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THE RESISTANCE LAW FOR ROUGH PLATES

DAS WIDERSTANDSGESETZ RAUHER PLATTEN

by

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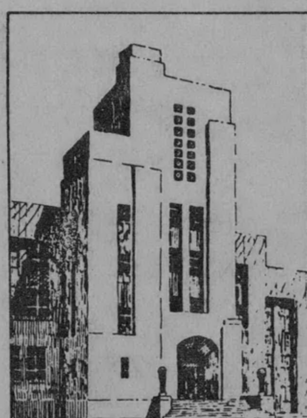
Göttingen



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THE RESISTANCE LAW FOR ROUGH PLATES

(Das Widerstandsgesetz rauher Platten.)

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INTRODUCTION

With the frictional resistance of smooth, plane plates situated lengthwise to the flow being known up to arbitrarily high Reynolds numbers from the work of Prof. von Kármán[1] and of the first author,[2] there still remains the problem of developing a similar theory for the frictional resistance of rough plates. Just as in the case of smooth plates the experimental basis of the theory for rough plates is fashioned from the results of research on flow through pipes, especially the research by Nikuradse on the flow through artifically roughened pipes as reported in detail in the Forschungsheft des V.D.I.[3]

The results of the theory on the frictional resistance of rough plates have already been reported by the first author to the 1932 Hamburg Conference on the Hydromechanical Problems of Ship Propulsion.[4] Inasmuch as the results presented there were brief, the respective calculations will be given here in somewhat greater detail.

1. In Figure 1 the coefficient of resistance λ is plotted against Reynolds number $\bar{u}d/\nu$ for the measurements of Nikuradse on pipes with various roughness ($\bar{u}$ = average velocity through the pipe, d = pipe diameter, $\frac{dp}{dx}$ = pressure drop, ρ = density, $\lambda = \frac{dp}{dx}\frac{d}{(\rho/2)\ \bar{u}^2}$). As long as the flow is laminar there is no influence of roughness present. At the critical Reynolds number the curves all rise together; then beginning with the largest roughness the curves all branch off in succession from the curve for smooth pipes. For relatively small roughness the coefficient of resistance of rough pipes is equal to that of smooth pipes over a considerable range of Reynolds numbers. Then follows an intermediate region where the coefficient of resistance increases with Reynolds numbers. Finally λ becomes constant (quadratic resistance law). This is the region of fully developed roughness. Physically these relations have essentially the following characteristics:

The determining factor in the effect of roughness on resistance is the ratio of average roughness height k to thickness of laminar sublayer δ_1 where δ_1 decreases with increasing Reynolds number $\bar{u}d/\nu$. If $k < \delta_1$, all the roughness elements lie within the thin laminar sublayer so that the roughness is hydraulically not noticeable and the resistance is equal to that of a smooth wall. If k is of the same order of magnitude as δ_1, the individual roughnesses project out of the laminar sublayer and produce eddyings which are associated with additional energy losses. Furthermore, as the thickness of the laminar sublayer diminishes with increasing Reynolds number, more and more roughness elements project out of the laminar sublayer. This increasing energy loss explains the rise in the resistance curve for the intermediate region. Finally, the laminar sublayer becomes so thin that all the roughness elements project out and the eddying causing the energy losses reaches a constant value. The

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[1]References are listed on page 13.

