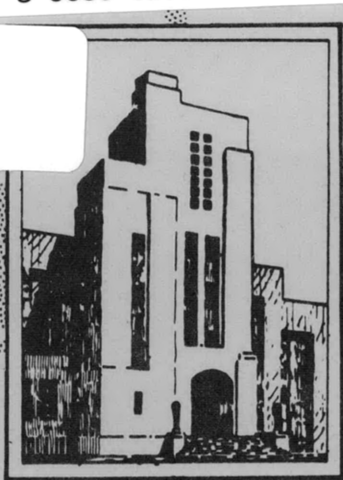

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HYDROMECHANICS

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APPLIED MATHEMATICS

THE OSCILLATIONS OF A FLOATING FORM ON THE SURFACE OF A HEAVY FLUID

(Kolebaniia plavaiushchego kontura na poverkhnosti tiazheloi' zhidkosti)

by

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ABSTRACT

This report gives a method of finding the exact solution of the problem on pitching of cylindrical ship forms. The solution is given for a wide range of contours by means of which all the hydrodynamic characteristics may be calculated.

1. The hydrodynamic statement of problems on pitching of cylindrical ships reduces to the problem of finding the harmonic function

$$\Phi(y, z, t) = \phi(y, z)\, e^{j\sigma t} \qquad (j = \sqrt{-1}) \qquad [1.1]$$

satisfying the boundary conditions

$$\frac{\partial \phi}{\phi z} - \nu\phi = 0 \quad \text{when } z = 0, \ |y| > a \quad \left(\nu = \frac{\sigma^2}{g}\right) \qquad [1.2]$$

$$\frac{\partial \phi}{\partial n} = v_n\,(l) \quad \text{on the contour } L \qquad [1.3]$$

and the asymptotic conditions

$$\begin{aligned} \phi &= \left(j\frac{g}{\sigma} r_0 + B_+\right) e^{\nu z - j\nu y} \quad \text{when } y \to +\infty, \\ \phi &= j\frac{g}{\sigma} r_0 e^{\nu z - j\nu y} + B e^{\nu z + j\nu y} \quad \text{when } y \to -\infty \end{aligned} \qquad [1.4]$$

where σ is the frequency of the oscillations, ν is the frequency parameter, g is the acceleration due to gravity, $v_n(l)e^{j\sigma t}$ is the normal component of the velocity of any point of the contour, $2a = b$ is the breadth of the contour L at the waterline, n is the outward normal to the contour, $\Phi^* = jg\sigma^{-1}r_0 e^{\nu z + j(\sigma t - \nu y)}$ is the velocity potential of an oncoming system of regular progressive waves, $2r_0$ is their height, B_+ and B_- are the complex amplitudes with respect to j, of the waves generated. In all terms having the exponential time factor $e^{j\sigma t}$, it is proper to consider only the real part.

In the present work we intend to give a method of finding the exact solution of the problem in the general case for a wide range of contours, by means of which all the hydrodynamic characteristics can be checked. This method was used in a particular form in our works.[1,2]

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[1]References are listed on page 21.

2. For finding the solution of the problem we shall use the function

$$w = \phi(y, z) + i\psi(y, z)$$

of the complex variable $x = y + iz$ where $i = \sqrt{-1}$ is an imaginary unity, not interchangeable with the imaginary unity $j = \sqrt{-1}$.

By means of the function $w(x)$ condition [1.2] can be written in the form

$$\mathrm{Im}\left(\frac{dw}{dx} + i\nu w\right) = 0 \qquad \text{when } z = 0,\ |y| > a \qquad [2.1]$$

Condition [2.1] permits us to expand the function $dw/dx + i\nu w$ in the upper half plane. As a result we obtain a holomorphic function in the whole x-plane outside the contour $L + \overline{L}$, where $\overline{L}$ is the mirror image of the contour L in the upper half plane.

In the neighborhood of an infinitely distant point the following expansion is valid:

$$\frac{dw}{dx} + i\nu w = \frac{a_0'}{x} + \frac{b_0'}{x^2} + \ldots \qquad [2.2]$$

where from condition [2.1] the coefficients a_0', b_0', . . . are real with respect to the imaginary unit i.

For building up the function, having finite values everywhere, we shall make use of the idea of L.C. Sedov which was developed by him in the problem of gliding,[3] and we shall introduce another function $f(x) = r + is$, connected to $w(x)$ by the equation

$$\frac{df}{dx} = \frac{dw}{dx} + i\nu w \qquad [2.3]$$

Obviously the function $f(x)$ is holomorphic everywhere outside the contour $L + \overline{L}$ and is limited to points of this contour.

On the basis of [2.1] for the function $f(x)$ we have the condition

$$\frac{\partial r}{\partial z} = 0 \qquad \text{when } z = 0,\ |y| > a \qquad [2.4]$$

We shall set up the boundary condition for the function r at points of the contour L. Multiplying both sides of Equation [2.3] by the quantity

$$\frac{dx}{dl} = \frac{dy}{dl} + i\frac{dz}{dl}$$

where dl is an element of the arc of the contour L, we get

$$\frac{df}{dl} = \frac{dw}{dl} + i\nu w\left(\frac{dy}{dl} + i\frac{dz}{dl}\right)$$

Separating the real and imaginary parts in this relationship we will have as a result

$$\frac{\partial r}{\partial l} = \frac{\partial \phi}{\partial l} - \nu \frac{dz}{dl} \phi - \nu \frac{dy}{dl} \psi, \qquad \frac{\partial r}{\partial n} = \frac{\partial \phi}{\partial n} + \nu \frac{dz}{dl} \psi - \nu \frac{dy}{dl} \phi \qquad [2.5]$$

On the contour L we have the relationship

$$\frac{\partial \phi}{\partial n} = -\frac{\partial \psi}{\partial l} = v_n (l) \qquad [2.6]$$

Therefore the function ψ is known with accuracy on the contour L up to the constant l

$$\psi = \psi_1 - \int_0^l v_n(l)\, dl \qquad [2.7]$$

where ψ_1 is the value of the function ψ at the point $y = a$, $z = 0$, and the integration here and in later integrals is carried out over the portion of the contour L from the point $y = a$, $z = 0$.

