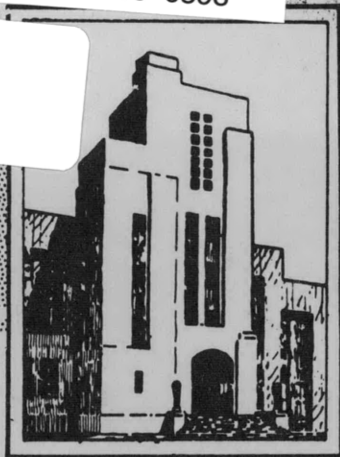

DEPARTMENT OF THE NAVY
DAVID TAYLOR MODEL BASIN

[redacted]

HYDROMECHANICS

o

AERODYNAMICS

o

STRUCTURAL MECHANICS

o

APPLIED MATHEMATICS

ON THE HYDRODYNAMICALLY DETERMINED THRUST AND TORQUE FLUCTUATIONS IN THE PROPELLING MACHINERY OF SHIPS

(Ueber die Hydrodynamisch bedingten Schub- und Drehmoment-Schwankungen in Schiffsantriebs-Anlagen)

by

S. Schuster, Dr. Eng., Berlin

Translated by E.N. Labouvie, Ph. D.

March 1959

Translation 275

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ABSTRACT

Periodic load variations to which ship propellers are subjected may cause the overloading of bearings and gears. By means of simplifying assumptions, it is possible to determine the characteristic time curve of these load fluctuations as a function of the most fundamental parameters and to deduce therefrom measures for the reduction of these fluctuations.

INTRODUCTION

The lists of new ships being built all over the world today comprise a surprising number of large units which are remarkable, moreover, on account of their great fullness, great performance loads, and the relatively high speeds to be attained. The preference of these ship types which is based on their economic superiority is essentially due to the choice of a single-screw drive which permits the regaining of a maximum amount of the energy lost to the water on account of the friction of the hull.

Due to the concentration of the total propulsive output of the ship on a single, large screw, the periodic fluctuations which occur during its operation are naturally more pronounced than in the case of the multiple-screw drives which were more commonly used before for ships of such size. In dimensioning the individual parts of a projected power plant, the marine engineer of today can hardly get along without a thorough knowledge of the vibrations which the propeller is expected to produce. The naval architect, too, must be familiar with the ways and means whereby he may alleviate the conditions confronting the marine engineer by improving the lines of the hull, the design of the propeller race, and the shape of the propeller.

In practice, we have been able most of all to cope with the partial problem of a suitable adaptation of the mass forces of large ship propellers to the stiffness requirements of the shaft lines. (See Reference 1, for instance.)** The more complicated partial problem concerning the hydrodynamic interaction between ship and propeller, too, has repeatedly been the subject of a thorough-going theoretical treatment. (See References 2 and 3, for instance.) The data which

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*Communication of the Versuchsanstalt für Wasserbau und Schiffbau in Berlin (Berlin Tank).

**References are listed on page 17.

in increasing measure have been obtained empirically for certain cases in the various ship model basins, as well as on trial trips, afford a basis for selecting the most suitable blade number and the most favorable afterbody design. (See References 4, 5, and 6, for instance.)

In the following we shall attempt to clarify in a very general way the mathematical relationship and the order of magnitude of the individual factors in order to be able to estimate for certain cases the thrust and torque fluctuations which are to be expected in actual practice.

DERIVING A GENERAL LAW FOR THE PROPELLER THRUST

The ship propeller by its rotation accelerates the water flowing into it. In doing so, the propeller acts on the water with the resultant force $\bar{P}$ and it experiences itself as a reaction an equally large hydrodynamic force $\bar{P}$ in opposite direction which is mechanically transmitted to the ship by the shaft, the bearings, and the gear unit. Inasmuch as the flow velocity v_p in way of the propeller is not constant over the propeller disk either in respect to magnitude or direction on account of the hull in front of it, each blade element is subject to a fluctuating load.

Quite generally, the force vector $\bar{P}$ on a z-bladed propeller may be regarded as the sum of the elementary forces $\bar{p}$* which vary in an arbitrary wake field $\psi\ (r, \phi)$ with the distance r from the center of the propeller shaft and with the angle of rotation ϕ taking into account the blade arrangement index j. For the arrangement shown in Figure 1, the following relation applies with $\bar{p} = \bar{p}\ (r, \phi)$ and with the index j for the blade arrangement at an arbitrary number of blades z

$$\bar{P} = \sum_{j=1}^{z} \sum_{r=0}^{R} \bar{p}(r, \phi_j) \qquad [1]$$

if R denotes the radius of the propeller disk.

A resolution of the coordinates leads to three forces X, Y, Z and to three moments M_x, M_y, M_z from among which the thrust $S = X$ referred to the propeller axis (x-direction) and the torque $M_d = M_x$ about the x-axis shall be further investigated in this case as the most important quantities. As "wake" we shall define the difference between the speed v and the flow velocity v_p in way of the propeller. The ratio

$$\psi = 1 - (v_p / v) \qquad [2]$$

of the wake to the speed is the so-called wake factor.

In analogy with the lift for aircraft wings with the local thrust coefficient c_s, the local stagnation pressure $q_u = \rho_u^2 / 2$ in the direction of rotation, and with the surface element

*Elementary force = force acting on a surface element.