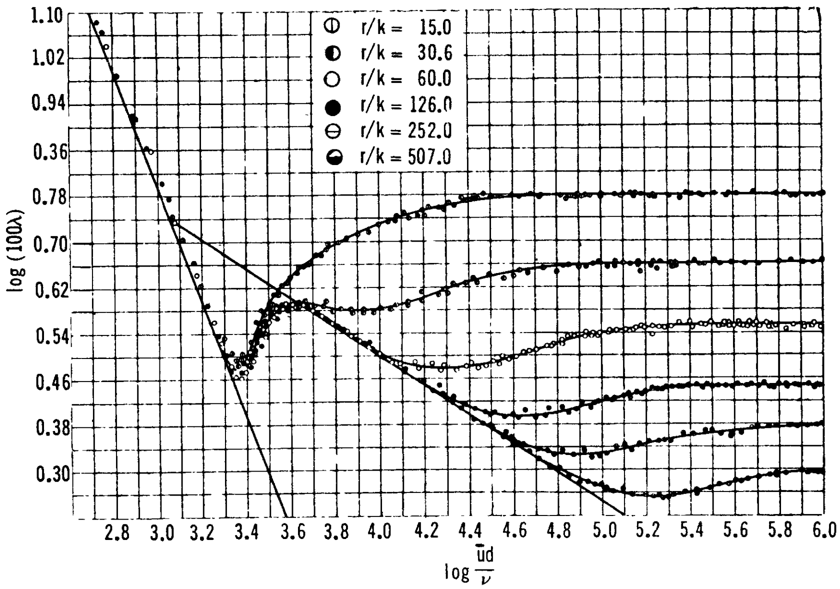

Figure 1 - Coefficient of Resistance λ of Rough Pipes as a Function of Reynolds Number $\bar{u}d/\nu$ for Various Relative Roughnesses k/r

coefficient of resistance is then independent of Reynolds number (fully developed roughness flow).

Since in the case of flow over a plate the boundary layer thickness and the accompanying laminar sublayer thickness increase along the plate, the ratio k/δ_1 for a plate of constant roughness height decreases along the plate from its greatest value near the leading edge. Thus on the forward part of the plate there is a region of fully developed roughness flow behind which is attached an intermediate region and, provided the plate is long enough, a region of hydraulically smooth flow.

The introduction of a dimensionless velocity $\phi = u/v^*$ and a dimensionless distance from the wall $\eta = yv^*/\nu$ has proved very appropriate for plotting the velocity profiles of smooth pipes. Here u denotes the actual velocity, y the actual distance from the wall, and $v^* = \sqrt{\tau/\rho}$ is a mathematical velocity (friction velocity) formed from the wall shearing stress τ. In terms of these dimensionless quantities, ϕ and η, the universal velocity law for smooth pipes is

$$\phi = A_1 + B \log \eta \qquad [1]$$

which gives a straight line when $\log \eta$ and ϕ are used as coordinates. The constants A_1 and B have here the values of $A_1 = 5.5$ and $B = 5.75$. Now if the relation $\phi = f(\log \eta)$ is also plotted for rough pipes, each relative roughness and Reynolds number gives a straight line for the dimensionless velocity profiles and furthermore all the straight lines are parallel to each other. The universal velocity law of rough pipes can therefore be written for $\eta = v^*k/\nu \cdot y/k$ in the form:

$$\phi = A + 5.75 \log \frac{y}{k} \qquad [2]$$

from which

$$A = \frac{u}{v^*} - 5.75 \log \frac{y}{k} \qquad [3a]$$

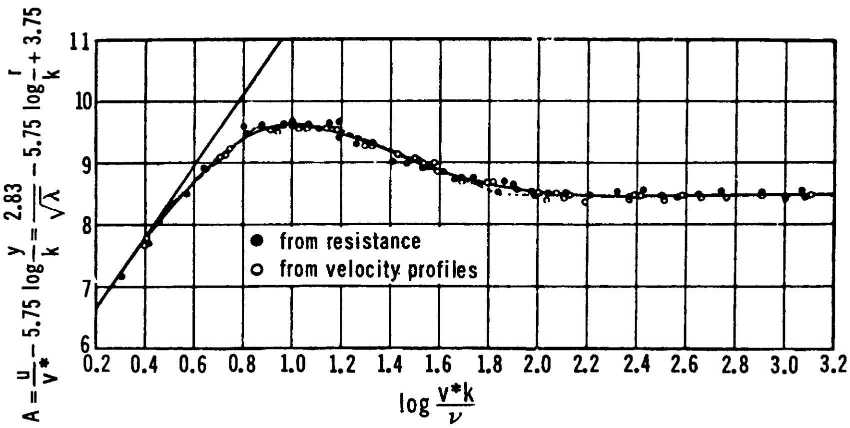

Figure 2 - $A = \frac{u}{v^*} - 5.75 \log \frac{y}{k} = \frac{2.83}{\sqrt{\lambda}} - 5.75 \log \frac{r}{k} + 3.75$ is a function of $\log \frac{v^*k}{v}$