We eliminate the unknown function ϕ from the relationship [2.5]. Thus, examining the first relationship as an equation relative to ϕ, we get

$$\phi = r + e^{\nu z} \left(\phi_1 - r_1 + \nu \int_0^l e^{-\nu z} r dz + \nu \int_0^l e^{-\nu z} \psi dy \right)$$

or

$$r + \nu e^{\nu z} \int_0^l e^{-\nu z} r dz = \phi - e^{\nu z} \left(\phi_1 - r_1 + \nu \int_0^l e^{-\nu z} \psi dy \right) \qquad [2.8]$$

where ϕ_1 and r_1 are the values of the functions ϕ and r at the point $(a, 0)$.

Now multiplying [2.8] by $\nu (dy/dl)$ and adding this to the second equation of [2.5], we get the condition for r on the contour L:

$$\frac{\partial r}{\partial n} + \nu \frac{dy}{dl} \left(r + \nu e^{\nu z} \int_0^l e^{-\nu z} r dz \right) = v_n + \nu \frac{dz}{dl} \psi - \nu \frac{dy}{dl} e^{\nu z}$$

$$\left(\phi_1 - r_1 + \nu \int_0^l e^{-\nu z} \psi dy \right) \qquad [2.9]$$

Let the function $x = y + iz = F(\tau)$ fulfill the conformal transformation of the outside of the contour $L + \bar{L}$ in the x-plane to the outside of the unit circle with the center at the origin of the coordinates in the τ-plane.

Further, let us introduce the auxiliary variable

$$\zeta = \xi + i\eta, \qquad \tau = e^{\zeta}, \qquad x = F(e^{\zeta}) = F_1(\zeta)$$

In the ζ-plane the contour L corresponds to the part of the axis η from $-\pi$ to 0, and the half lines $\eta = 0$ and $\eta = -\pi$ $(\xi \geq 0)$ correspond to the parts of the coordinate axis $z = 0$ and $|y| > a$.

It is easily seen that in the ζ-plane, condition, [2.4] has the form

$$\frac{\partial r}{\partial \eta} = 0 \quad \text{when } \eta = 0 \quad \text{and } \eta = -\pi \qquad [2.10]$$

Let us transform condition [2.9]. For this we note that when $\xi = 0$, the equations are valid

$$\frac{\partial r}{\partial n} = \frac{\partial r}{\partial \xi}\left|\frac{d\zeta}{dx}\right|, \qquad \frac{dy}{dl} = -\frac{dy}{d\eta}\left|\frac{d\zeta}{dx}\right|, \qquad \frac{dz}{dl} = -\frac{dz}{d\eta}\left|\frac{d\zeta}{dx}\right| \qquad [2.11]$$

Therefore, multiplying both sides of Equation [2.9] by the coefficient of the quantity $dx/d\zeta$, we get the condition when $\xi = 0$ for the values $-\pi < \eta < 0$:

$$\frac{\partial r}{\partial \xi} - \nu \frac{dy}{d\eta}\left(r + \nu e^{\nu z}\int_0^{\eta} e^{-\nu z} r\, dz\right) = \Phi_n(\eta) \qquad [2.12]$$

where

$$\Phi_n(\eta) = v_n \left|\frac{dx}{d\zeta}\right| - \nu \frac{dz}{d\eta}\psi + \nu \frac{dy}{d\eta} e^{\nu z}\left(\phi_1 - r_1 + \nu \int_0^{\eta} e^{-\nu z}\psi\, dy\right) \qquad [2.13]$$

For finding the function f let us represent it in the form of the following series:

$$f = \sum_{n=1}^{\infty} a_n \tau^{-n} + a_0 \ln \tau = \sum_{n=1}^{\infty} a_n e^{-n\xi + in\eta} + a_0(\xi + i\eta) \qquad [2.14]$$

where from condition [2.10] it follows that all the a_n are real coefficients. In order to satisfy condition [2.12], let us form the expression for r and $dr/d\xi$ when $\xi = 0$:

$$r = \sum_{n=1}^{\infty} a_n \cos n\eta, \qquad \frac{dr}{d\xi} = -\sum_{n=1}^{\infty} n a_n \cos n\eta + a_0 \qquad [2.15]$$

and let us expand $\Phi_n(\eta)$ in a Fourier cosine series in the interval $(-\pi, 0)$:

$$\Phi_n(\eta) = \frac{b_0}{2} + \sum_{n=1}^{\infty} b_n \cos n\eta \qquad [2.16]$$

Using these expansions we can represent condition [2.12] in the form

$$-\sum_{n=1}^{\infty} na_n \cos n\eta - \nu \sum_{m=1}^{\infty} a_m \frac{dy}{d\eta}\left(\cos m\eta + \nu e^{\nu z}\int_0^{\eta} e^{-\nu z}\cos m\eta dz\right) + a_0$$

$$= \frac{b_0}{2} + \sum_{n=1}^{\infty} b_n \cos n\eta \qquad [2.17]$$

Further, let us expand the following function in a Fourier cosine series in the interval $(-\pi, 0)$

$$-\nu \frac{dy}{d\eta}\left(\cos m\eta + \nu e^{\nu z}\int_0^{\eta} e^{-\nu z}\cos m\eta dz\right) = \frac{A_{0m}}{2} + \sum_{n=1}^{\infty} A_{nm}\cos n\eta \qquad [2.18]$$

where

$$A_{nm} = -\frac{2\nu}{\pi}\int_{-\pi}^{0}\left(\cos m\eta + \nu e^{\nu z}\int_0^{\eta}\cos m\eta dz\right)\frac{dy}{d\eta}\cos n\eta d\eta \qquad [2.19]$$

