Translation 316

DAVID TAYLOR MODEL BASIN
WASHINGTON D.C.

DEPARTMENT OF THE NAVY

HYDROMECHANICS

AERODYNAMICS

STRUCTURAL MECHANICS

APPLIED MATHEMATICS

MOTION OF A THIN WING NEAR THE FREE SURFACE OF A LIQUID

(Rukh Tonkogo Krila Poblizu Vil′Noyi Poverkhni Ridini)

by

A.N. Panchenkov

Translated by J.N. Newman

[redacted]

HYDROMECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

September 1963

Translation 316

MOTION OF A THIN WING NEAR THE FREE SURFACE OF A LIQUID

(Rukh Tonkogo Krila Poblizu Vil'Noyi Poverkhni Ridini

by

A.N. Panchenkov

Prikladna Mekhanika
Akademia Nauk Ukrainskoi R.S.R.
Institut Mekhanikii
Vol. 8, No. 4
Kiev – 1962
pp. 433–442

Translated by J.N. Newman

September 1963 Translation 316

ABSTRACT

A new method is described for the approximate solution of integral equations which arise in the research of M.V. Keldysh and M.A. Lavrentyev on the problem of the motion of a thin wing near the free surface of a liquid. The intensity of the distribution of eddies is defined in the form of a series. The values of $\gamma_0(s)$, $\gamma_1(s)$, ..., $\gamma_6(s)$ and the formula for the lifting force are given for two special cases—a plane plate and a circular arc profile.

In the limit of infinite Froude numbers, the results obtained are in good agreement, for any depth of submersion of the wing, with experimental data and with N.E. Kochin's method.

The problem of the motion of a thin wing with a linear boundary condition on the surface has been examined in the work of M.V. Keldysh and M.A. Lavrentyev.[1] They obtained a solution of the problem for the case when the depth of submergence of the wing is large. The present paper proposes a generalized solution to this problem which is suitable for any relative submergence.

Beneath the free surface of a heavy fluid moving with steady horizontal velocity V_0, let there be a submerged wing having the form of the curve C.

The motion of the fluid is assumed to be potential and to be steady with respect to the moving system of coordinates. The (x, y) coordinate system is fixed with respect to the wing. The x-axis is situated in the free surface and directed downstream. The y-axis is directed vertically upwards and passes through the middle of the wing.

We prescribe the boundary conditions:

1. On the free surface of the fluid

$$I_m\left(i\frac{d^2w}{dz^2}-\frac{g}{v^2}w\right)=0 \qquad [1]$$

2. On the contour of the submerged wing

$$\frac{\partial\varphi}{\partial n}=-v_0\sin\theta \qquad [2]$$

The complex velocity potential function will be sought in the form

$$w(z)=\int_C \omega_s(z)\,\gamma(s)\,ds \qquad [3]$$

[1]References are listed on page 12.

where $\omega_s(z)$ is the potential of a unit vortex, situated at the point s on the contour C, and $\gamma(s)$ is the intensity of the vortex sheet.

It follows that the normal component of the fluid velocity along the line C will be given by the expression

$$v_n = \int_C C(\sigma, s)\,\gamma(s)\,ds,$$

where

$$C(\sigma, s) = Re\left[\frac{d\omega_s[z(\sigma)]}{dz}\,ie^{i\theta(\sigma)}\right]$$

In accordance with the boundary conditions [1] and [2], we obtain a Fredholm integral equation of the first kind

$$\int_C C(\sigma, s)\,\gamma(s)\,ds = v_0 \sin\theta(\sigma) \qquad [4]$$

For the accurate determination of $\gamma(s)$ including small second-order quantities with respect to α (the angle of attack), the integral equation (Equation [4]) can be reduced to the form

$$\int_{-a}^{a} \{K(x-s) - [f(x) - f(s)]\,K_1(x-s) - f'(x)\,I(x-s)\}\,\gamma(s)\,ds = 2\pi v_0 f'(x) \qquad [5]$$

where

$$K(x) + iI(x) = 2\pi i\,\frac{d\omega_0(x)}{dx} = \frac{1}{x} + \frac{1}{x-2hi}$$

$$-\frac{2gi}{v_0^2}\,e^{-\frac{gi}{v_0^2}x}\int_{-\infty}^{x}\frac{e^{\frac{gi}{v_0^2}\xi}}{\xi-2hi}\,d\xi \qquad [6]$$

and

$$K_1(x) = Re\left[-\frac{i}{(x-2hi)^2} - \frac{2g}{v_0^2}\,e^{-\frac{gi}{v_0^2}xi}\int_{-\infty}^{x}\frac{e^{\frac{gi}{v_0^2}\xi}\,d\xi}{(\xi-2hi)^2}\right] \qquad [7]$$

Here h is the depth of submergence and $f(x)$ is the equation of the wing surface.