The hereby defined quantity A which is obtained from the velocity profile in accordance with Equation [3a] has proved to be a function of roughness height k and friction velocity v^* combined in a Reynolds number v^*k/ν†. On the other hand A may be calculated from the resistance law owing to the connection between the resistance law and the velocity profile. When the work of Nikuradse is more closely studied, there results

$$A = \frac{2.83}{\sqrt{\lambda}} - 5.75 \log \frac{r}{k} + 3.75 \qquad [3b]$$

In Figure 2, A is plotted as a function of $\log v^*k/\nu$. The values of A fall on a smooth curve for the hydraulically smooth pipe, the intermediate region and the region of fully developed roughness, this curve, for the purposes of our calculation, being close to a polygonal line. The heavily drawn straight line corresponds to the hydraulically smooth pipe which from Equations [1] and [2]

$$A = 5.5 + 5.75 \log \frac{v^*k}{\nu} \qquad [4a]$$

The intermediate region is separated into two subregions, namely,

$$0.85 < \log \frac{v^*k}{\nu} < 1.15: A = 9.58 \text{ (Intermediate Region I)} \qquad [4b]$$

$$1.15 < \log \frac{v^*k}{\nu} < 1.85: A = 11.50 - 1.62 \log \frac{v^*k}{\nu} \text{ (Intermediate Region II)} \qquad [4c]$$

And for fully developed roughness flow

$$\log \frac{v^*k}{\nu} > 1.85: \; A = 8.48 \qquad [4d]$$

By means of these relations the law governing the turbulent velocity profiles of rough pipes is sufficiently specified. The calculation of the resistance of rough plates now proceeds in a way completely analogous to the earlier calculation for smooth plates.

First of all, we will now carry out the calculation for fully developed roughness flow.

†From a dimensional analysis of the situation near a smooth wall it can be concluded that the thickness of the laminar sublayer δ_1 = number $\cdot \nu/v^*$. Thus v^*k/ν is the same as k/δ_1 except for a numerical factor.

2a. Fully Developed Roughness. The resistance W of a plate of length x and of unit width is equal to the integral of the wall shearing stress τ along the plate or

$$W = \int_0^x \tau \, dx \qquad [5]$$

On the other hand, by the principle of momentum, the integral of the wall shearing stress τ is equal to the loss of momentum of the velocity profile at position x:

$$W = \int_0^x \tau \, dx = \rho \int_0^\delta u(U - u)\, dy = \rho U^2 \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad [6]$$

where U is the undisturbed velocity outside of the boundary layer. From Equations [2] and [4d] the universal velocity law reads for fully developed roughness flow

$$\frac{u}{v*} = \phi = 8.48 + 5.75 \log \frac{y}{k} = 2.50 \ln 29.9 \frac{y}{k} = a \ln (1 + b\theta)$$

where

$$\theta = \frac{y}{k}, \; a = 2.50, \; b = 29.9$$

and a unit quantity not affecting the relation is added to help in a later integration. Any value at the outer edge of the boundary layer is indicated by a subscript 1; thus

$$\theta_1 = \frac{\delta}{k}, \; \phi_1 = \frac{U}{v*}$$

which gives in Equation [6]

$$W = \rho k U^2 \int_0^{\theta_1} \left[\frac{\phi}{\phi_1} - \left(\frac{\phi}{\phi_1}\right)^2\right] d\theta$$

With the abbreviation

$$F(\theta_1) = \int_0^{\theta_1} \left[\frac{\phi}{\phi_1} - \left(\frac{\phi}{\phi_1}\right)^2\right] d\theta \qquad [7]$$

the resistance is then given by the formula

$$W = \rho k U^2 F(\theta_1) \qquad [8]$$

From a differentiation of Equation [5]

$$\frac{dW}{dx} = \tau = \rho v*^2 = \rho \frac{U^2}{\phi_1^2}$$

Inserting for dW/dx its value from Equation [8] gives

$$\rho \frac{U^2}{\phi_1^2} = \rho k U^2 \frac{dF}{d\theta_1} \frac{d\theta_1}{dx}$$

and with additional abbreviation

$$G(\theta_2) = \int_0^{\theta_2} \phi_1^2 \frac{dF}{d\theta_1} d\theta_1 \qquad [9]$$

there results for plate length x

$$x = k G(\theta_2) \qquad [10]$$

By means of Equations [8] and [10] the resistance law has been found in parametric form such that resistance W and plate length x are represented as functions of parameters $\theta_{1,2} = \delta/k$.