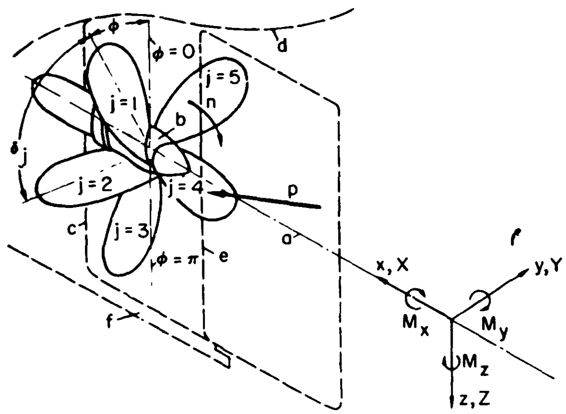

Figure 1 – Field of Forces Acting on a Five-Bladed Ship Propeller

a	Propeller axis
b	Five-bladed propeller of the index (arrangement) $j = 1$ to 5
c	Propeller post
d	Stern contour
e	Leading edge of the rudder
f	Stern heel
c, d, e, f	Propeller race
n	Propeller rpm
ϕ	Angle of rotation
δ_j	Center angle of propeller
p	Elementary force acting on a (unit) surface element of a blade
x, y, z	Rectangular coordinates
X, Y, Z	Components of the resultant force in the x-, y-, and z-direction
M_x, M_y, M_z	Torques about the x-, y-, and z-axis

$dF = ldr$, the elementary thrust dS_j (thrust on a blade element of the jth blade) may be written in the form

$$dS_j = c_s \, q_u \, dF = 2\rho \, \pi^2 n^2 c_s l r^2 dr \qquad [3]$$

where l is the blade length at a distance r from the center of the propeller shaft (Figures 2-4),

ρ is the density of the water,

n is the number of revolutions, and

u is the circumferential velocity of the blades at the distance r.

The elementary torque dM_j on the blade element in analogy with the wing resistance with c_m as the local moment coefficient is found to be

$$dM_j = c_m q_u r dF = 2\rho \, \pi^2 n^2 c_m l r^3 dr \qquad [4]$$

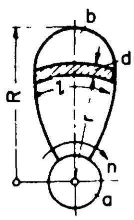

Figure 2 – Blade Notations

a Boss
b Blade
r Distance from the center of the propeller shaft
R Radius of the propeller disk
l Length of the Blade
n Number of revolutions

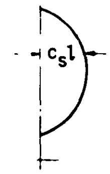

Figure 3 – Thrust Coefficient Area

c_s Local thrust Coefficient
l Length of the blade

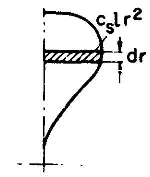

Figure 4 – Representation of the Area $\int_0^R c_s l r^2 dr$

Figures 2-4 – Schematic Representation Illustrating the Derivation of the Elementary Forces Acting on the Blade of a Marine Propeller

For clarity we shall first give further consideration to the thrust only in the following section. The total thrust $S(\phi)$, according to

$$S(\phi) = z\, S_j(\psi_M) + \sum_{j=1}^{z} \Delta S_j(\Delta\psi_a) + \sum_{j=1}^{z} \Delta S_j(\psi_t) \qquad [5]$$

may be thought of as having resulted from three components. The first component on the right side of Equation [5] corresponds to the values in the case of a mean ψ_M of the wake factor taken over the entire wake. The second component results from the thrust fluctuations $\Delta S_j(\Delta\psi_a)$ due to the axial fluctuations $\Delta\psi_a$ of the wake factor. The third component is caused by the thrust fluctuations $\Delta S_j(\psi_t)$ due to a tangential component ψ_t of the velocity which lies in the propeller plane. From Equation [5] we obtain, with Equation [3] and the abbreviation $\nu = r/R$, the relation

$$S(\phi) = 2\rho\pi^2 n^2 R^3 \cdot \left[z \int_0^1 c_s l \nu^2 d\nu + \sum_{j=1}^{z} \int_0^1 c_s l \nu^2 \frac{\Delta c_s(\Delta\psi_a)}{c_s}\, d\nu + \sum_{j=1}^{z} \int_0^1 c_s l \nu^2 \frac{\Delta c_s(\psi_t)}{c_s}\, d\nu \right] \qquad [6]$$

if $\Delta c_s\ (\Delta\psi_a)$ and $\Delta c_s\ (\psi_t)$ denote the thrust coefficients corresponding to the thrust quantity $\Delta S_j\ (\Delta\psi_a)$ and $\Delta S_j\ (\psi_t)$, respectively.

SIMPLIFICATION OF THE THRUST FORMULA

For a given case, Equation [6] may be solved graphically if the wake field and the open-water propeller diagram are known.[6*] For a first approximation, however, considerable simplifications are permitted.

1. On the basis of numerical examples it is to be assumed that the amplitudes of the thrust fluctuations due to the tangential wake variation during one revolution in general are no greater than 25 percent of the amplitudes of the thrust fluctuations due to the axial wake variation. Hence, since the two amplitudes are displaced in phase, no effect whatsoever is to be expected at the peaks (ϕ = 0 deg and 180 deg, $\Delta\psi_a = \Delta\psi_{a\ max}$, $\psi_t = 0$) if the tangential wake variation is taken into account whereas at the hollows (ϕ = 120 deg and 240 deg) an amplitude increase of 20 percent at the most may be expected. The frequency does not vary. Therefore, there would probably be no fundamental change in the result if the third term in Equation [6] were neglected.