With an accuracy of first order with respect to α we have the integral equation

$$\int_{-a}^{a} K(x-s)\,\gamma(s)\,ds = 2\pi v_0 f'(x) \qquad [8]$$

Putting Equation [8] in nondimensional form:

$$\int_{-1}^{+1} \bar{K}(\bar{x}-\bar{s})\,\bar{\gamma}(\bar{s})\,d\bar{s} = 2\pi \bar{f}'(\bar{x})$$

$$x = a\bar{x}; \qquad s = a\bar{s}; \qquad f'(x) = a\bar{f}'(\bar{x});$$

$$K(x-s) = \frac{1}{a}\bar{k}(\bar{x}-\bar{s}); \qquad \gamma(s) = \bar{\gamma}(\bar{s})\,v_0; \qquad [9]$$

$$\bar{h} = \frac{h}{2a}$$

where a is the half-chord of the wing.

The kernel $\bar{K}(\bar{x})$ in Equation [9] can be represented in the form

$$\bar{K}(\bar{x}) = Re\left[\frac{1}{\bar{x}} + \frac{1}{\bar{x}-4\bar{h}i} - \frac{4ghi}{v_0^2}\int_0^\infty \frac{e^{-\lambda i\alpha}}{\bar{x}-4\bar{h}(\alpha+i)}\,d\alpha\right]$$

where

$$\lambda = \frac{2gh}{v_0^2} \qquad [10]$$

For the solution of Equation [9], we develop the kernel $K(x)$ in powers of the parameter

$$\tau = \sqrt{2\bar{h}^2+1} - \sqrt{2}\,\bar{h}$$

As a result of expanding the right-hand side of Equation [10] in increasing powers of the parameter τ, we have

$$K(x) + iI(x) = \frac{1}{x} + \frac{\tau i}{\sqrt{2}}\left\{\sum_{n=0,2,4}^{\infty} \tau^n \left\{\sum_{k=0}^{\frac{n}{2}} \frac{(n-k)(n-k-1)\ldots(k+1)}{(n-2k)!}\right.\right.$$

$$\times\left(\frac{x}{\sqrt{2}}\right)^{n-2k}\left[i^{n-2k} + 2\lambda\int_0^\infty \frac{e^{-i\lambda\alpha}}{(\alpha+i)^{n+1-2k}}\,d\alpha\right]$$

$$-\sum_{n=1,3}^{\infty} \tau^n \sum_{k=0}^{\frac{n-1}{2}} \frac{(n-k)(n-k-1)\ldots(k+1)}{(n-2k)!}\left(\frac{x}{\sqrt{2}}\right)^{n-2k}$$

$$\left.\left.\times\left[i^{n-2k} - 2\lambda\int_0^\infty \frac{e^{-i\lambda\alpha}}{(\alpha+i)^{n+1-2k}}\,d\alpha\right]\right\}\right\}$$

The integral which appears in this expression can be represented in the form of the series

$$\int_0^{+\infty} \frac{e^{-i\lambda\alpha}}{(\alpha+i)^{n+1}}\,d\alpha = i^{-n}\left\{\frac{1}{n}+\frac{\lambda}{n(n-1)}+\frac{\lambda^2}{n(n-1)(n-2)}+\ldots+\right.$$

$$\left.+\frac{\lambda^{n-1}}{n(n-1)\ldots 2\cdot 1}-\frac{\lambda^n e^{-\lambda}}{n!}\left[E_i(\lambda)+i\pi\right]\right\}$$

Then

$$K(x)+iI(x)=\frac{1}{x}+\frac{i\tau}{\sqrt{2}}\left\{\sum_{n=0,2,4}^{\infty}\tau^n\sum_{k=0}^{\frac{n}{2}}\frac{(n-k)(n-k-1)\ldots(k+1)}{(n-2k)!}\right.$$

$$\times\left(\frac{x}{\sqrt{2}}\right)^{n-2k}\left[i^{n-2k}+\frac{2\lambda}{n-2k}+\frac{2\lambda^2}{(n-2k)(n-1-2k)}\right. \qquad [11]$$

$$\left.+\ldots-\frac{2\lambda^{n-2k}e^{-\lambda}}{(n-2k)!}(E_i(\lambda)+i\pi)\right]$$

$$-\sum_{n=1,3}^{\infty}\tau^n\sum_{k=0}^{\frac{n-1}{2}}\frac{(n-k)(n-k-1)\ldots(k+1)}{(n-2k)!}\left(\frac{x}{\sqrt{2}}\right)^{n-2k}\left[i^{n-2k}\right.$$

$$\left.\left.-\frac{2\lambda}{n-2k}+\frac{2\lambda^2}{(n-2k)(n-1-2k)}+\ldots-\frac{2\lambda^{n-2k}e^{-\lambda}}{(n-2k)!}(E_i(\lambda)+i\pi)\right]\right\}$$

$$=\frac{1}{x}+\sum_{n=1}^{\infty}\left[K_n(x)+iI(x)\right]\tau^n$$

We will look for the function $\bar{\gamma}(\bar{s})$ in the form of a series in increasing powers of τ

$$\bar{\gamma}(\bar{s})=\gamma_0(\bar{s})+\tau\gamma_1(\bar{s})+\ldots+\tau^n\gamma_n(\bar{s})\ldots \qquad [12]$$

Substituting the expansions, Equations [11] and [12], in Equation [9], we obtain a system of recursive equations

$$\int_{-1}^{+1}\frac{\gamma_0(\bar{s})\,d\bar{s}}{\bar{x}-\bar{s}}=2\pi f'(\bar{x})$$

$$\int_{-1}^{+1}\frac{\gamma_n(\bar{s})}{\bar{x}-\bar{s}}\,d\bar{s}=-\int_{-1}^{+1}\left[\sum_{m=1}^{n}K_m(\bar{x}-\bar{s})\,\gamma_{n-m}(\bar{s})\right]d\bar{s} \qquad [13]$$

for $h>0$.