Integrals [7] and [9] have to be evaluated if numerical results are to be obtained later. When the abbreviation

$$1 + b\theta_{1,2} = z$$

is introduced there results

$$F(z) = \frac{1}{b}\left[\frac{z+1}{\ln z} - 2\frac{z-1}{(\ln z)^2}\right]$$

$$G(z) = \frac{a^2}{b}[z \ln z - 4z + 4 - \ln z + 4 E_i(\ln z) - 4 \ln \ln z]^{\dagger} \qquad [11]$$

The numerical calculation of the local and total resistance coefficients as functions of x/k can be performed by means of these two equations (see paragraph 3).

2b. Intermediate Region I. The formulas for the intermediate regions are constructed exactly like those for fully developed roughness. The universal velocity law for this region is from Equations [2] and [4b] $\phi = 9.58 + 5.75 \log y/k = 2.50 \ln(1 + 46.3\,\theta) = a \ln(1 + b_1\theta)$ where $b_1 = 46.3$. Inserting for this region

$$W = \rho k U^2 F_1(\theta_1) \text{ and } x = k G_1(\theta_1) \qquad [12]$$

gives immediately from the preceeding formulas

$$F_1 = \frac{b}{b_1} F = 0.646\, F; \quad G_1 = \frac{b}{b_1} G = 0.646\, G \qquad [13]$$

so that the whole calculation for this region is reduced to that for fully developed roughness flow.

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†Here E_i denotes the logarithmic integral which is defined as $E_i(x) = \int_{\infty}^{-x} \frac{e^{-u}}{u} du$

2c. Intermediate Region II. The calculation for this region turns out to be somewhat different from the preceeding methods for the other velocity laws. From Equations [2] and [4c] there is for this region

$$\phi = 11.50 - 1.62 \log \frac{v^*k}{\nu} + 5.75 \log \frac{y}{k} = 2.50 \, (\ln \psi + 4.60)$$

where

$$\psi = \frac{y}{k} \left(\frac{v^*k}{\nu}\right)^{-0.282} \dagger \qquad [14]$$

A one may be added moreover so that

$$\phi = a \ln (1 + b_2 \psi) \qquad [15]$$

where $b_2 = 98.8$. Since

$$\psi = \frac{y}{k} \phi_1^{0.282} \left(\frac{Uk}{\nu}\right)^{-0.282}$$

there results from Equation [6]

$$W = \rho U^2 k \left(\frac{Uk}{\nu}\right)^{0.282} F_2 (\psi_1) \qquad [16]$$

where

$$F_2 (\psi_1) = \phi_1^{-0.282} \left(\frac{1}{\phi_1} \int_0^{\psi_1} \phi \, d\psi - \frac{1}{\phi_1^2} \int_0^{\psi_1} \phi^2 \, d\psi\right) \qquad [17]$$

In a way wholly analogous to that for fully developed roughness flow, the formula is obtained for plate length x

$$x = k \left(\frac{Uk}{\nu}\right)^{0.282} G_2 (\psi_2) \qquad [18]$$

where

$$G_2 (\psi_2) = \int_0^{\psi_2} \phi_1^2 \frac{dF_2}{d\psi_1} d\psi_1 \qquad [19]$$

The calculation of Integrals [17] and [19] gives for $1 + b_2 \psi_{1,2} = t$

$$F_2 (t) = \frac{a^{-0.282}}{b_2} (\ln t)^{-0.282} \left[\frac{t+1}{\ln t} - 2 \frac{t-1}{(\ln t)^2}\right] \qquad [20]$$

$$G_2 (t) = \frac{a^{1.718}}{b_2} \int_1^t \left[\ln t - 3.282 - \frac{1.282}{t} + \frac{4.564}{\ln t} - \frac{4.564}{t(\ln t)}\right] (\ln t)^{-0.282} \, dt$$

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†Translator's Note: To prevent possible confusion of notation, ψ is substituted for the χ of the original paper; and later in this paper t for u and α, β, and γ for subscripts I, II, and III of the original paper.

Further calculation of this last integral must depend on numerical or graphical methods.