2. If we confine ourselves to the two components (thrust S and torque M_d) under consideration, the variation in the lift distribution on the blades during one revolution is of secondary importance; hence, the integration of the elementary forces over the distance r may be obviated with sufficient approximation by setting up a concentrated force at the distance $r = 0.7\ R$.

3. In a uniform flow into the propeller each blade receives the same thrust component known from the open-water propeller test. On account of the simplification according to paragraph 2., we are moreover permitted to equate without hesitation the relative thrust variation $\Delta S_j/S_j$ of a blade passing through a nonuniform wake field to the relative thrust variation $\Delta S/S$ of the entire propeller if the coefficient of advance varies. With the familiar expressions $K_S = S/\rho n^2 D^4$ for the total thrust coefficient and $\Lambda = v_p/nD$ for the coefficient of advance, both referred to the propeller diameter D, we then obtain

$$\frac{\Delta c_s}{c_s} = -\frac{\Delta K_S}{K_S} = -\frac{K'_S}{K_S}\ \Delta\Lambda \qquad [7]$$

*In the so-called open-water propeller test, the thrust and torque of a model propeller is measured which runs freely in the towing tank of a model basin and which is driven at a constant rpm by means of a special device while the speed of advance varies. The nondimensional plotting of the test results, viz. of the thrust coefficient K_S, of the moment coefficient K_M, and of the propeller efficiency $\eta_P = \frac{K_S}{K_M}\ \frac{\Lambda}{2\pi}$ against the coefficient of advance Λ is called an open-water propeller diagram in naval architecture. (For further details regarding experimental research with ship models, see Reference 7, for instance.)

if linearity by sections is assumed and if the tendency toward change is taken to account.

In this case, c_s denotes the thrust coefficient of the blade element, K_S the thrust coefficient for the entire propeller, Δc_s and ΔK_S the variation of c_s and K_S, respectively, $\Delta\Lambda$ the variation of the coefficient of advance, and K_S' the ratio $\Delta K_S/\Delta\Lambda$.

In the nonuniform wake field the coefficient of advance of the blade, assumed to be concentrated at the distance 0.7 R, varies according to

$$\Delta\Lambda(\psi) = -\Lambda_{0.7R}\,\Delta\psi \qquad [8]$$

In this case, the index $0.7R$ refers to quantities at a distance $0.7R$ from the center of the propeller shaft and we have

$$\Delta\psi = \Delta\psi_{a;\,0.7R}(\phi) \approx \psi(\phi) - \psi_M$$

$$\psi_M = \frac{1}{2\pi}\int_0^{2\pi} \psi\, d\phi$$

$$\Lambda_{0.7R} = 0.7\,\Lambda = 0.7\, v_p/nD$$

With the simplifications of paragraphs 1 to 3, Equation [6] reduces to

$$S(\phi) \approx S_M \left\{ 1 - C_p \frac{1}{z} \sum_1^z [\psi(\phi) - \psi_M] \right\} \qquad [9]$$

if S_M denotes the constant thrust for the mean value of the wake and

$$C_p = \frac{\Lambda}{0.7}\,\frac{K'_S}{K_S} \qquad [10]$$

denotes a constant factor for the conversion of the wake fluctuations into thrust fluctuations of the propeller.

INTRODUCTION OF SUBSTITUTE FUNCTIONS

Equation [8] is not yet suitable for a physical interpretation. First, the wake distribution $\Delta\psi(\phi)$ over a circle with radius $r = 0.7R$ which has been determined experimentally for several cases must be expressed approximately in an analytic manner as a generally valid function by making use of parameters for adaptation to the particular cases. Since the wake distribution and thus the quantity $\Delta\psi(\phi)$ shows the period 2π and maxima at $\phi = 0$ and π, it is advisable to adjust its curve by the overlapping of periodic functions. For a sufficient approximation at least two such functions are required, one of which must follow a symmetrical course while the other must follow an unsymmetrical course with respect to $\phi = \pi$. Hence, we shall set up the expression

$$\psi = C_S\,(y_1 + y_2) \qquad [11]$$

In this case,

$$y_1 = \cos^{2\mu_1} \phi_j \qquad [12]$$

and

$$y_2 = k \cos^{2\mu_2 + 1} \phi_j \qquad [13]$$

represent the two periodic functions, and

$$C_S = \frac{\psi_0 - \psi_{\min}}{1 + k} \qquad [14]$$

designates a ship constant. Furthermore, μ_1 and μ_2 are positive integral exponents, ϕ_j is the angle of rotation of a blade of index j, ψ_0 and ψ_π are the values of the wake factor for $\phi = 0$ and $\phi = \pi$, respectively, $\psi_{\min}$ is the minimum value of ψ, and

$$k = \frac{\psi_0 - \psi_\pi}{\psi_0 + \psi_\pi + 2\psi_{\min}} \qquad [15]$$

is the "overlapping scale." The ship constant C_S confines the substitute function to the range $0 \leqq y_1 + y_2 \leqq 1$, and the overlapping scale according to $0 \leqq k \leqq 1$ controls the reduction of the wake peak on the heel of the stern of the hull ($\phi = \pi$). The exponents μ_1 and μ_2 vary the distribution of the wake factor which drops more or less abruptly from ψ_0 to $\psi_{\min}$ between the peaks (at $\phi = 0$ and π).