Thus the determination of the function $\gamma_n(s)$ reduces to solving the singular integral equation

$$\int_{-1}^{+1} \frac{\gamma_n(\bar{s})\, d\bar{s}}{\bar{x}-\bar{s}} = \psi_n(\bar{x})$$

Taking account of the end condition for $\gamma_n(\bar{s})$ at the point $\bar{x}=+1$, this equation will have the solution[1,2]

$$\gamma_n(\bar{s}) = \frac{1}{\pi^2}\sqrt{\frac{1-\bar{x}}{1+\bar{x}}}\int_{-1}^{+1}\sqrt{\frac{1+\bar{s}}{1-\bar{s}}}\,\frac{\psi_n(\bar{s})\, d\bar{s}}{\bar{s}-\bar{x}} \qquad [14]$$

The integral that appears in Equation [14] can be calculated by means of an effective method, defining the function $f'(x)$ in arbitrary form.[4] This is especially simple to carry out if $f'(x)$ is given by a polynomial in x, which is true for certain practical cases. For carrying out these operations, $\gamma_0(\bar{x})$, $\gamma_1(\bar{x})$, ... $\gamma_n(\bar{x})$ can be expressed as polynomials in x. The integrals for these can be determined from the formulas

$$D_n = \int_{-1}^{+1}\sqrt{\frac{1+s}{1-s}}\,\frac{s^n ds}{s-x};$$

$$D_0 = \pi; \qquad D_n = xD_{n-1} + A_{n+1};$$

$$A_0 = \pi; \qquad A_{2k} = A_{2k-1} = \frac{1\cdot 3\ldots(2k-1)}{2^k k!}\pi;$$

$$B_n = \int_{-1}^{+1}\sqrt{\frac{1-x}{1+x}}\,x^n dx; \qquad B_0 = \pi; \qquad B_{2k} = -B_{2k-1} = \frac{1\cdot 3\ldots(2k-1)}{2^k k!}\pi$$

The lift force for a thin wing can be expressed in terms of $\gamma(\bar{s})$ as:[1]

$$P = -\varrho a v_0^2 \int_{-1}^{+1} \bar{\gamma}(\bar{s})\, d\bar{s} \qquad [15]$$

Including terms of second order in Equation [15], one obtains the form

$$P = -\varrho a v_0^2 \int_{-1}^{+1} (1+\bar{u}^*)\bar{\gamma}(\bar{s})\, d\bar{s} \qquad [16]$$

where $\bar{u}^*$ is the nondimensional perturbation velocity on the wing and

$$\bar{u}^* = \frac{1}{2\pi}\int_{-1}^{+1} I(\bar{x}-\bar{s})\bar{\gamma}(\bar{s})\, d\bar{s} \qquad [17]$$

We shall examine some individual examples.

1. *For a flat plate*

$$\frac{d\bar{y}}{d\bar{x}} = -\alpha$$

We shall confine the calculation to a few terms of the expansion [12]. Using Equations [11], [13], and [14], we obtain:

$$\gamma_0(\bar{x}) = -2R\alpha; \qquad \gamma_1(\bar{x}) = 2RK_1(\lambda)\alpha$$

$$\gamma_2(\bar{x}) = 2R\left[\frac{3}{2}K_2(\lambda) - K_1^2(\lambda) + K_2(\lambda)\bar{x}\right]\alpha$$

$$\gamma_3(\bar{x}) = 2\alpha R\left[K_3(\lambda) - 2K_1(\lambda)K_2(\lambda) + \frac{3}{2}K_3^{\cdot}(\lambda) + K_1^3(\lambda) + \bar{x}K_3^{\cdot}(\lambda)\right]$$

$$\gamma_4(\bar{x}) = 2\alpha R\left\{\frac{3}{2}K_4'(\lambda) + \frac{25}{8}K_4^{\cdot}(\lambda) - 2K_1(\lambda)K_3'(\lambda)\right.$$

$$-2K_3^{\cdot}(\lambda)K_1(\lambda) - \frac{5}{4}K_2^2(\lambda) + \frac{7}{2}K_1^2(\lambda)K_2(\lambda) - K_1^4(\lambda) + \frac{1}{2}K_3'(\lambda)$$

$$+\left[K_4'(\lambda) + \frac{7}{2}K_4^{\cdot}(\lambda) - 2K_3^{\cdot}(\lambda)\dot{K}_1(\lambda) - K_2^2(\lambda) - \frac{3}{2}K_1(\lambda)K_2(\lambda)\right]\bar{x}$$