3. All the necessary formulas are now known for plotting curves of the local coefficient of resistance c_f' and the total coefficient of resistance c_f as a function of Reynolds number Ux/ν and of dimensionless roughness x/k. If the plate is so long that it extends over more than one of the designated regions of paragraph 1, it will be necessary then to connect up each of the separate regions for which the appropriate formulas have been found. Owing to the large value of shearing stress τ at the leading edge of a rough plate, the dimensionless parameter v_*k/ν is very large there, decreasing then downstream along the plate. The forward part of the plate is in a region of fully developed roughness flow which is followed by Intermediate Regions II and I and finally by the portion of the plate farthest removed from the leading edge which is effectively hydraulically smooth. Piecewise then as stated in Paragraph 1.

$$\begin{aligned} \infty > \frac{v_*k}{\nu} &\geq 70.8 && \text{Fully developed roughness} \\ 70.8 \geq \frac{v_*k}{\nu} &\geq 14.1 && \text{Intermediate Region II} \\ 14.1 \geq \frac{v_*k}{\nu} &\geq 7.08 && \text{Intermediate Region I} \\ 7.08 \geq \frac{v_*k}{\nu} &\geq 0 && \text{Hydraulically smooth} \end{aligned} \qquad [21]$$

A plate of uniform roughness will now be considered which is of such length that the end of the plate lies in Intermediate Region I. Then there is a constant

$$\frac{Uk}{\nu} = C$$

over all of the region to be integrated.

To begin, the value of parameter $z_\alpha = 1 + b\theta_{1,\alpha}$ is calculated which is the value at the end of the fully developed roughness region. With $v_*k/\nu = 70.8$ it follows that

$$\frac{Uk}{\nu} = \left(\frac{v_*k}{\nu}\right)\left(\frac{U}{v_*}\right) = 70.8\ \phi_{1,\alpha} = C$$

$$\phi_{1,\alpha} = 2.5 \ln z_\alpha = \frac{C}{70.8}$$

$$z_\alpha = e^{C/177}$$

The part of the resistance due to the region of fully developed roughness becomes then

$$W_{\text{fully rough}} = \rho U^2 k F(z_\alpha)$$

and the corresponding plate length

$$x_{\text{fully rough}} = k\,G\,(z_\alpha)$$

where F and G are the functions given by Equation [11] and the accompanying numerical table.

z / t	$c_f' \times 10^3$	$F(z)$	$G(z)$	$F_2(t)$	$G_2(t)$	$2\frac{F(z)}{G(z)} \times 10^3$	$2\frac{F_2(t)}{G_2(t)} \times 10^3$
10^2	15.1	0.4213	3.643×10^1	0.0631	5.195	23.1	24.3
2×10^2	11.4	0.7946	9.445×10^1	0.1144	1.318×10^1	16.8	17.4
3×10^2	9.84	1.150	1.615×10^2	0.1621	2.227×10^1	14.3	14.6
5×10^2	8.28	1.832	3.140×10^2	0.2520	4.228×10^1	11.7	11.9
7×10^2	7.46	2.490	4.832×10^2	0.3379	6.417×10^1	10.3	10.5
10^3	6.71	3.453	7.519×10^2	0.4609	9.909×10^1	9.18	9.30
2×10^3	5.54	6.490	1.766×10^3	0.8432	2.261×10^2	7.35	7.46
3×10^3	4.99	9.407	2.878×10^3	1.205	3.637×10^2	6.54	6.63
5×10^3	4.41	1.503×10^1	5.270×10^3	1.891	6.571×10^2	5.70	5.76
7×10^3	4.08	2.048×10^1	7.841×10^3	2.548	9.678×10^2	5.22	5.27
10^4	3.77	2.843×10^1	1.195×10^4	3.500	1.454×10^3	4.76	4.82
2×10^4	3.26	5.390×10^1	2.660×10^4	6.501	3.177×10^3	4.05	4.09
3×10^4	3.01	7.845×10^1	4.232×10^4	9.354	5.002×10^3	3.71	3.74
5×10^4	2.73	1.260×10^2	7.564×10^4	1.482×10^1	8.826×10^2	3.33	3.36
7×10^4	2.57	1.722×10^2	1.106×10^5	2.008×10^1	1.281×10^4	3.11	3.14
10^5	2.41	2.400×10^2	1.654×10^5	2.774×10^1	1.896×10^4	2.90	2.93
2×10^5	2.15	4.581×10^2	3.581×10^5	5.208×10^1	4.048×10^4	2.56	2.57
3×10^5	2.01	6.693×10^2	5.611×10^5	7.538×10^1	6.295×10^4	2.39	2.40
5×10^5	1.86	1.080×10^3	9.888×10^5	1.202×10^2	1.095×10^5	2.18	2.20
7×10^5	1.77	1.481×10^3	1.433×10^6	1.637×10^2	1.575×10^5	2.07	2.08
10^6	1.68	2.070×10^3	2.124×10^6	2.273×10^2	2.313×10^5	1.95	1.97
2×10^6	1.52	3.975×10^3	4.514×10^6	4.304×10^2	4.870×10^5	1.76	1.77
3×10^6	1.44	5.824×10^3	7.022×10^6	6.257×10^2	7.518×10^5	1.66	1.67
5×10^6	1.34	9.434×10^3	1.224×10^6	1.004×10^3	1.297×10^6	1.54	1.55
7×10^6	1.29	1.297×10^4	1.760×10^7	1.372×10^3	1.856×10^6	1.47	1.48
10^7	1.23	1.817×10^4	2.588×10^7	1.909×10^3	2.713×10^6	1.40	1.41
2×10^7	1.13	3.505×10^4	5.463×10^7	3.642×10^3	5.654×10^6	1.28	1.29
3×10^7	1.08	5.150×10^4	8.444×10^7	5.312×10^3	8.685×10^6	1.22	1.22
5×10^7	1.02	8.367×10^4	1.460×10^8	8.561×10^3	1.489×10^7	1.15	1.15
7×10^7	0.98	1.152×10^5	2.093×10^8	1.173×10^4	2.124×10^7	1.10	1.10
10^8	0.94	1.618×10^5	3.062×10^8	1.638×10^4	3.093×10^7	1.06	1.06