For the purpose of simplification, the substitute functions are, with the aid of binomial series, developed with respect to multiples of ϕ_j whose coefficients are tabulated in Table 1. With these coefficients, $a_{2\mu_1}$ and $a_{2\mu_2 + 1}$ the following holds true

$$y_1 = \sum_{0}^{\mu_1} a_{2\mu_1} \cos\,(2\mu_1\,\phi_j) \qquad [16]$$

or

$$y_2 = \sum_{0}^{\mu_2} a_{2\mu_2 + 1} \cos\,[(2\mu_2 + 1)\,\phi_j] \qquad [17]$$

TABLE 1

Values of the Coefficients $a_{2\mu_1}$ and $a_{2\mu_2+1}$ Compare Equations [16] and [17], Respectively

Exponent μ_1 / Coefficient $a_{2\mu_1}$	0	1	2	3	4	5
a_0	1.000	0.500	0.375	0.313	0.274	0.246
a_2	0	0.500	0.500	0.469	0.438	0.410
a_4	0	0	0.125	0.187	0.219	0.234
a_6	0	0	0	0.031	0.036	0.088
a_8	0	0	0	0	0.008	0.019
a_{10}	0	0	0	0	0	0.002
Exponent μ_2 / Coefficient $a_{2\mu_2+1}$	0	1	2	3	4	5
a_1	1	0.750	0.623	0.547	0.492	0.451
a_3	0	0.250	0.313	0.328	0.328	0.322
a_5	0	0	0.062	0.109	0.141	0.161
a_7	0	0	0	0.016	0.035	0.054
a_9	0	0	0	0	0.004	0.011
a_{11}	0	0	0	0	0	0.001

respectively, if the summation sign indicates that all values of μ_1 are to be inserted consecutively into Equation [16] and all values of μ_2 into Equation [17].

The splitting up of the variable ϕ_j into a sum

$$\phi_j = \phi + \delta_j \qquad [18]$$

with the blade center angle

$$\delta_j = j\,\frac{2\pi}{z} \qquad [19]$$

(according to Figure 1), which is required for an interpretation of Equation [9], first produces a complication which, however, can be eliminated again by writing y_1 and y_2 as real parts of complex exponential functions in which the terms with ϕ and δ_j form products. Expressed forthwith in the form required for Equation [9] as a sum over the blades, we obtain, with $\Re$ as a sign for the real-part formation and $i = \sqrt{-1}$, the following expressions:

$$\sum_{1}^{z} y_1 = \sum_{1}^{z}\sum_{0}^{\mu_1} a_{2\mu_1} \Re\left[e^{i2\mu_1(\phi + \delta_j)}\right]$$

$$= \Re\left[\sum_{0}^{\mu_1}\left(a_{2\mu_1} e^{i2\mu_1\phi} \sum_{1}^{z} e^{i\frac{2\mu_1}{z} j2\pi}\right)\right] \qquad [20]$$

and

$$\sum_{1}^{z} y_2 = \Re\left[\sum_{0}^{\mu_2}\left(a_{2\mu_2+1} e^{i(2\mu_2+1)\phi} \sum_{1}^{z} e^{i\frac{2\mu_2+1}{z} j2\pi}\right)\right] \qquad [21]$$

These expressions possess the characteristic that they can only exist under certain conditions and even then only in a simpler manner. As a matter of fact, if $2\mu_1/z = m_1$ represents an integral number, we obtain

$$e^{i\frac{2\mu_1}{z} j2\pi} = e^{im_1 j2\pi} = 1$$

and thus

$$\sum_{1}^{z} y_1 = \Re\left(\sum_{1}^{z}\sum_{0}^{\mu_1} a_{2\mu_1} e^{i2\mu_1\phi}\right) = z\sum_{0}^{\mu_1} a_{2\mu_1} \cos(2\mu_1\phi) \qquad [22]$$

Correspondingly, we obtain for $(2\mu_2 + 1)/z = m_2$ as an integral number the simpler form

$$\sum_{1}^{z} y_2 = z\sum_{0}^{\mu_2} a_{2\mu_2+1} \cos[(2\mu_2 + 1)\phi] \qquad [23]$$

If however $2\mu_1$ and z or $2\mu_2 + 1$ and z, respectively, are prime to each other, the following equation applies according to the summation formula for geometrical series

$$\Re\left(e^{ih\phi}\sum_{1}^{z} e^{i\frac{2\pi h}{z}j}\right) = \Re\left[e^{i\left(h\phi + \frac{2\pi h}{z}\right)} \frac{1 - e^{i\frac{2\pi h z}{z}}}{1 - e^{i\frac{2\pi h}{z}}}\right]$$

if h is generally substituted for $2\mu_1$ or $2\mu_2 + 1$, respectively. After a few transformations, this leads to

$$y = \frac{\cos\left(h\phi + \frac{\pi h}{z} + \pi h\right)\sin(\pi h)}{\sin\left(\frac{\pi h}{z}\right)} = 0$$

with $y = y_1$ for $h = 2\mu_1$ and $y = y_2$ for $h = 2\mu_2 + 1$ or else to

$$\sum_{1}^{z} y_1 = z a_0 \qquad [24]$$

and

$$\sum_{1}^{z} y_2 = 0 \qquad [25]$$

respectively.