Now substituting expansion [2.18] in [2.17] and equating the coefficients of $\cos n\eta$ on both sides of Equation [2.17], we get the infinite system of equations for the coefficients a_n and the equation defining a_0:

$$a_n = \sum_{m=1}^{\infty} C_{nm} a_m + B_n \quad (n = 1, 2, 3, \ldots) \qquad [2.20]$$

$$a_0 = \frac{1}{2}\sum_{m=1}^{\infty}\left(b_0 - a_m A_{0m}\right) \qquad [2.21]$$

Here

$$C_{nm} = \frac{A_{nm}}{n}, \qquad B_n = -\frac{b_n}{n} \qquad [2.22]$$

The coefficients A_{nm} depend on the frequency parameter ν and on the geometric properties of the contour L. The contour $L + \overline{L}$ appears to be symmetric both with respect to axis y and to axis z, and its characteristic parameters are the ratio T/a, where

T is the draft of the contour L, and the coefficient of fullness of the area limited by this contour. Only these two parameters essentially influence the value of the hydrodynamic characteristics.

For symmetric contours the mapping function $x = F(\tau)$ has the following form in the neighborhood of an infinitely distant point:

$$x = k_0\tau + \frac{k_1}{\tau} + \frac{k_2}{\tau^3} + \dots \qquad [2.23]$$

where k_n are real numbers. One of the simpler functions

$$x = \frac{a + T}{2}\tau + \frac{a - T}{2\tau} \qquad [2.24]$$

results in a conformal mapping outside the ellipse with semi-axis a and T on the outside of the unit circle $\tau = e^{i\eta}$.

If in [2.23] we retain only one term, containing τ^{-3}, then we get a function realizing the conformal representation of the exterior of the family of smooth symmetric contours on the outside of the unit circle in the τ-plane:

$$x = y + iz = k_0\tau + k_1\tau^{-1} + k_2\tau^{-3} \qquad [2.25]$$

Assuming $\tau = e^{i\eta}$ ($\xi = 0$) and separating the real and imaginary parts in [2.25], we obtain the equation of the contour in parametric form:

$$y = (k_0 + k_1)\cos\eta + k_2\cos 3\eta, \qquad z = (k_0 - k_1)\sin\eta - k_2\sin 3\eta \qquad [2.26]$$

Let us introduce the dimensionless parameters p and q by means of the relationship

$$k_0 = \frac{T}{1 + p + q}, \qquad k_1 = -pk_0, \qquad k_2 = qk_0 \qquad [2.27]$$

then let us put $\eta = 0$, in Equation [2.26], and we will have

$$\frac{1 + p + q}{1 - p + q} = \frac{T}{a} \qquad [2.28]$$

Further, calculating the area enclosed by the contour L, and making use of the symbol for the coefficient of fullness, $\beta = S/2aT$, we get the equation:

$$\beta = \frac{\pi}{4}\frac{1 - p^2 - q^2}{(1 + p + q)^2}\frac{T}{a} \qquad [2.29]$$

The values of the parameters p and q in terms of T/a and β are determined by relationships [2.28] and [2.29].

In Figure 1 the outlines of the contours are shown for various values of β when $T/2a = 0.5$.

Obviously, if we retain the next term containing τ^{-5} in expression [2.23], we obtain a wider class of contours. However, for problems of pitching, a range of contours which are described by two parameters is adequate.

From all that has been said it follows that the expression [2.25] can be used as the basis of calculation of the coefficients A_{nm} for a wide range of contours of principal configurations. Let us calculate these coefficients for an elliptic contour $L + \overline{L}$ and let us carry out the analysis of the solvability of the system of infinite equations [2.20]. From [2.24] wé have $y = a \cos \eta$, $z = T \sin \eta$ for the coordinates of the elliptic contour. Therefore expression [2.19] takes the form

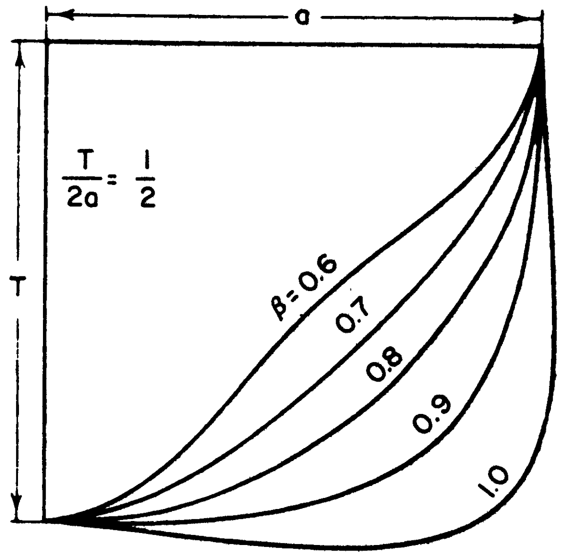

Figure 1

$$A_{nm} = -\frac{2\lambda}{\pi} \int_0^\pi \left(\cos m\eta - \alpha e^{-\alpha \sin \eta} \int_0^\eta e^{\alpha \sin t} \cos mt \cos t\, dt \right) \cos n\eta \sin \eta\, d\eta \qquad [2.30]$$

where

$$\lambda = \nu a, \qquad \alpha = \lambda \frac{T}{a}\,. \qquad [2.31]$$

First of all we shall show that $\lim A_{nm} = 0$ when $T/a \to \infty$. For this let us consider the integral

$$J = \alpha e^{-\alpha \sin \eta} \int_0^\eta e^{\alpha \sin t} \cos t \cos mt\, dt \qquad [2.32]$$

The substitution $\sin\eta - \sin t = u/\alpha$ transforms it into the form

$$J = \int_0^{\alpha \sin\eta} e^{-u} \cos mt\,(u)\,du$$

Hence without difficulty we find out that $\lim J = \cos m\eta$ when $T/a \to \infty$ for the values $0 < \eta < \pi$; therefore $A_{nm} \to 0$ when $T/a \to \infty$.