In Intermediate Region II, parameter ψ_1 is utilized together with θ_1 in equation

$$\psi_1 = \theta_1 \left(\frac{v^*k}{\nu}\right)^{-0.282}$$

At the beginning of Intermediate Region II where $\frac{v^*k}{\nu} = 70.8$ then

$$\psi_{1,\alpha} = 0.302\ \theta_{1,\alpha}$$

and

$$1 + b_2\psi_{1,\alpha} = t_\alpha = 1 + 0.302\ \frac{b_2}{b}\left(e^{C/177} - 1\right)$$

and, if the unnecessary unit values are dropped,

$$t_\alpha = 0.302\ \frac{b_2}{b}\ e^{C/177} = e^{C/177} = z_\alpha$$

where $b/b_2 = 29.9/98.8 = 0.302$ from Paragraphs 2a and 2c.

The value of parameter $t_\beta = 1 + b_2\psi_{1,\beta}$ at the end of Intermediate Region II corresponding to $v^*k/\nu = 14.1$ is obtained by a similar calculation as that before and

$$t_\beta = e^{C/35.3}$$

From this the part of the resistance contributed by Intermediate Region II is

$$W_{IR\text{-}II} = \rho U^2 k\, C^{0.282}\, [F_2(t_\beta) - F_2(t_\alpha)]$$

and the corresponding plate length

$$x_{IR\text{-}II} - x_{\text{fully rough}} = k\, C^{0.282}\, [G_2(t_\beta) - G_2(t_\alpha)]$$

where F_2 and G_2 are given by Equation [20] and in the numerical table.

Finally the end points of Intermediate Region I are obtained in a like manner. The value of parameter z_β at the beginning of this region becomes for $v^*k/\nu = 14.1$

$$z_\beta = e^{C/35.3} = t_\beta$$

and at the end where $v^*k/\nu = 7.08$

$$z_\gamma = e^{C/17.7}$$

The part of the resistance due to Intermediate Region I amounts to

$$W_{IR\text{-}I} = \rho\, U^2 k\, [F_1(z_\gamma) - F_1(z_\beta)]$$

and the corresponding plate length

$$x_{IR\text{-}I} - x_{IR\text{-}II} = k\, [G_1(z_\gamma) - G_1(z_\beta)]$$

where functions F_1 and G_1 are given by Equations [13] and [11]. By a summation of the parts of the resistance the total resistance of a rough plate whose end lies in Intermediate Region I becomes

$$W = \rho k U^2 \{F(z_\alpha) + C^{0.282} [F_2(t_\beta) - F_2(t_\alpha)] + F_1(z) - F_1(z_\beta)\}$$

and the corresponding plate length

$$x = k \{G(z_\alpha) + C^{0.282} [G_2(t_\beta) - G_2(t_\alpha)] + G_1(z) - G_1(z_\beta)\}$$