Since in the summation over μ the summation variable μ is supposed to assume for the individual terms all positive integral values from 0 to μ, Equation [22] develops into

$$\sum_{0}^{z} y_1 = z\left[a_0 + a_2\cos(2\phi) + a_4\cos(4\phi) + \ldots + a_{2\mu_1}\cos(2\mu_1\phi)\right] \qquad [26]$$

For a specific number of blades z there are in this equation the terms which satisfy the condition $2\mu_1/z = m_1$. If this is not the case for any one of the terms, i.e. if only the constant term a_0 remains, then y_1 becomes independent of ϕ. The same thing applies to y_2.

Equation [9] thus obtains for the thrust distribution the interpretable form

$$S(\phi) \approx S_M\left[1 + C_p\left(C_s\Phi - \psi_M\right)\right] \qquad [27]$$

with C_p according to Equation [10], C_S according to Equation [14], and Φ as the expression

$$\Phi = \sum_{0}^{\mu_1} a_{2\mu_1} \cos(2\mu_1\phi) + k \sum_{0}^{\mu_2} a_{2\mu_2+1} \cos[(2\mu_2+1)\phi] \qquad [28]$$

in which k is given by Equation [15] and $2\mu_1 = m_1 z$ and $2\mu_2 + 1 = m_2 z$ apply with m_1 and m_2 as integral numbers. If this integral-number condition does not apply, we obtain according to Equations [24] and [25]

$$\Phi = a_0 = \text{const} \qquad [29]$$

and from this we get $C_S a_0 = \psi_M$.

INTERPRETATION OF THE SIMPLIFIED THRUST FORMULA

From the simplified thrust formula, Equation [27], we can now obtain a number of important relationships:

a. Thrust fluctuations occur if a ship propeller with $z = 2\mu_1/m_1$ blades is used on an afterbody of the currently customary shape with propeller post and rudder in the central longitudinal plane. Let us assume that $\mu_1 = 3$, for instance, i.e. that the exponent of the even cos-function y_1 have the value of 6 and the factors of the angle ϕ in the explicit function y_1 assume the values 0, 2, 4, 6. Let us further assume that $\mu_2 = 0$, i.e. that the odd cos-function y_2 exists only for the practically insignificant case $z = 1$. In that case, thrust fluctuations occur if the propeller possesses 2, 3, 4, or 6 blades. In the case of propellers with some other number of blades (5, 7, 8, 9, and more, for instance), no thrust fluctuations are to be expected on the basis of the approximation formula with $\mu_1 = 3$, $\mu_2 = 0$.

b. The amplitudes of the thrust fluctuations are proportional to the constants C_p and C_S and increase in the same ratio as the rise of the thrust coefficient over the advance coefficient is increased by altering the blade or profile shape of the propeller, for instance, or as the range ψ_0 to ψ_{min} in the propeller plane is increased by altering the propeller race and the afterbody lines or by displacing the propeller in the race.

c. The variations of the amplitudes of the thrust fluctuations are inversely proportional to the dissymmetry factor k. For $k \to 1$ (cut-away heel of the stern), the fluctuations to be expected are about half as great as for $k \to 0$ (thick deadwood in front of the propeller).

d. For $\phi = 0$ (i.e. when the very top of a blade passes through the plane of the propeller post), the fluctuations always reach a maximum, if any occur, since $\cos(2\mu_1\phi) = 1$ in that case.

e. The fluctuations become smaller, the smoother the wake field is for an equal range $\psi_0 - \psi_{min}$, since μ_1 remains small in that case. At $\mu_1 = 2$, for instance, the three-bladed propeller becomes theoretically free from thrust fluctuations whereas at $\mu_1 = 5$ (applicable to wake distribution curves with narrow peaks which are deflected in a very bag-like fashion) only the seven-bladed propeller acts like a propeller with an infinite number of blades as far as thrust fluctuations are concerned. Any change in μ_1 and μ_2 influences only the ordinates of the thrust distribution over ϕ, but not the maxima which lie at multiples of $\pi/2$. Such a change, therefore, cannot have any effect on the magnitude of the thrust fluctuations on ship propellers with two or four blades.

f. The additional term y_2 reducing the wake distribution in a downward direction can only come into play in the case of propellers with an odd number of blades and only in the case where $z = 2\mu_2 + 1$. The effect remains slight and may be disregarded for a first approximation since the dissymmetry of the wake distribution is taken into account in a larger measure by the overlapping scale k contained in C_S.

g. Finally, it is noteworthy that in Equation [26] which was investigated especially in regard to the effect of the number of blades, there occurs neither the number of blades z nor the center angle δ_j associated with the latter (except in the existence condition).

