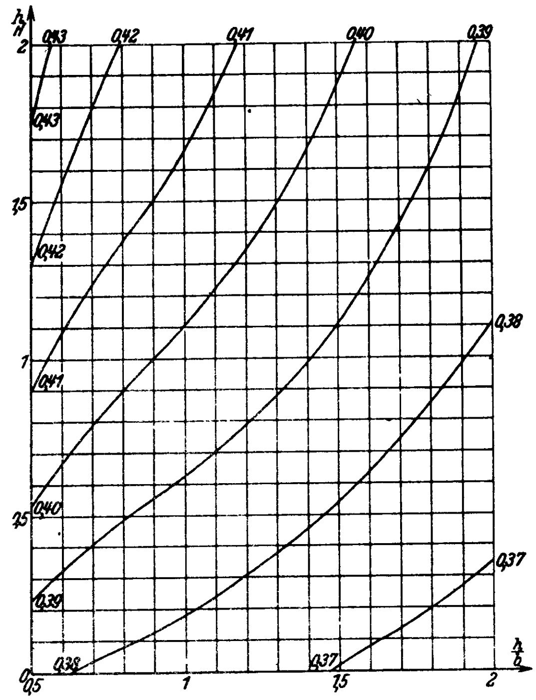


Figure 13 - Constant Added Mass Factors
for Horizontal Oscillations

## FINAL REMARKS

These investigations were performed with the aim to explain discrepancies between measured (97-98 cpm) and calculated frequencies (calculated without added masses) (152 cpm).

A repeated computation based on the results of the experiments here discussed furnished a value of 96.5 cpm - a very satisfactory agreement.

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