Also it is not difficult to calculate the values of A_{nm} when $\alpha = 0$ $(T/a = 0)$:

$$A_{nm}{}^{(1)} = -\frac{2\lambda}{\pi} \int_0^{\pi} \cos m\eta \cos n\eta \sin \eta \, dn \qquad [2.33]$$

The coefficients $A_{nm}{}^{(1)}$ are different from zero only when the indices n and m are either both even or both odd. We have

$$A^{(1)}_{2s+1,\ 2l+1} = \frac{2\lambda}{\pi} \left[\frac{1}{4(l+s+1)^2 - 1} + \frac{1}{4(l - s^2) - 1} \right]$$

$$A^{(1)}_{2s,\ 2l} = \frac{2\lambda}{\pi} \left[\frac{1}{4(l+s)^2 - 1} + \frac{1}{4(l-s)^2 - 1} \right] \qquad [2.34]$$

For convenience in computing the coefficients A_{nm} for any α we shall separate expression [2.30] into two parts

$$A_{nm} = A_{nm}{}^{(1)} + A_{nm}{}^{(2)} \qquad [2.35]$$

where the values $A_{nm}{}^{(1)}$ are defined by Formulas [2.34] and the values $A_{nm}{}^{(2)}$ by the formula

$$A_{nm}{}^{(2)} = -\frac{2\lambda}{\pi} \int_0^{\pi} \left[\frac{d}{d\alpha} (e^{-\alpha \sin\eta}) \cos n\eta \int_0^{\eta} \frac{d}{dt} (e^{\alpha \sin t}) \cos t\,dt \right] d\eta \qquad [2.36]$$

We shall use the expansion

$$\frac{d}{dt}(e^{\alpha \sin t}) = -2 \sum_{k=1}^{\infty} (-1)^k 2k I_{2k}(\alpha) \sin 2kt$$

$$+ 2 \sum_{k=0}^{\infty} (-1)^k (2k+1) I_{2k+1}(\alpha) \cos (2k+1)\eta \qquad [2.37]$$

where $I_n(\alpha)$ is a Bessel function with imaginary argument,

$$(-1)^k I_{2k}(\alpha) = \frac{1}{\pi} \int_0^{\pi} \text{ch}\,(\alpha \sin x) \cos 2kx\, dx$$

$$(-1)^k I_{2k+1}(\alpha) = \frac{1}{\pi} \int_0^{\pi} \text{sh}\,(\alpha \sin x) \sin (2k+1)\, x\, dx$$

Substituting the expansion [2.37] in [2.36] and performing the necessary calculations, we get the following expression for the coefficients $A_{nm}{}^{(2)}$:

$$A_{nm}{}^{(2)} = -\sum_{k=1}^{\infty} (-1)^k\, 2kI_{2k}(\alpha)$$

$$\times \frac{d}{d\alpha}\left[\frac{E_{n+2k+m}^{(2)} + E_{n-2k-m}^{(2)} - 2E_n{}^{(2)}}{2k+m} + \frac{E_{n+2k-m}^{(2)} + E_{n-2k+m}^{(2)} - 2E_n{}^{(2)}}{2k-m}\right]$$

$$-\sum_{k=0}^{\infty} (-1)^k (2k+1)\, I_{2k+1}(\alpha)$$

$$\times \frac{d}{d\alpha}\left[\frac{E_{2k+1+m+n}^{(1)} + E_{2k+1+m-n}^{(1)}}{2k+1+m} + \frac{E_{2k+1-m+n}^{(1)} + E_{2k+1-m-n}^{(1)}}{2k+1-m}\right] \qquad [2.38]$$

Here

$$E_n{}^{(1)} = \frac{1}{\pi}\int_0^{\pi} e^{-\alpha \sin \eta} \sin n\eta\, d\eta, \qquad E_n{}^{(2)} = \frac{1}{\pi}\int_0^{\pi} e^{-\alpha \sin \eta} \cos n\eta\, d\eta \qquad [2.39]$$

where $E_{2p}{}^{(1)} = 0$, $E_{2p+1}{}^{(2)} = 0$ and therefore the indices n and m of the coefficients $A_{nm}{}^{(2)}$ are either both even or both odd. The coefficients $E_{2p+1}{}^{(1)}$ and $E_{2p}{}^{(2)}$ are expressed by means of the Bessel function I_n and the Lommel-Weber function Ω_n. We have

$$E_{2p+1}{}^{(1)} = -(-1)^p I_{2p+1}(\alpha) - \Omega_{2p+1}(i\alpha), \qquad E_{2p}{}^{(2)} = (-1)^p I_{2p}(\alpha) + i\Omega_{2p}(i\alpha)$$

where

$$\Omega_{2p}(z) = \frac{1}{\pi} \int_0^{\pi} \sin (z \sin x) \cos 2 px\, dx \qquad [2.40]$$

$$\Omega_{2p+1}(z) = -\frac{1}{\pi} \int_0^{\pi} \cos (z \sin x) \sin (2p + 1)\, x\, dx$$

Thus the system of Equations [2.20] is broken up into two independent systems of infinite equations—one for $a_n c$ with odd indices and the other for $a_n c$ with even indices:

$$a_{2s+1} = \sum_{l=0}^{\infty} C_{sl}^{(1)} a_{2l+1} + B_{2s+1} \qquad (s = 0, 1, 2, \ldots) \qquad [2.41]$$

$$a_{2s} = \sum_{l=1}^{\infty} C_{sl}^{(2)} a_{2l} + B_{2s} \qquad (s = 1, 2, 3, \ldots) \qquad [2.42]$$

where

$$C_{sl}^{(1)} = \frac{A_{2s+1,\, 2l+1}}{2s + 1}, \qquad C_{sl}^{(2)} = \frac{A_{2s,\, 2l}}{2s} \qquad [2.43]$$

Let us investigate under what conditions the sums $|C_{s_0}^{(i)}| + |C_{s_1}^{(i)}| + \ldots$ for all values of s remain less than one and the same number, less than unity. And therefore under these conditions the systems [2.41] and [2.42] belong to a number of completely regular systems, which are well-known and convenient for computation. First of all, because of the passing to the limit in [2.32], we have $(|C_{s_0}^{(i)}| + |C_{s_1}^{(i)}| + \ldots) = 0$ for $T/a \to \infty$. Therefore, for sufficiently large T/a the systems [2.41] and [2.42] appear to be completely regular.