EVALUATION

In plotting certain thrust distributions, numerical values must be inserted into Equation [27]. Figure 5 shows a wake field of lines of equal velocity for a freighter of 12 000 m^3 displacement and a fullness of 69 percent with a speed of approximately 17 knots which was recorded in the Berlin Tank (Versuchsanstalt für Wasserbau und Schiffbau). In Figure 6, the wake distribution referred to 0.7 R has been adjusted with the aid of the substitute functions y_1 and y_2. The formula for the reduced thrust distribution Φ, Equation [28], was developed in Table 2 for propellers with two up to an infinite number of blades and graphically represented in Figures 7-9 after inserting the coefficients from Table 1. Figures 10 and 11 show the variations of the amplitudes and frequencies of the thrust fluctuation with the number of blades.

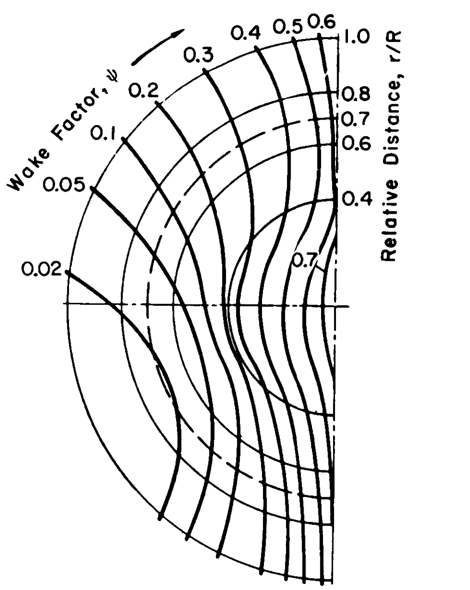

Figure 5 – Wake Field of Lines of Equal Velocity for a Freighter, in Polar Coordinates

It must be conceded, of course, that a purely stationary consideration and the first approximation with but two periodic functions, which is discussed here for reasons of simplification, cannot yield any quantitatively correct results; however, the variation of the curves

TABLE 2

Development of the Formulas for the Reduced Thrust Φ According to Equation [28] for Various Values of the Number of Blades z and of the Exponent μ_1 Using an Exponent $\mu_2 = 0$ as a Basis

No. of Blades	Summation Term According to Equation [28] for				
z	$\mu_1 = 0$	$\mu_1 = 1$	$\mu_1 = 2$	$\mu_1 = 3$	$\mu_1 = 4$
2	a_o	$a_2 \cos 2\phi$	$a_4 \cos 4\phi$	$a_6 \cos 6\phi$	$a_8 \cos 8\phi$
3	a_o	—	—	$a_6 \cos 6\phi$	—
4	a_o	—	$a_4 \cos 4\phi$	—	$a_8 \cos 8\phi$
5	a_o	—	—	—	—
6	a_o	—	—	$a_6 \cos 6\phi$	—
7	a_o	—	—	—	—
8	a_o	—	—	—	$a_8 \cos 8\phi$
9	a_o	—	—	—	
∞	a_o	—	—	—	—

Examples for the formation of Φ:

1. For $z = 8$ and $\mu_1 = 0$, we obtain $\Phi = a_o$
2. For $z = 2$ and $\mu_1 = 3$, we get $\Phi = a_o + a_2 \cos 2\phi + a_4 \cos 4\phi + a_6 \cos 6\phi$
3. For $z = 8$ and $\mu_1 = 4$, we obtain $\Phi = a_o + a_8 \cos 8\phi$

by varying the parameters, which is clearly apparent from the figures, corresponds nevertheless to the actual conditions. In this way, the empirical fact was confirmed that the fluctuation amplitudes do not vary continuously with the number of blades. On the one hand, an increase in the number of blades brings about a reduction of the amplitudes while an odd number of blades, on the other hand, results in smaller amplitudes than the adjacent even numbers of blades. Mathematically, both effects can be explained on the same basis in a reduction of the terms of the series in the sequence $\mu_1 = 0, 1, 2, 3. \ldots$ Of course, an essential difference in the magnitude of the thrust-fluctuation amplitudes can only exist in the case of small numbers of blades since for large z-values the summation which goes over into an integration must yield the mean value ψ_M in every case.

Variations of the exponents μ_1 and μ_2 do not change the fundamental result. Thus, it appears to have been proved that within the framework of todays customary propellers and afterbody forms of seagoing vessels, the four-bladed propeller represents the most unfavorable and the five-bladed propeller the most favorable solution with respect to thrust and torque fluctuations. Naturally, this rating does not extend to the remaining characteristics as well. It is very well possible, for instance, that disadvantages with respect to efficiency, the tendency toward cavitation or the exciting of ship vibrations prohibit the use of a five-bladed propeller in individual cases.