These equations may be still simplified considerably. By means of a calculation which it is not necessary to reproduce here it can be shown that all the terms cancel out when the individual regions are joined together. This calculation gives

$$F(z_\alpha) = C^{0.282} F_2(t_\alpha)$$

$$C^{0.282} F_2(t_\beta) = F_1(z_\beta)$$

$$G(z_\alpha) = C^{0.282} G_2(t_\alpha)$$

$$C^{0.282} G_2(t_\beta) = G_1(z_\beta)$$

By means of these equations the resistance and length of a plate whose end lies in Intermediate Region I becomes

$$W = \rho k U^2 F_1(z)$$

$$\frac{Ux}{\nu} = \frac{Uk}{\nu} G_1(z) \qquad [22a]$$

If the end of the plate lies in Intermediate Region II or in the region of fully developed roughness, the corresponding formulas are respectively

$$W = \rho k U^2 F_2(t); \; \frac{Ux}{\nu} = \frac{Uk}{\nu} G_2(t)$$

and [22b, c]

$$W = \rho k U^2 F(z); \; \frac{Ux}{\nu} = \frac{Uk}{\nu} G(z)$$

From the final formulas just now found it follows that the resistance depends only on dimensionless plate length Ux/ν and dimensionless roughness Uk/ν.

The applicable formula for the local coefficient of resistance $c_f' = \frac{\tau}{(\rho/2)U^2}$ for any region as a result of $\tau/\rho = v^{*2}$ and $U/v^* = \phi_1$ is

$$c_f' = \frac{2}{\phi_1^2} \qquad [23]$$

whereas the formula for the total coefficient of resistance $c_f = \frac{W}{(\rho/2)\, U^2 x}$ becomes from Equation [22] respectively

$$c_f = 2\frac{F_1(z)}{G_1(z)} = 2\frac{F_2(t)}{G_2(t)} = 2\frac{F(z)}{G(z)} \qquad [24]$$

depending on whether the end of the plate lies in Intermediate Region I, Intermediate Region II or in the region of fully developed roughness. The functions c_f', $F(z)$, $G(z)$, $F_2(t)$, $G_2(t)$, $2\frac{F(z)}{G(z)}$ and $2\frac{F_2(t)}{G_2(t)}$ are presented in the accompanying numerical table. The inclusion of $G_1(z)$ and $\frac{F_2(t)}{G_2(t)}$ is not necessary since $G_1(z) = 0.646\, G(z)$ and $\frac{F_1(z)}{G_1(z)} = \frac{F(z)}{G(z)}$ fromEquation [13].

From this numerical table curves of c_f' and c_f can immediately be plotted as functions of Reynolds number Ux/ν for specified values of Uk/ν. Such diagrams are reproduced in Figures 3 and 4. In addition a family of curves for x/k = constant has also been plotted. A curve of x/k = constant gives the behavior of the coefficient of resistance at the same position on the plate for various velocities while a curve of Uk/ν = constant represents the behavior at various positions on the plate for the same velocity.

It has been assumed throughout our calculations that the boundary layer has turbulent flow from the very beginning of the plate. The inclusion of an initial laminar flow would still further complicate the relationships.

The resistance law just derived for rough plates has chiefly validity for a very specific type of roughness, namely a smooth surface to which sand grains have been densely attached and where the Nikuradse pipe results have been taken as the basis. The densities involved were about 4600 grains per cm^2 for grain size $k = 0.01$ cm, 1130 grains per cm^2 for $k = 0.02$ cm, 590 grains per cm^2 for $k = 0.04$ cm and 150 grains per cm^2 for $k = 0.08$ cm. For other types of roughnesses, for example, other shapes of roughness elements or other roughness densities the resistance law of the rough plate will be different.

A single roughness parameter (the relative roughness) will in all likelihood no longer answer the purpose in continued investigations of the roughness problem. A second parameter, perhaps the roughness density, ought probably to be introduced in order for all types of roughnesses met in practice to be amenable to theory. Extensive additional researches are being planned at the Kaiser Wilhelm Institute for Flow Research at Göttingen. For this purpose a longer channel with a rectangular cross-section is to be built of which one side wall is smooth and the other side wall is rough and removable. By this experimental arrangement the resistance of plates with a great variety of roughnesses can be investigated in a relatively simple way as to the effect of Reynolds number. Moreover an actual section of a ship's hull can be mounted in the channel and the corresponding resistance determined.

Hence hope is justified that questions asked in practice involving roughness problems can soon be answered to a substantial degree.