Also let us show that the systems [2.41] and [2.42] appear to be completely regular for small values of T/a. Using expansion [2.18] when $\eta = 0$ and Formulas [2.34] and [2.43], it is possible to show that when $T/a = 0$

$$\sum_{l=0}^{\infty} |C_{sl}^{(1)}| = \frac{4\lambda}{\pi(2s + 1)} \left[1 - \frac{1}{4(2s + 1)^2 - 1}\right] \qquad (s = 0, 1, 2 \ldots)$$

$$\sum_{l=1}^{\infty} |C_{sl}^{(2)}| = \frac{4\lambda}{\pi s} \left(1 - \frac{1}{16s^2 - 1} - \frac{1}{8s^2 - 2}\right) \qquad (s = 1, 2, 3, \ldots)$$

Hence

$$\sum_{l=0}^{\infty} |C_{sl}^{(1)}| \leq \frac{8\lambda}{3\pi}, \qquad \sum_{l=1}^{\infty} |C_{sl}^{(2)}| \leq \frac{92\lambda}{30\pi}$$

Here the equality sign holds in the first sum when $s = 0$ and in the second sum when $s = 1$. Thus for small values of T a the indicated sums are less than one and the same number, less than unity, as long as $\lambda < 30/92\ \pi$.

If the system of coefficients B_n is limited in modulus to its own set, that is, regardless of the index n, which, obviously, takes place in our case since $B_n = -b_n/n$, where b_n appear as the coefficients of the Fourier series, then, as follows from the theory of completely regular systems, systems [2.41] and [2.42] have a uniquely determined solution for the unknown coefficients, where for the solution of completely regular systems it is possible to use the method of successive approximations.

Further, let us show that for small and large values of T/a and any λ the systems [2.41] and [2.42] satisfy the conditions of solvability by means of infinite determinants. For this let us write system [2.41], for instance, in some other form:

$$a_{2s+1} = \sum_{l=0}^{\infty}{}' C_{sl}' a_{2l+1} + B_{2s+1}' \qquad (s = 0, 1, 2, \ldots)$$

where

$$C_{ll}' = 0, \qquad C_{sl}' = \frac{A_{2s+1,\ 2l+1}}{2s+1-A_{2s+1,\ 2s+1}}, \qquad B_{2s+1}' = -\frac{b_{2s+1}}{2s+1-A_{2s+1,\ 2s+1}}$$

It is easy to see that the series $|B_1'| + |B_3'| + |B_5'| + \ldots$ converges and that for small and large values of T/a the double series made up of the sum of the coefficients $C_{sl}'^2$ converges for all λ. Since $C_{ll}' = 0$ for any l, we are convinced that the sufficient conditions are fulfilled for the solvability of system [2.41] by means of infinite determinants. The solvability of equations [2.42] is proved by similar means.

3. If the function $f(x)$ is determined, then considering the relationship [2.3] as a differential equation relative to $w(x)$ and trying to satisfy conditions [1.4] partially, we find

$$w(x) = e^{-i\nu x}\left[j\frac{g}{\sigma}r_0(1-ij) + A^1 + iA_2 + \int_{+\infty}^{x}\frac{df}{dx}e^{i\nu x}\,dx\right] \qquad [3.1]$$

where A_1 and A_2 are constants of integration.

From Formula [3.1] we get, that at great distances from the contour L the wave motion is defined by the functions

$$w(x) = \left[j \frac{g}{\sigma} r_0 (1 - ij) + A_1 + i A_2 \right] e^{-i\nu x} \qquad \text{when } x \to +\infty$$

$$w(x) = \left[j \frac{g}{\sigma} r_0 (1 - ij) + B_1 + i B_2 \right] e^{-i\nu x} \qquad \text{when } x \to -\infty \qquad [3.2]$$

where

$$B_1 + i B_2 = A_1 + i A_2 + \int_{+\infty}^{-\infty} \frac{df}{dx} e^{i\nu x} dx$$

In this formula the path of integration is along a curve connecting the points $x = +\infty$ and $x = -\infty$ and located in the lower half plane below the contour L.

Therefore, it is easy to see that the indicated path of integration may be replaced by the contour C, encompassing the contour $L + \overline{L}$ and directed in a clockwise direction:

$$B_1 + i B_2 = A_1 + i A_2 + \int_C \frac{df}{dx} e^{i\nu x} dx$$

Here replacing the function f by the expansion [2.14] and then letting $\tau = \zeta^{-1}$, we shall have the following expressions:

$$B_1 + B_2 = A_2 + i A_2 - a_0 \int_K e^{i\nu x} \frac{d\zeta}{\zeta} + \sum_{n-1}^{\infty} n a_n \int_K \zeta^{n-1} e^{i\nu x} d\zeta$$

Here K is a closed contour containing the point $\zeta = 0$ and directed in a counterclockwise direction.