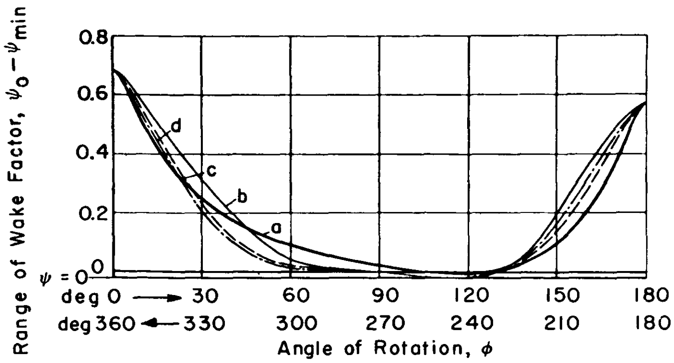

Figure 6 – Adaptation of the Measured Wake Distribution at a Distance from the Center of the Propeller Shaft $r = 0.7\ R$ by Means of Periodic Functions

ψ_o is the value of the wake factor ψ at the angle of rotation $\phi = 0$

ψ_{min} is the minimum value of the wake factor

a is the measured curve drawn from Figure 5

b is the curve calculated according to Equation [11] with the exponents $\mu_1 = 3$ and $\mu_2 = 0$

c is the curve calculated according to Equation [11] with the exponents $\mu_1 = 4$ and $\mu_2 = 0$

d is the curve calculated according to Equation [11] with the exponents $\mu_1 = 4$ and $\mu_2 = 2$

The calculations for the curves *b* to *d* are based on an overlapping scale $k = \dfrac{\psi_o - \psi_\pi}{\psi_o + \psi_\pi - 2\psi_{min}} = 0.1$ and on a ship constant $C_S = \dfrac{\psi_o - \psi_{min}}{1 + k} = 0.6$

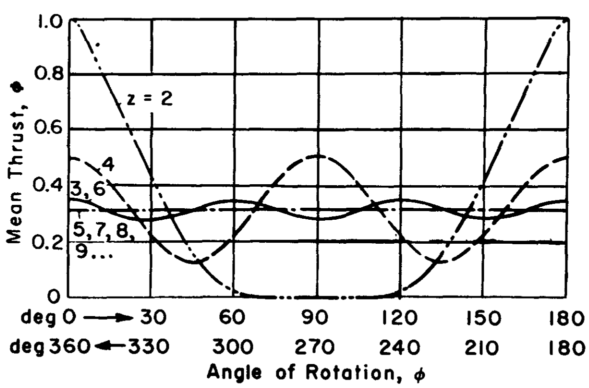

Figure 7 – Thrust Distribution in the Wake According to Curve *b* of Figure 6

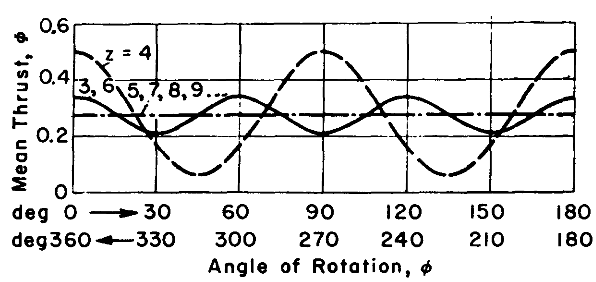

Figure 8 – Thrust Distribution in the Wake According to Curve *c* of Figure 6

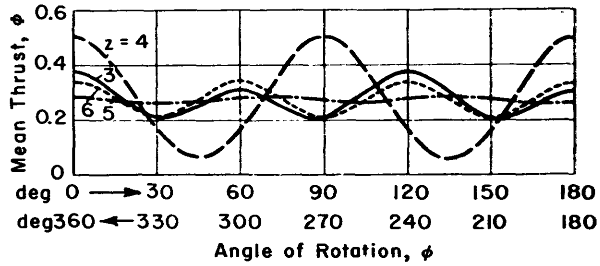

Figure 9 – Thrust Distribution in the Wake According to Curve d of Figure 6

Figures 7-9 – Distribution of the Reduced Thrust Φ over the Angle of Rotation ϕ for Ship Propellers of Varying Numbers of Blades z for a Wake Distribution According to Figure 6

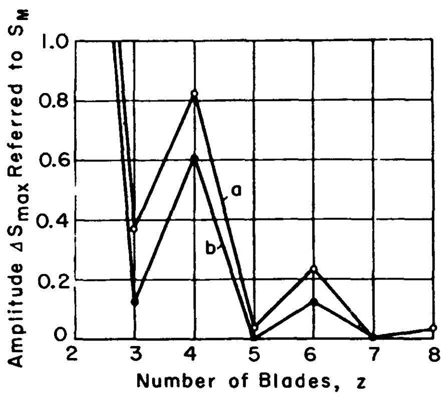

Figure 10 – The Thrust-Fluctuation Amplitude ΔS_{max} Referred to the Mean Thrust S_M as a Function of the Number of Blades z

a – For the approximation with the exponents $\mu_1 = 3$ and $\mu_2 = 0$

b – For the approximation with the exponents $\mu_1 = 4$ and $\mu_2 = 2$

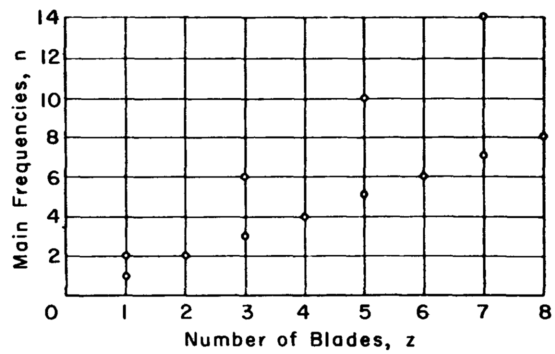

Figure 11 – The Main Frequencies of the Thrust Fluctuations in Multiples of the Number of Revolutions n as a Function of the Number of Blades z