$$\left.+\left[\frac{5}{2}K_4^{\cdot}(\lambda) - K_3^{\cdot}(\lambda)K_1(\lambda)\right]\bar{x}^2 + K_4^{\cdot}(\lambda)\bar{x}^3\right\}$$

where

$$R = \sqrt{\frac{1-\bar{x}}{1+\bar{x}}}.$$

Then the expression for the lift force can be obtained in the form:

$$P_{\bar{h}} = 2\pi\varrho a v_0^2\alpha\left\{1 - \tau K_1(\lambda) - [K_2(\lambda) - K_1^2(\lambda)]\tau^2\right.$$

$$-[K_3'(\lambda) - 2K_1(\lambda)K_2(\lambda) - K_3^{\cdot}(\lambda) + K_1^3(\lambda)]\tau^3$$

$$-\left[K_4'(\lambda) + \frac{9}{4}K_4^{\cdot}(\lambda) - \frac{3}{4}K_2^2(\lambda) - \frac{3}{2}K_3^{\cdot}(\lambda)K_1(\lambda)\right. \qquad [18]$$

$$\left.\left.+\frac{7}{2}K_1^2(\lambda)K_2(\lambda) - \frac{3}{4}K_1(\lambda)K_2(\lambda) + \frac{1}{2}K_3'(\lambda) - K_1^4(\lambda)\right]\tau^4 - \ldots\right\}$$

where

$$K_1(\lambda) = \frac{2\lambda\pi}{\sqrt{2}} e^{-\lambda},$$

$$K_2(\lambda) = \frac{1}{2} + \frac{2\lambda}{\sqrt{2}} [1 - \lambda e^{-\lambda} E_i(\lambda)],$$

$$K_3'(\lambda) = K_1(\lambda); \qquad K_3''(\lambda) = -\frac{\lambda^3}{\sqrt{2}} \pi e^{-\lambda},$$

$$K_4'(\lambda) = 1 + \frac{2\lambda}{\sqrt{2}} [1 - \lambda e^{-\lambda} E_i(\lambda)], \text{ and}$$

$$K_4''(\lambda) = -\frac{1}{4}\left\{1 + \frac{2\lambda}{\sqrt{2}}\left[\frac{1}{3} + \frac{\lambda}{6} + \frac{\lambda^2}{6} - \frac{\lambda^3 e^{-\lambda}}{6} E_i(\lambda)\right]\right\}.$$

We shall calculate the hydrodynamic characteristics for the condition

$$Fr_b = \frac{v}{\sqrt{2ga}} \to \infty.$$

In this case $\gamma_1(s) = \gamma_3(s) = 0$, and $\gamma_0(s)$, $\gamma_2(s)$ ...... have the form

$$\gamma_0(\bar{x}) = -2R\alpha;$$

$$\gamma_2(\bar{x}) = 2R\alpha\left(\frac{3}{4} + \frac{1}{4}\bar{x}\right);$$

$$\gamma_4(\bar{x}) = 2R\alpha\left(\frac{13}{32} - \frac{1}{8}\bar{x} - \frac{5}{8}\bar{x}^2 - \frac{1}{4}\bar{x}^3\right);$$

$$\gamma_6(\bar{x}) = 2R\alpha\left(-\frac{17}{32} - \frac{39}{32}\bar{x} - \frac{17}{16}\bar{x}^2 + \frac{7}{16}\bar{x}^4 + \frac{1}{8}\bar{x}^5\right).$$

then

$$P_{\bar{h}} = 2\pi\varrho a v_0^2 \alpha\left(1 - \frac{1}{2}\tau^2 - \frac{1}{4}\tau^4 + \frac{21}{64}\tau^6 + \dots\right)$$

$$\psi_{n\pi} = \frac{P_{\bar{h}}}{P_\infty} = 1 - \frac{1}{2}\tau^2 - \frac{1}{4}\tau^4 + \frac{21}{64}\tau^6 + \dots \qquad [19]$$

Equations [16] and [17] can be used to obtain expressions accurate to second order with respect to α. Retaining the first term of the expansion, Equation [11], we have

$$I(\bar{x}) = \left[\frac{1}{\sqrt{2}} - \frac{2\lambda}{\sqrt{2}} e^{-\lambda} E_i(\lambda)\right]\tau$$

Then

$$\bar{u}^* = -\alpha\left[\frac{1}{\sqrt{2}} - \frac{2\lambda}{\sqrt{2}} e^{-\lambda} E_i(\lambda)\right]\tau\,\{1 - K_1(\lambda)\tau - [K_2(\lambda) - K_1^2(\lambda)]\tau^2\} \qquad [20]$$