Let us introduce the notation

$$D_n = D_n^{(1)} + i D_n^{(2)} = \int_K \zeta^{n-1} e^{i\nu x} d\zeta \qquad [3.3]$$

Then the preceding equality has the form

$$B_1 + iB_2 = A_1 + iA_2 - a_0\,[D_0^{(1)} + iD_0^{(2)}] + \sum_{n=1}^{\infty} na_n\,[D_n^{(1)} + iD_n^{(2)}] \qquad [3.4]$$

Separating the real parts in [3.2], we find that to completely satisfy condition [1.4], it is proper to set

$$A_1 = jA_2 = B_+$$
$$B_1 = -jB_2 = B_- \qquad [3.5]$$

Therefore, in [3.4] replacing i by j and later by $-j$, we obtain the expression for B_+ and B_-:

$$B_+ = \frac{1}{2}\left\{a_0\,[D_0^{(1)} + jD_0^{(2)}] - \sum_{n=1}^{\infty} na_n\,[D_n^{(1)} + jD_n^{(2)}]\right\}$$
$$B_- = -\frac{1}{2}\left\{a_0\,[D_0^{(1)} - jD_0^{(2)}] - \sum_{n=1}^{\infty} na_n\,[D_n^{(1)} - jD_n^{(2)}]\right\} \qquad [3.6]$$

These formulas permit us to calculate B_+ and B_-, if, besides a_n, D_n is also known. For calculating the latter let us consider the smooth contours, for which according to [2.25] the mapping function has the form

$$x = k_0\tau + k_1\tau^{-1} + k_2\tau^{-3} = k_1\zeta + \frac{k_0}{\zeta} + k_2\zeta^3$$

Let us substitute this expression in [3.3] and then set

$$\zeta = \left(\frac{k_0}{k_1}\right)^{1/2} t = \left(-\frac{1}{p}\right)^{1/2} t$$

As a result we shall have

$$D_n = \left(-\frac{1}{p}\right)^{1/2\,n} \int_{K'} t^{n-1} \exp\left[\frac{i\mu}{2}\left(t + \frac{1}{t} + \gamma t^3\right)\right] dt$$

where

$$\mu = 2\nu\sqrt{k_0 k_1} = 2\nu k_0 (-p)^{1/2}, \qquad \gamma = \frac{k_0 k_2}{k_1^2} = qp^{-2} \qquad [3.7]$$

Further, expanding $\left(\frac{1}{2}\, i\mu\gamma t^3\right)$ as a series in powers of t we find

$$D_n = \left(-\frac{1}{p}\right)^{1/2\,n} \sum_{k=0}^{\infty} \left(\frac{\mu\gamma}{2}\right)^k \frac{i^k}{k!} \int_{K'} t^{n+3k-1} \exp\left[\frac{i\mu}{2}\left(t + \frac{1}{t}\right)\right] dt$$

The integral sums in this form are easily determined by means of Bessel functions, and finally, we get the following expression:

$$D_n = 2\pi \left(-\frac{1}{p}\right)^{1/2\,n} \sum_{k=0}^{\infty} \left(\frac{\mu\gamma}{2}\right)^k \frac{i^{n+4k+1}}{k!} J_{n+3k}(\mu) \qquad [3.8]$$

Let us note several particular cases. First of all when $T/a \to 1$ ($p \to 0$), expression [3.8] takes the form

$$D_n = 2\pi i \left(\frac{i\mu_0}{2}\right)^n \sum_{k=0}^{\infty} \left(\frac{i^4 \mu_0^4 q}{2^4}\right)^k \left[\frac{1}{k!\,(n+3k)\,!}\right] \qquad \left(\mu_0 = 2\nu k_0 = \frac{2\nu T}{1+q}\right) \qquad [3.9]$$

Further, considering the case of the elliptic contour and letting $q = 0$ ($\gamma = 0$) in [3.8], we get

$$D_n = 2\pi i^{n+1} \left(\frac{a+T}{a-T}\right)^{1/2\,n} J_n\left(\nu\sqrt{a^2 \quad T^2}\right) \qquad \text{when } a > T \qquad [3.10]$$

$$D_n = 2\pi i^{n+1} \left(\frac{a+T}{T-a}\right)^{1/2\,n} I_n\left(\nu\sqrt{T^2 - a^2}\right) \qquad \text{when } a < T \qquad [3.11]$$

In particular, for $T = a$ we have

$$D_n = 2\pi i^{n+1} \frac{(\nu T)^n}{n!} \qquad [3.12]$$

which also follows from the more general Formula [3.9].

Finally, let us note that for the elliptic contour the following form can be given to Formulas [3.6]

$$B_+ = \pi j a_0 J_0 (\nu\sqrt{a^2 - T^2}) - \pi \sum_{n=1}^{\infty} n a_n \left(\frac{a+T}{a-T}\right)^{\frac{1}{2}n} j^{n+1} J_n (\nu\sqrt{a^2 - T^2})$$

$$B_- = \pi j a_0 J_0 (\nu\sqrt{a^2 - T^2}) + \pi \sum_{n=1}^{\infty} n a_n \left(\frac{a+T}{a-T}\right)^{\frac{1}{2}n} (-j)^{n+1} J_n (\nu\sqrt{a^2 - T^2}) \qquad [3.13]$$

in which for $T > a$ and $T = a$ it is necessary to perform a substitution according to [3.11] and [3.12].

Now let us determine the constants r_1, ϕ_1, and ψ_1, upon which the coefficients a_n and hence, B_+ and B_-, depend linearly. We shall let $\eta = 0$ in expression [2.15] for r, then we shall get the following equation:

$$r_1 = a_1 + a_2 + a_3 + \ldots \qquad [3.14]$$

Further, letting $x = a$ in [3.1], then replacing the function f by the expansion [2.14] and separating the real and imaginary parts, we get the equations for ϕ_1 and ψ_1:

$$\phi_1 = \left(j \frac{g}{\sigma} r_0 + A_+\right) e^{-j\nu a} + a_0 P_0 - \sum_{n=1}^{\infty} n a_n P_n \qquad [3.15]$$

$$\psi_1 = \left(\frac{g}{\sigma} r_0 - jB_-\right) e^{-j\nu a} + a_0 Q_0 - \sum_{n=1}^{\infty} n a_n Q_n \qquad [3.16]$$

where

$$P_n + iQ_n = e^{-i\nu a} \int_{+\infty}^{1} e^{i\nu x(\tau)} \tau^{-n-1} d\tau = -e^{-i\nu a} \int_0^1 \zeta^{n-1} e^{i\nu x(\zeta)} d\zeta \qquad [3.17]$$