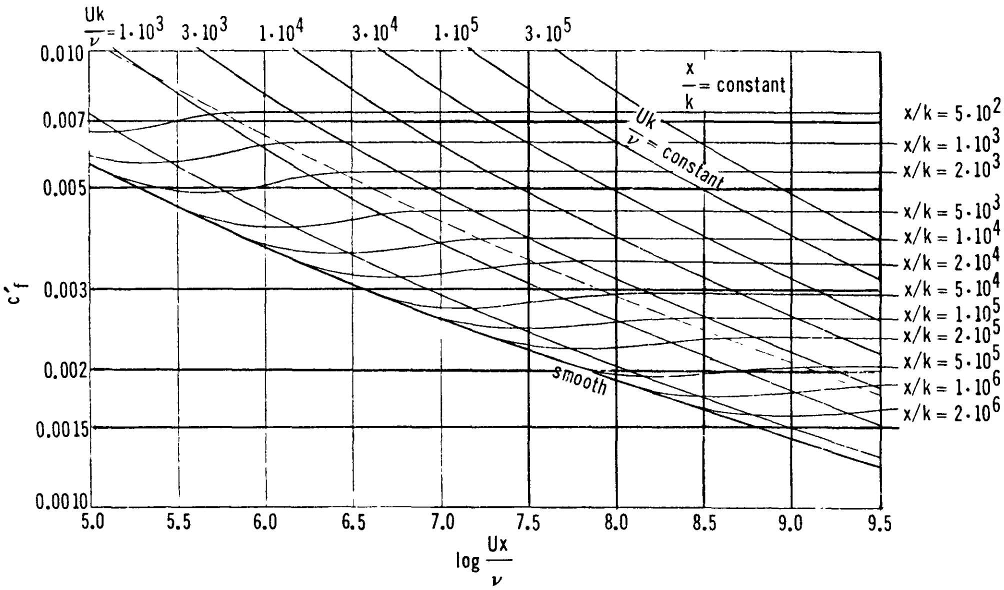

Figure 3 - Local Coefficient of Resistance $c_f{}'$ of Rough Plates as a Function of Reynolds Number Ux/ν for Various Roughnesses Uk/ν

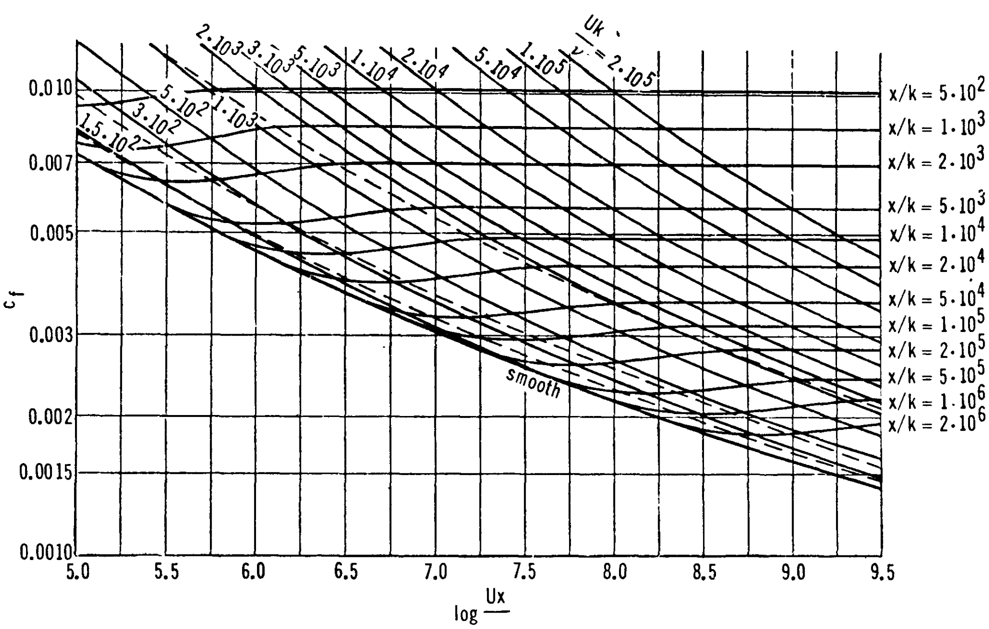

Figure 4 - Total Coefficient of Resistance c_f of Rough Plates as a Function of Reynolds Number Ux/ν for Various Roughnesses Uk/ν

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The results of the theory on the frictional resistance of rough plates is given in detail in this report.

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