APPLICATION OF THESE FINDINGS TO THE PROBLEM OF TORQUE FLUCTUATIONS

In establishing a formula for the torque distribution corresponding to Equation [27], we merely have to introduce a ratio K_M'/K_M in place of the ratio of the thrust coefficient K_S'/K_S. In this case, $K_M = M_d/\rho\pi^2 n^2 D^5$ denotes the total-torque coefficient and $K_M' = \Delta K_M/\Delta\Lambda$ the variation of K_M referred to a variation of the advance coefficient by $\Delta\Lambda$. The distribution of of the torque fluctuations ΔM_d is similar to that of the thrust fluctuations ΔS. In order of magnitude, the ratio between the amplitudes is as follows:

$$\frac{\Delta M_d}{M_d} = \frac{K_M'}{K_M}\frac{K_S}{K_S'}\frac{\Delta S}{S} \qquad [30]$$

Thus, to a four-bladed propeller of the Wageningen series B 4 with the pitch ratio 1 and the blade area ratio 0.55 the following values apply if $\Lambda = 0.96$: $K_S = 0.1$, $K'_S = 0.44$, $K_M = 0.027$, $K'_M = 0.06$ and thus $\Delta M_d/M_d = 0.75\,\Delta S/S$.

CONCLUSIONS TO BE DRAWN WITH RESPECT TO APPLYING THESE FINDINGS

The foregoing considerations point up the mathematical relationship between the thrust and torque fluctuations. Special problems to be solved would be to investigate more closely the relationship between the ship form, the design of the propeller race, and the location of the propeller with respect to the propeller race, on the one hand, and of the wake distribution, on the other, and to gain an insight into the effects of the thrust and torque fluctuations.

As to the first problem concerning the dependence of the wake distribution on the geometric ratios, we shall merely point out here briefly that the wake range $\psi_o - \psi_{min}$ referred to 0.7 R for modern single-screw vessels reaches values of 0.4 to 0.7 during service speeds. For the ship propeller there exists in each case an optimum location within the propeller race which does not apply generally. If the propeller is displaced in the propeller race, the variation of

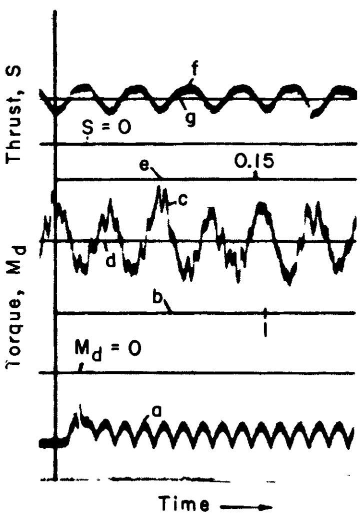

Figure 12 – Oscillogram for a Four-Bladed Propeller

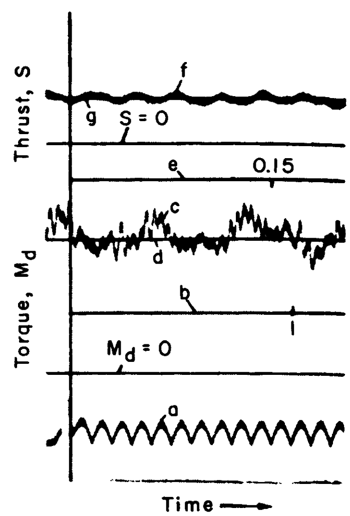

Figure 13 – Oscillogram for a Five-Bladed Propeller

Figures 12 and 13 – Portions of Oscillograms of the Thrust and Torque Fluctuations from Model Tests with a Freighter Travelling at a Speed of 15 Knots under Conditions Similar to Those in the Calculated Example

a Time pulse
b Scale for the number of revolutions
c Torque distribution
d Mean torque value
e Scale of time
f Thrust distribution
g Mean thrust value

the distance from the propeller post exercises a considerably greater effect than a variation of the distance from the leading edge of the rudder. Enlarging an existing propeller race without altering at the same time the contour of the afterbody is of questionable effect since the wake distribution in this case may become worse under certain circumstances.

As far as the effects of the thrust and torque fluctuations are concerned, it should be borne in mind that the formulas for the fluctuations of the forces and moments established here apply at the propeller. The manner in which these excitations are transmitted in the form of mechanical vibrations depends on the weight ratio, the stiffness ratio, and the ratio of attenuation of the power plant as a whole. In such places where these interferences may become dangerous, i.e. in the shaft, at the thrust bearing and in the gear unit, thrust and torque fluctuations can no longer be measured except as fractions of the magnitude which they presumably possess on the propeller. On account of the great mass moment of inertia of the ship propeller, especially the torque fluctuations are largely reduced. While in model tests, for instance, in the transmission gear thrust and torque fluctuations of ± 25 percent to ±40 percent of the mean value are regarded as normal for four-bladed propellers and fluctuations of ±13 percent to ±20 percent for five-bladed propellers (see Figures 12 and 13), we usually find that in the case of the prototypes–for which, to be sure, only few measurements have become known thus far–the thrust fluctuations for four-bladed propellers lie at ±20 percent and the torque fluctuations at ±10 percent. With five-bladed propellers, thrust fluctuations falling below 10 percent of the mean value have been measured for sister ships whereas the torque fluctuations lay within the accuracy of measurement.

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