For the calculation of the function P_n and Q_n first of all let us establish a recurrence formula. Considering the class of contours with smooth outlines and integrating in [3.17] by parts, we find

$$k_1 (P_{n+1} + iQ_{n+1}) + 3k_2 (P_{n+3} + iQ_{n+3}) - k_0 (P_{n-1} + iQ_{n-1})$$

$$+ \frac{n}{i\nu} (P_n + iQ_n) = \frac{i}{\nu} \qquad [3.18]$$

Hence all the functions P_n and Q_n are expressed in terms of these same functions with indices $n = 0, 1, 2, 3$. In particular, for the elliptic contour ($k_2 = 0$) the functions P_n and Q_n are expressed only in terms of P_0, Q_0, P_1, and Q_1, where for $T = 0$ these functions are expressed by means of Hankel functions.

We have

$$P_0 + iQ_0 = -\frac{\pi}{2} ie^{-i\lambda} H_0^{(1)} (\lambda), \qquad P_1 + iQ_1 = \frac{\pi}{2} e^{-i\lambda} H_1^{(1)} (\lambda) + \frac{i}{\lambda} \quad (\lambda = \nu a) \qquad [3.19]$$

We get the simplest expressions for the functions P_n and Q_n for the contour having the form of a semicircle. In this case $x(\tau) = a\tau$ and therefore

$$P_n + iQ_n = \lambda^n e^{-i\lambda} \int_{+\infty}^{\lambda} \frac{e^{it} dt}{t^{n+1}}$$

The corresponding recurrence formula has the form

$$P_n + iQ_n = \frac{i\lambda}{n} (P_{n-1} + iQ_{n-1}) - \frac{1}{n}$$

Hence all P_n and Q_n are expressed only in terms of P_0 and Q_0. The latter are expressed in terms of the sine and cosine integrals

$$P_0 + iQ_0 = e^{-i\lambda} (ci\,\lambda + isi\,\lambda)$$

Therefore

$$P_n + iQ_n = -\frac{1}{n} - \frac{i\lambda}{n(n-1)} - \frac{(i\lambda)^2}{n(n-1)(n-2)} - \ldots + \frac{(i\lambda)^n}{n!} e^{-i\lambda} (ci\,\lambda + isi\,\lambda) \qquad [3.20]$$

Let us return to the system of Equations [3.14]–[3.16]. The coefficients a_n and, therefore, B_+ and B_- depend linearly on νr_1, $\nu\phi_1$, and $\nu\psi_1$. Hence the determinant $\Delta(\nu)$ of the system [3.14]–[3.16] with respect to the unknowns r_1, ϕ_1, and ψ_1 is equal to unity when $\nu = 0$. Since $\Delta(\nu)$ is a continuous function from $\nu \geq 0$, then $\Delta(\nu) > 0$ for sufficiently small ν. Therefore, for small ν the quantities r_1, ϕ_1 and ψ_1 are determined uniquely.

4. Now let us investigate the general formulas for calculating the hydrodynamic forces. First of all, for the pressure at any point in the fluid we have

$$p = -\rho j \sigma \phi (y, z) e^{j\sigma t}$$

Here we have not taken into account the hydrostatic pressure, the resultant and moment of which are easily calculated.

Let Y, Z, and M_x denote the projections of the hydrodynamic forces and their moment acting on the contour L. Then for these we have

$$Y = -\int_L p \cos (n, y)\, dl, \qquad Z = -\int_L p \cos (n, z)\, dl$$

$$M_x = -\int_L p\, [y \cos (n, z) - z \cos (n, y)]\, dl$$

or

$$Y = -\rho j \frac{g}{\sigma} e^{j\sigma t} \int_L \nu\phi\, dz, \qquad Z = \rho j \frac{g}{\sigma} e^{j\sigma t} \int_L \nu\phi\, dy$$

$$M_x = \rho j \frac{g}{\sigma} e^{j\sigma t} \int_L \nu\phi\, (y\, dy + z\, dz)$$

For the following calculation we use the relationships, following from [2.5]

$$\nu \frac{dz}{dl} \phi = \frac{\partial\phi}{\partial l} - \nu \frac{dy}{dl} \psi - \frac{\partial r}{\partial l}, \qquad \nu \frac{dy}{dl} \phi = \frac{\partial\phi}{\partial n} + \nu \frac{dz}{dl} \psi - \frac{\partial r}{\partial n}$$

Therefore the preceding formulas may be written:

$$Y = -\rho j \frac{g}{\sigma} e^{j\sigma t} \int_L \left(\frac{\partial \phi}{\partial l} - \nu \frac{dy}{dl} \psi - \frac{\partial r}{\partial l} \right) dl$$

$$Z = \rho j \frac{g}{\sigma} e^{j\sigma t} \int_L \left(\frac{\partial \phi}{\partial n} + \nu \frac{dz}{dl} \psi - \frac{\partial r}{\partial n} \right) dl \qquad [4.1]$$

$$M_x = \rho j \frac{g}{\sigma} e^{j\sigma t} \int_L \left[\left(\frac{\partial \phi}{\partial l} z + \frac{d\phi}{\partial n} y \right) - \nu \left(z \frac{dy}{dl} - \frac{dz}{dl} y \right) \psi - \left(\frac{\partial r}{\partial l} z + \frac{\partial r}{\partial n} y \right) \right] dl$$

First let us make the calculation of the vertical component of the hydrodynamic forces. From condition [2.4] we have

$$\int_L \frac{\partial r}{\partial n} dl = \frac{1}{2} \int_{L + \bar{L}} \frac{\partial r}{\partial n} dl = -\frac{1}{2i} \int_{L + \bar{L}} df$$

where the integration is performed in the clockwise direction.