$$P_{\bar{h}} = 2\pi\varrho a v_0^2 \alpha \Big\{ 1 - \Big[K_1(\lambda) + \alpha \Big(\frac{1}{\sqrt{2}} - \frac{2\lambda}{\sqrt{2}} e^{-\lambda} E_i(\lambda) \Big) \Big] \tau$$
$$- \Big[K_2(\lambda) - K_1^2(\lambda) - 2\alpha K_1(\lambda) \Big(\frac{1}{2} - \frac{2\lambda}{\sqrt{2}} e^{-\lambda} E_i(\lambda) \Big) \Big] \tau^2$$
$$- \Big[K_3'(\lambda) - 2K_1(\lambda) K_2(\lambda) - K_3''(\lambda) + K_1^3(\lambda) - \alpha \Big(2K_2(\lambda) - 3K_1^2(\lambda) \Big) \Big(\frac{1}{\sqrt{2}}$$
$$- \frac{2\lambda}{\sqrt{2}} e^{-\lambda} E_i(\lambda) \Big) \Big] \tau^3 - \Big[K_4'(\lambda) + \frac{9}{4} K_4''(\lambda)$$
$$- \frac{3}{4} K_2^2(\lambda) - \frac{3}{2} K_3''(\lambda) K_1(\lambda) + \frac{7}{2} K_1^2(\lambda) K_2(\lambda) - \frac{3}{4} K_1(\lambda) K_2(\lambda)$$
$$+ \frac{1}{2} K_3'(\lambda) - K_1^4(\lambda) + \alpha \Big(4K_1(\lambda) K_2(\lambda) - 3K_1^3(\lambda) - K_3'(\lambda)$$
$$- K_3''(\lambda) \Big) \Big(\frac{1}{\sqrt{2}} - \frac{2\lambda}{\sqrt{2}} e^{-\lambda} E_i(\lambda) \Big) \Big] \tau^4 + \dots \Big\} \qquad [21]$$

The asymptotic expression for the lift force as $Fr_b \to \infty$ is given by the expression

$$P_{\bar{h}} = 2\pi\varrho a v_0^2 \alpha \Big[1 - \frac{\alpha}{\sqrt{2}} \tau - \frac{1}{2} \tau^2 + \frac{\alpha}{\sqrt{2}} \tau^3 - \frac{1}{4} \tau^4 + \frac{21}{64} \tau^6 - \frac{37}{64} \frac{\alpha}{\sqrt{2}} \tau^7 + \dots \qquad [22]$$

2. *For the case of a thin circular arc* with chord 2a and central angle 2β we have the equation

$$\frac{d\bar{y}}{d\bar{x}} = -\Big(\alpha + \beta \frac{\bar{x}}{a} \Big)$$

Carrying out the necessary calculations, we obtain

$$\gamma_0(\bar{x}) = -2R(\alpha + \beta + \beta\bar{x})$$
$$\gamma_1(\bar{x}) = 2RK_1(\lambda) \Big(\alpha + \frac{1}{2} \beta \Big)$$
$$\gamma_2(\bar{x}) = 2R \Big\{ \Big[\frac{3}{2} K_2(\lambda) - K_1^2(\lambda) + K_2(\lambda) \bar{x} \Big] \Big(\alpha + \frac{1}{2} \beta \Big) - K_2(\lambda) \frac{1}{4} \beta \Big\} \qquad [23]$$
$$\gamma_3(\bar{x}) = 2R \Big\{ \Big[K_3'(\lambda) - 2K_1(\lambda) K_2(\lambda) + \frac{3}{2} K_3''(\lambda)$$
$$+ K_1^3(\lambda) + \bar{x} K_3''(\lambda) \Big] \Big(\alpha + \frac{1}{2} \beta \Big) - \frac{1}{4} \beta [K_1(\lambda) K_2(\lambda) + K_3''(\lambda)] \Big\}$$

Then the formula for $P_{\bar{h}}$ can be written as:

$$P_{\bar{h}} = 2\pi\varrho\alpha v_0^2(\alpha+\alpha_0)\Big\{1 - \tau K_1(\lambda) - \Big[\Big(1-\frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}\Big)K_2(\lambda) - K_1^2(\lambda)\Big]\tau^2$$
$$- \Big[K_3'(\lambda) - 2K_1(\lambda)K_2(\lambda) - K_3''(\lambda) + K_1^3(\lambda)$$
$$- \frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}K_1(\lambda)K_2(\lambda) - K_3''(\lambda)\Big]\tau^3 + \ldots\Big\} \qquad [24]$$

For $Fr_b \to \infty$ the limits of the functions $\gamma_0(s)$, $\gamma_1(s)$, ... can be written as

$$\gamma_0(\bar{x}) = -2R\Big[\Big(\alpha+\frac{1}{2}\beta\Big) + \beta\Big(\frac{1}{2}+\bar{x}\Big)\Big]$$
$$\gamma_2(\bar{x}) = 2R\Big[\Big(\alpha+\frac{1}{2}\beta\Big)\Big(\frac{3}{4}+\frac{1}{2}\bar{x}\Big) - \frac{1}{8}\beta\Big]$$
$$\gamma_4(\bar{x}) = 2R\Big\{\Big(\alpha+\frac{1}{2}\beta\Big)\Big(\frac{13}{32}-\frac{1}{8}\bar{x}-\frac{5}{8}\bar{x}^2-\frac{1}{4}\bar{x}^3\Big)$$
$$+ \Big(\frac{3}{16}\bar{x}^2+\frac{11}{32}\bar{x}+\frac{5}{64}\Big)\beta\Big\} \qquad [25]$$
$$\gamma_6(\bar{x}) = 2R\Big[\Big(\alpha+\frac{1}{2}\beta\Big)\Big(-\frac{17}{32}-\frac{39}{32}\bar{x}-\frac{17}{16}\bar{x}^2$$
$$+\frac{7}{16}\bar{x}^4+\frac{1}{8}\bar{x}^5\Big) + \beta\Big(\frac{94}{256}+\frac{92}{128}\bar{x}-\frac{1}{4}\bar{x}^2-\frac{5}{16}\bar{x}^3-\frac{5}{32}\bar{x}^4\Big)\Big]$$

Then

$$P_{\bar{h}} = 2\pi\varrho\alpha v_0^2(\alpha_0+\alpha)\Big[1-\frac{1}{2}\Big(1-\frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}\Big)\tau^2$$
$$-\frac{1}{4}\tau^4 + \Big(\frac{21}{64}-\frac{49}{128}\frac{\alpha_0}{\alpha_0+\alpha}\Big)\tau^6 + \ldots\Big] \qquad [26]$$

$$\psi_x = 1 - \frac{1}{2}\Big(1-\frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}\Big)\tau^2 - \frac{1}{4}\tau^4 + \Big(\frac{21}{64}-\frac{49}{128}\frac{\alpha_0}{\alpha_0+\alpha}\Big)\tau^6 + \ldots \qquad [27]$$

where α_0 is the angle of zero lift for the circular arc $\alpha_0 = \frac{1}{2}\beta$.

For $\alpha = 0$

$$\psi_{\pi} = 1 - 0{,}25\tau^2 - 0{,}25\tau^4 - 0{,}054\tau^6 + \ldots \qquad [28]$$

On completing the reduction of the formulas for $\bar{u}^*$, $P_{\bar{h}}$, and ψ_{π} with an accuracy up to second order in α,

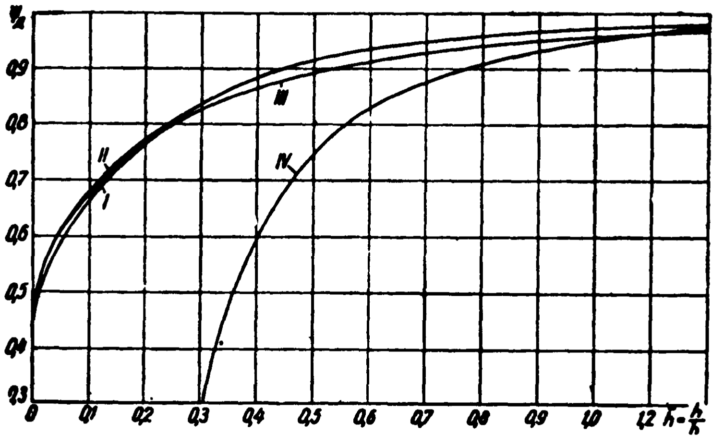
$$\begin{aligned}\bar{u}^* = -(\alpha_0+\alpha)\left[\frac{1}{\sqrt{2}} - \frac{2\lambda}{\sqrt{2}}e^{-\lambda}E_i(\lambda)\right]\tau\Big\{1 - K_1(\lambda)\tau \\ - \left[\left(1 - \frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}\right)K_2(\lambda) - K_1^2(\lambda)\right]\tau^2 + \ldots\Big\}\end{aligned} \qquad [29]$$

$$\begin{aligned}P_{\bar{h}} = 2\pi\varrho a v_0^2(\alpha_0+\alpha)\Big\{1 - \left[K_1(\lambda) + (\alpha_0+\alpha)\left(\frac{1}{\sqrt{2}} - \frac{2\lambda}{\sqrt{2}}e^{-\lambda}E_i(\lambda)\right)\right]\tau \\ - \Big[K_2(\lambda)\left(1 - \frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}\right) - K_1^2(\lambda) - 2(\alpha_0+\alpha)K_1(\lambda)\Big(\frac{1}{\sqrt{2}} \\ - \frac{2\lambda}{\sqrt{2}}e^{-\lambda}E_i(\lambda)\Big)\Big]\tau^2 - \Big[K_3'(\lambda) - 2K_1(\lambda)K_2(\lambda) - K_3''(\lambda) + K_1^3(\lambda) \\ - \frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}K_1(\lambda)K_2(\lambda) - K_3''(\lambda) - 2K_2(\lambda)\left(1 - \frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}\right) \\ - 3K_1^2(\lambda)\left(\frac{1}{\sqrt{2}} - \frac{2\lambda}{\sqrt{2}}e^{-\lambda}E_i(\lambda)\right)(\alpha_0+\alpha)\Big]\tau^3 + \ldots\Big\}\end{aligned} \qquad [30]$$

$$\begin{aligned}\psi_{\pi} = 1 - (\alpha_0+\alpha)\frac{\tau}{\sqrt{2}} - \frac{1}{2}\left(1 - \frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}\right)\tau^2 \\ + \frac{1}{2}(\alpha_0+\alpha)\left(1 - \frac{1}{2}\frac{\alpha_0}{\alpha_0+\alpha}\right)\times \\ \times\tau^3 - \frac{1}{4}\tau^4 + \frac{1}{8\sqrt{2}}\alpha_0\tau^5 + \left(\frac{21}{64} - \frac{49}{128}\frac{\alpha_0}{\alpha_0+\alpha}\right)\tau^6 + \ldots\end{aligned} \qquad [31]$$

The results obtained are in good agreement both with experimental data and with results using the general method of N. E. Kochin.[6,3]

For comparison, the figure shows the results of calculations for $\psi_{Д}$ from Equation [28] (Curve I), experimental data[5] (Curve III), results of the solution from Reference 6 (Curve II), and the solution of M. V. Keldysh and M. A. Lavrentyev[1] (Curve IV).