Further, using expression [2.14] and the theorem of residues we find

$$\int_L \frac{\partial r}{\partial n} dl = \pi a_0$$

and, therefore,

$$Z = \rho j \frac{g}{\sigma} e^{j\sigma t} \left[\int_L \left(\frac{\partial \phi}{\partial n} + \nu \frac{dz}{dl} \psi \right) dl - \pi a_0 \right] \qquad [4.2]$$

The integral sum in Formula [4.2] is also easy to calculate. Actually, at points of the contour we have

$$\frac{\partial\phi}{\partial n} = v_2 \cos(n, y) + v_3 \cos(n, z) + v_4 [y \cos(n, z) - z \cos(n, y)]$$

$$= - v_2 \frac{dz}{dl} + v_3 \frac{dy}{dl} + v_4 \left(y \frac{dy}{dl} + z \frac{dz}{dl} \right) \qquad [4.3]$$

$$\psi = \psi_1 - \int_0^l \frac{\partial\phi}{\partial n} dl = \psi_1 + v_2 z - v_3 (y - a) - \frac{v_4}{2} (y^2 - a^2 + z^2)$$

Here v_2, v_3 and v_4 are the complex amplitudes, with respect to j, of the linear and angular velocities.

Substituting [4.3] in [4.2], we find the simple formula

$$Z = - \rho j \frac{g}{\sigma} e^{j\sigma t} (\pi a_0 + b v_3 + \nu S v_3) \qquad [4.4]$$

where $b = 2a$ is the breadth of the contour L at the waterline, and S is the area bounded by this contour.

Now let us calculate Y. Performing the integration using [4.3], we have

$$Y = - \rho j \frac{g}{\sigma} e^{j\sigma t} (\phi_{-1} - \phi_1 + r_1 - r_{-1}) + \rho j \sigma e^{j\sigma t} [S v_2 - a b v_3$$

$$+ (h S - \frac{2}{3} a^3) v_4 - \psi_1 b] \qquad [4.5]$$

where ϕ_{-1} and r_{-1} are the values of the functions ϕ and r at the point $y = - a$ and $z = 0$, h is the depth of submergence of the center of gravity of the surface. The quantity r_{-1} is easily determined. Letting $\eta = - \pi$ in expression [2.15] for r, we get

$$r_{-1} = - a_1 + a_2 - a_3 + a_4 - \dots$$

Therefore

$$r_1 - r_{-1} = 2 (a_1 + a_3 + a_5 + \dots) \qquad [4.6]$$

The value of the function ϕ at the point $y = - a$ and $z = 0$ is determined with the help of quantities analogous to the functions P_n.

$$\phi_{-1} = j\frac{g}{\sigma} r_0 e^{j\nu a} + B_- e^{-j\nu a} + a_0 P_0' - \sum_{n=1}^{\infty} n a_n P_n' \qquad [4.7]$$

where

$$P_n' + iQ_n' = e^{i\nu a} \int_{-\infty}^{-1} \frac{e^{i\nu x(\tau)}}{\tau^{n+1}}\, d\tau \qquad [4.8]$$

The moment of the hydrodynamic forces is calculated by a similar method. We shall present this calculation in the transformed form

$$M_x = \frac{1}{\nu} Y + \rho j \frac{g}{\sigma} e^{j\sigma t} \frac{1}{2} \operatorname{Im} \int_{L+\bar{L}} \bar{x}\, df + M_0$$

where

$$M_0 = \rho j \frac{g}{\sigma} e^{j\sigma t} \int_L \left(y \frac{\partial \phi}{\partial n} - \nu z \frac{dy}{dl} \psi + \nu \frac{dz}{dl} y \psi \right) dl$$

For the class of contours with smooth outlines we have

$$\bar{x} = y - iz = k_0 \bar{\tau} + k_1 \bar{\tau}^{-1} + k_2 \bar{\tau}^{-3}$$

and since for a unit circle $\bar{\tau} = \tau^{-1}$, then $\bar{x} = k_0 \tau^{-1} + k_1 \tau + k_2 \tau^3$.

Using this expression, and also [2.14], we find

$$M_x = \frac{1}{\nu} Y + \pi \rho j \frac{g}{\sigma} e^{j\sigma t} (k_1 a_1 + 3 k_2 a_3) + M_0 \qquad [4.9]$$

where from [4.3] the value M_0 is determined from the formula

$$M_0 = \rho j \frac{g}{\sigma} e^{j\sigma t} \Big[S(1 + 3\nu h) v_2 - 2\nu a S v_3 + \Big(2\nu I_y + 2\nu I_z - \nu a^2 S + hS - \frac{2a^3}{3} \Big) v_4 - 2\nu \psi_1 S \Big] \qquad [4.10]$$

Here I_y and I_z represent the moments of inertia of the surface S with respect to the y- and z-axes.

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II. Brode, Mildred H., tr.
III. Fisher, David, tr.
IV. Thorpe, Alice C., tr.

David Taylor Model Basin. Translation 283.
THE OSCILLATIONS OF A FLOATING FORM ON THE SURFACE OF A HEAVY FLUID (Kolebaniia plavaiushchego kontura na poverkhnosti tiazheloi' zhidkosti), by M.D. Haskind. February 1959. 23p. refs. (Translated by Mildred H. Brode David Fisher, and Alice C. Thorpe; from Prikladnaia Matematika i Mekhanika, Vol. 17, 1953, p. 165-178.) UNCLASSIFIED

This report gives a method of finding the exact solution of the problem on pitching of cylindrical ship forms. The solution is given for a wide range of contours by means of which all the hydrodynamic characteristics may be calculated.

1. Cylindrical shells – Pitch – Mathematical analysis
2. Cylindrical shells – Hydrodynamic characteristics – Mathematical analysis
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