REFERENCES

1. Keldysh, M. V., and Lavrentyev, M. A., "On the Motion of a Wing Beneath the Surface of a Heavy Fluid," (Russian) Proc. Conference on the Theory of Wave Resistance, Central Aero-Hydrodynamical Institute, Moscow (1937).

2. Tricomi, F., "Integral Equations," (Russian Translation) Izd-vo Inostr. Lit., 1960 (English Edition-Interscience, 1958).

3. Kochin, N. E., "On the Wave Resistance and Lift of Bodies Submerged in a Fluid," (Russian) Proc. Conference on the Theory of Wave Resistance, Central Aero-Hydrodynamical Institute, Moscow (1937). (English Translation - Society of Naval Architects and Marine Engineers, Tech & Res. Bull. No. 1-8, 1951).

4. Pikhteev, G. N., "On the Calculation of Certain Integrals with Regular Cauchy-type Kernels," (Russian) PMM, Vol. 24, No. 6, 1960. (English Translation-Journal of Applied Mathematics and Mechanics, Vol. 24, No. 6, 1960).

5. Chudinov, S. D., "On the Lift of a Submerged Wing of Finite Span," (Russian) Trudy V.N.I.T.O.S.S. (All-Union Scientific Engineering and Technical Society of Shipbuilders), Vol. 6, No. 2 (1955).

6. Panchenkov, A. N., "Approximate Calculation of the Lift on a Submerged Wing Near a Free Surface," (Russian), Journal of Applied Mechanics and Technical Physics (P.M.T.F.), No. 4 (1960).

INITIAL DISTRIBUTION

Copies

8 CHBUSHIPS
  3 Tech Lib (Code 210L)
  1 Appl Res (Code 340)
  1 Prelim Des (Code 420)
  1 Sub (Code 525)
  1 Lab Mgt (Code 320)
  1 Scientific & Res (Code 442)

3 CHBUWEPS
  1 Aero & Hydro Br (Code RAAD-3)
  1 Ship Instal & Des (Code SP-26)
  1 Loads & Dyn Sec (Code RAAD-22)

4 CHONR
  1 Nav Analysis (Code 405)
  1 Math Br (Code 432)
  2 Fluid Dyn (Code 438)

1 ONR, New York

1 ONR, Pasadena

1 ONR, Chicago

1 ONR, Boston

1 ONR, London

1 CDR, USNOL, White Oak

2 DIR, USNRL (Code 5520)
  1 Mr. Faires

1 CDR, USNOTS, China Lake

1 CDR, USNOTS, Pasadena

1 CDR, USNAVMISCEN, Point Mugu

1 DIR, Natl BuStand
  Attn: Dr. Schubauer

10 CDR,DDC, Attn: TIPDR

1 DIR, APL, JHU, Silver Spring

1 DIR, Fluid Mech Lab, Columbia Univ

1 DIR, Fluid Mech Lab, Univ of Calif, Berkeley

5 DIR, DL, SIT

1 DIR, Exptl Nav Tank, Univ of Mich

Copies

1 DIR, Scripps Inst of Oceanography, Univ of Calif

1 DIR, Penn St. Univ, University Park

1 DIR, WHOI

1 Admin, Webb Inst of Nav Arch

1 DIR, Iowa Inst of Hydrau Research

1 DIR, St Anthony Falls Hydrau Lab

3 Head, NAME, MIT, Cambridge
  1 Prof Abkowitz
  1 Prof Kerwin

1 Inst of Mathematical Sciences, NYU, New York

2 Hydronautics, Inc., Pindell School Rd., Howard County, Laurel, Md.

2 Dept of Engin, Nav Architecture, Univ of Calif
  1 Dr. J. Wehausen

1 Dr. Willard J. Pierson, Jr., Col of Engin, NYU, New York

1 Dr. Finn Michelsen, Dept of Nav Arch, Univ of Mich, Ann Arbor

1 Prof. Richard MacCamy, Carnegie Tech, Pittsburgh 13

1 Dr. T. Y. Wu, Hydro Lab, CIT, Pasadena

1 Dr. Hartley Pond, 14 Elliot Dr., New London, Conn.

1 Dr. J. Kotik, TRG, 2 Aerial Way, Syosset, N.Y.

1 Prof. Byrne Perry, Dept of Civil Eng., Stanford Univ, Palo Alto, Calif.

1 Prof. B.V. Korvin-Kroukovsky, East Randolph, Vt

1 Prof. L.N. Howard, Dept of Math, MIT

3 Dept of Aero & Astro, MIT
  1 Prof Ashley
  1 Prof Landahl
  1 Mrs. Widnall

1 DIR, Inst for Fluid Dyn & Appl Math, Univ of Md

1 DIR, Hydrau Lab, Univ of Colorado

1 Pres, Oceanics, Inc., Technical Industrial Park, Plainview, N.Y.

David Taylor Model Basin. Translation 316.
MOTION OF A THIN WING NEAR THE FREE SURFACE OF A LIQUID (Rukh Tonkogo Krila Poblizu Vil'Noyi Poverkhni Ridini), by A.N. Panchenkov. Sep 1963. 13p. refs. (Translated by J.N. Newman from Prikladna Mekhanika, Akademia Nauk Ukrainskoi R.S.R. Institut Mekhanikii, Vol. 8, No. 4, Kiev – 1962, pp. 433–442.)

A new method is described for the approximate solution of integral equations which arise in the research of M.V. Keldysh and M.A. Lavrentyev on the problem of the motion of a thin wing near the free surface of a liquid. The intensity of the distribution of eddies is defined in the form of a series. The values of $\gamma_0(s)$, $\gamma_1(s), \ldots, \gamma_6(s)$ and the formula for the lifting force are given for two special cases—a plane plate and a circular arc profile.

1. Thin wings--Motion--Mathematical analysis
2. Thin wings--Motion--Submerged conditions
I. Panchenkov, A.N.

David Taylor Model Basin. Translation 316.
MOTION OF A THIN WING NEAR THE FREE SURFACE OF A LIQUID (Rukh Tonkogo Krila Poblizu Vil'Noyi Poverkhni Ridini), by A.N. Panchenkov. Sep 1963. 13p. refs. (Translated by J.N. Newman from Prikladna Mekhanika, Akademia Nauk Ukrainskoi R.S.R. Institut Mekhanikii, Vol. 8, No. 4, Kiev – 1962, pp. 433–442.)

A new method is described for the approximate solution of integral equations which arise in the research of M.V. Keldysh and M.A. Lavrentyev on the problem of the motion of a thin wing near the free surface of a liquid. The intensity of the distribution of eddies is defined in the form of a series. The values of $\gamma_0(s)$, $\gamma_1(s), \ldots, \gamma_6(s)$ and the formula for the lifting force are given for two special cases—a plane plate and a circular arc profile.

1. Thin wings--Motion--Mathematical analysis
2. Thin wings--Motion--Submerged conditions
I. Panchenkov, A.N.

David Taylor Model Basin. Translation 316.
MOTION OF A THIN WING NEAR THE FREE SURFACE OF A LIQUID (Rukh Tonkogo Krila Poblizu Vil'Noyi Poverkhni Ridini), by A.N. Panchenkov. Sep 1963. 13p. refs. (Translated by J.N. Newman from Prikladna Mekhanika, Akademia Nauk Ukrainskoi R.S.R. Institut Mekhanikii, Vol. 8, No. 4, Kiev – 1962, pp. 433–442.)

A new method is described for the approximate solution of integral equations which arise in the research of M.V. Keldysh and M.A. Lavrentyev on the problem of the motion of a thin wing near the free surface of a liquid. The intensity of the distribution of eddies is defined in the form of a series. The values of $\gamma_0(s)$, $\gamma_1(s), \ldots, \gamma_6(s)$ and the formula for the lifting force are given for two special cases—a plane plate and a circular arc profile.

1. Thin wings--Motion--Mathematical analysis
2. Thin wings--Motion--Submerged conditions
I. Panchenkov, A.N.

David Taylor Model Basin. Translation 316.
MOTION OF A THIN WING NEAR THE FREE SURFACE OF A LIQUID (Rukh Tonkogo Krila Poblizu Vil'Noyi Poverkhni Ridini), by A.N. Panchenkov. Sep 1963. 13p. refs. (Translated by J.N. Newman from Prikladna Mekhanika, Akademia Nauk Ukrainskoi R.S.R. Institut Mekhanikii, Vol. 8, No. 4, Kiev – 1962, pp. 433–442.)

A new method is described for the approximate solution of integral equations which arise in the research of M.V. Keldysh and M.A. Lavrentyev on the problem of the motion of a thin wing near the free surface of a liquid. The intensity of the distribution of eddies is defined in the form of a series. The values of $\gamma_0(s)$, $\gamma_1(s), \ldots, \gamma_6(s)$ and the formula for the lifting force are given for two special cases—a plane plate and a circular arc profile.

1. Thin wings--Motion--Mathematical analysis
2. Thin wings--Motion--Submerged conditions
I. Panchenkov, A.N.

In the limit of infinite Froude numbers, the results obtained are in good agreement, for any depth of submersion of the wing, with experimental data and with N.E. Kochin's method.

In the limit of infinite Froude numbers, the results obtained are in good agreement, for any depth of submersion of the wing, with experimental data and with N.E. Kochin's method.

In the limit of infinite Froude numbers, the results obtained are in good agreement, for any depth of submersion of the wing, with experimental data and with N.E. Kochin's method.

In the limit of infinite Froude numbers, the results obtained are in good agreement, for any depth of submersion of the wing, with experimental data and with N.E. Kochin's method.