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CYLINDRICAL SHELL WITH A CIRCULAR HOLE

by

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Translated from Ukrainian by O. Lomacky

March 1965

Translation 319

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EFFECT OF A REINFORCING RING ON THE STRESSES IN A
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(Presented by G. N. Savin, Member Academy of Sciences,
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ABSTRACT

The author solves the problem of stresses in a cylindrical shell with a circular hole, the edge of which is reinforced with a thin elastic ring. The complex function of the stresses σ differs from the analogous functions of A. I. Lurye only by the real addends. The coefficients of the function are determined with the aid of boundary conditions of the junction of shell and ring and are expressed by formulas. The effect of the rigidity of the reinforcing ring on the magnitude of the normal stresses at two points is investigated. By reinforcing the edge of the hole in the shell, a considerable decrease may be attained in the stress concentration coefficient.

Let us consider a cylindrical shell subjected to the uniform internal pressure p_0 . The lateral surface of the shell contains a small circular opening, the edge of which is reinforced by a thin elastic ring. If the opening is absent, the state of stress in the shell is given by the stress resultants $S_1 = ph$ and $S_2 = qh$, where $2p = q = \frac{p_0 R}{h}$ and R is the radius of the cylinder. It is known¹ that the solution of the problem consists of the determination of the complex stress function σ which establishes additional stress field in the shell that arises in the vicinity of the opening. This additional stress field is due to the presence of the hole and the reinforcing ring.

Within the accuracy of the square of the small parameter $\beta = \frac{\sqrt[4]{3(1-\nu^2)}}{2\sqrt{Rh}}$ we shall take the imaginary and the real parts of the function σ in the form:

¹References are listed on page 5.

$$Im\sigma = \frac{2A_0}{\pi} \left(\ln \frac{\rho}{\rho_0} + \gamma' \right) - \frac{C_0}{\pi} - \left(C_0 + \frac{2F_1}{\rho^2} \right) \frac{\cos 2\lambda}{\pi} +$$

$$+ \beta^2 \left[\frac{2A_1}{\pi} \left(\ln \frac{\rho}{\rho_0} + \gamma' \right) + \frac{B_1}{2} - \frac{A_0 \rho^2}{4} + \left(\frac{C_0 \rho^2 - A_0 \cdot^2}{4} - \frac{C_1}{\pi} \right) (1 + \cos 2\lambda) - \right.$$

$$\left. - \frac{2F_2}{\pi \rho^2} \cos 2\lambda \right], \quad [1]$$

$$Re\sigma = w = \frac{\beta^2}{\pi} \left\{ \frac{\pi A_1}{2} - 2B_1 \gamma' + D_1 - \frac{F_1}{2} + 2A_2 \rho^2 \ln \frac{\rho}{\rho_0} - 2B_1 \ln \frac{\rho}{\rho_0} + \right.$$

$$\left. + A_0 \left(2\rho^2 \ln \frac{\rho}{\rho_0} + 2\rho^2 \gamma' - \rho^2 \right) + C_0 \rho^2 \left(\frac{1}{4} - \ln \frac{\rho}{\rho_0} - \gamma' \right) + \right. \quad (1)$$

$$\left. + \left[A_0 \rho^2 \left(\ln \frac{\rho}{\rho_0} + \gamma' \right) + C_0 \rho^2 \left(\frac{1}{6} - \ln \frac{\rho}{\rho_0} - \gamma' \right) + D_1 - \frac{2E_2}{\rho^2} - \frac{4H_2}{\rho^2} \right] \cos 2\lambda - \right.$$

$$\left. - \left(\frac{C_0}{12} \rho^2 + \frac{F_1}{2} + \frac{4H_2}{\rho^2} + \frac{24M_3}{\rho^4} \right) \cos 4\lambda \right\},$$

where (ρ, λ) are polar coordinates (pole in the center of the opening, polar axis is directed along the cylinder generator); $\gamma' = \ln \frac{\gamma \rho_0 \beta}{\sqrt{2}}$; $\ln \gamma = 0,577216$; and ρ_0 is the radius of the opening. Our function σ differs from the function of A. I. Lurye only by the additional real term $\frac{2}{\pi} \beta^2 A_2 \rho^2 \ln \frac{\rho}{\rho_0}$. For the determination of 12 unknown coefficients of function σ , we have the boundary conditions of continuity between the shell and the thin ring on the contour $\rho = \rho_0$. In a special case when one of the principal axes of inertia of the ring cross section is located on the middle surface of the shell, the boundary conditions are written as follows:

$$S = -\frac{B}{\rho_0^4} \frac{\partial^3}{\partial \lambda^3} \left(u_\lambda - \frac{\partial u_\rho}{\partial \lambda} \right) + \frac{E_1 F}{\rho_0^2} \left(\frac{\partial u_\lambda}{\partial \lambda} + u_\rho \right),$$

$$S_{\rho\lambda} = -\frac{B}{\rho_0^4} \frac{\partial^2}{\partial \lambda^2} \left(u_\lambda - \frac{\partial u_\rho}{\partial \lambda} \right) - \frac{E_1 F}{\rho_0^2} \frac{\partial}{\partial \lambda} \left(\frac{\partial u_\lambda}{\partial \lambda} + u_\rho \right), \quad [2]$$

$$M_\rho = \frac{C}{\rho_0^2} \frac{\partial}{\partial \lambda} \left(\frac{\partial^2 w}{\partial \rho \partial \lambda} - \frac{1}{\rho_0} \frac{\partial w}{\partial \lambda} \right) - \frac{A}{\rho_0^2} \left(\frac{\partial w}{\partial \rho} + \frac{1}{\rho_0} \frac{\partial^2 w}{\partial \lambda^2} \right),$$

$$Q_\rho^* = Q_\rho^0 + \frac{1}{\rho_0^3} \frac{\partial}{\partial \lambda} \left[C \left(\frac{\partial^2 w}{\partial \rho \partial \lambda} - \frac{1}{\rho_0} \frac{\partial w}{\partial \lambda} \right) + A \frac{\partial}{\partial \lambda} \left(\frac{\partial w}{\partial \rho} + \frac{1}{\rho_0} \frac{\partial^2 w}{\partial \lambda^2} \right) \right],$$

where S_ρ , $S_{\rho\lambda}$, Q_ρ^* , M_ρ are stress resultants and moments in the shell, which can be obtained from function σ by the use of the known formulas;¹

u_ρ , and u_λ are the displacement components in polar coordinates, the corresponding combinations of these displacements are also determined from σ ;¹ A, B, C, and F are the rigidities of the ring in flexure (in two planes), torsion and extension, respectively; and E_1 is the Young's modulus of the ring. The shears $Q_\rho^0 = \frac{1}{2} \rho_0 \rho_0$ are in equilibrium with pressure p_0 that acts on the surface of the opening.

By substituting Equation [1] and the corresponding expressions for u_ρ and u_λ in Equation [2], we obtain four expressions for the function σ at $\rho = \rho_0$. In these expressions we equate the corresponding coefficients in β and β^2 ; namely, by means of equating separately the free members and separately the coefficients attached to $\sin 2\lambda$ and $\cos 2\lambda$, or $\cos 4\lambda$. As a result, we obtain a system of equations which enables us to determine the following:

$$\begin{aligned}
 A_0 &= \frac{\alpha \pi \rho_0^2 (\rho + q)}{4Eh} \cdot \frac{(1 + \nu) \alpha_* - (1 - \nu) \delta_4}{(1 + \nu) \alpha_* + (1 + \nu) \delta_4}, \\
 C_0 &= \frac{\alpha \pi \rho_0^2 (\rho - q)}{2Eh \alpha_0} [\alpha_*^2 + \alpha_* (\delta_4 + 4\delta_3) - 12\delta_3 \delta_4] (1 + \nu), \\
 F_1 &= - \frac{\alpha \pi \rho_0^4 (\rho - q)}{8Eh \alpha_0} [(1 + \nu) \alpha_*^2 + 4\alpha_* (3 + \nu) \delta_3 - \alpha_* (1 - \nu) \delta_4 - 12(1 + \nu) \delta_3 \delta_4], \\
 A_1 &= \frac{\pi \rho_0^2}{2} \left(A_0 - \frac{1}{2} C_0 \right) \frac{\alpha_* (1 + \nu) - (1 - \nu) \delta_4}{(1 + \nu) (\alpha_* + \delta_4)}, \\
 C_1 &= \frac{\pi \rho_0^2}{2\alpha_0} (C_0 - A_0) (\alpha_*^2 + 4\alpha_* \delta_3 + \alpha_* \delta_4 - 12\delta_3 \delta_4) (1 + \nu), \\
 F_2 &= - \frac{\pi \rho_0^4}{8\alpha_0} (C_0 - A_0) [\alpha_*^2 (1 + \nu) + 4\alpha_* (3 + \nu) \delta_3 - \alpha_* (1 - \nu) \delta_4 - \\
 &\quad - 12(1 + \nu) \delta_3 \delta_4],
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 A_2 &= \frac{\alpha \pi \rho_0^2}{2Eh^2} \rho_0 R - \left(A_0 - \frac{1}{2} C_0 \right), \\
 B_1 &= \frac{\rho_0^2}{2(1 - \nu + \delta_1)} \left\{ (1 + \nu - \delta_1) \left[\left(2\gamma' - \frac{1}{2} \right) C_0 + 2(1 - 2\gamma') A_0 \right] - \right. \\
 &\quad \left. - (3 + \nu - \delta_1) \frac{\alpha \pi \rho_0^2 q}{Eh} \right\}, \\
 D_1 &= \frac{\rho_0^2}{2\alpha_3} (\alpha_1 A_0 - \alpha_2 C_0), \quad H_2 = \frac{\rho_0^2}{48\alpha_{10}} (\alpha_6 \rho_0^2 C_0 - \alpha_6 F_1), \\
 E_2 &= - \frac{\rho_0^4}{12\alpha_9} (\alpha_3 A_0 + \alpha_4 C_0) - 2H_2, \quad M_2 = \frac{\rho_0^4}{1440\alpha_{10}} (\alpha_7 \rho_0^2 C_0 + \alpha_8 F_1),
 \end{aligned} \tag{4}$$

where

$$\begin{aligned}
\alpha &= \sqrt{12(1-\nu^2)}, \quad \alpha_* = 12(1-\nu)\rho_0^2/h^2, \\
\alpha_0 &= (1+\nu)\alpha_*^2 + \alpha_* (3+\nu)(4\delta_3 + \delta_4) + 12(3-\nu)\delta_3\delta_4, \\
\alpha_1 &= (1-\nu)[4\gamma'(1-\nu) + (5-\nu)] + \delta_1[(1-\nu)(4\gamma' + 3) + 2(3+\nu)] + \\
&\quad + \delta_2[2(1-\nu)(2\gamma' + 1) + (3+\nu)] - 3\delta_1\delta_2(4\gamma' + 1), \\
\alpha_2 &= (1-\nu)\left[4\gamma'(1-\nu) + \frac{1}{3}(13-\nu)\right] + \delta_1\left[4(1-\nu)\gamma' + \frac{1}{3}(25-\nu)\right] + \\
&\quad + \delta_2\left[4(1-\nu)\gamma' + \frac{1}{3}(13-\nu)\right] - \delta_1\delta_2(12\gamma' + 1), \\
\alpha_3 &= -6\gamma'(1-\nu)^2 - (9-4\nu-3\nu^2) + \delta_1(6\gamma'(\nu-3) + (3\nu-13)) + \\
&\quad + \delta_2[6\gamma'(1+\nu) + (3\nu+5)] + 9\delta_1\delta_2(2\gamma' + 1), \\
\alpha_4 &= 6\gamma'(1-\nu)^2 + 2(\nu^2 - \nu + 4) + \delta_1[6\gamma'(3-\nu) + 2(5-\nu)] - \\
&\quad - \delta_2[6\gamma'(1+\nu) + 2(2+\nu)] - 6\delta_1\delta_2(3\gamma' + 1), \\
\alpha_5 &= (1-\nu)(1-3\nu) - \delta_1(5+3\nu) + \delta_2(1-3\nu) - 45\delta_1\delta_2, \\
\alpha_6 &= 4[3(1-\nu^2) + \delta_1(15+3\nu) + 3\delta_2(3+\nu) + 45\delta_1\delta_2], \\
\alpha_7 &= (19\nu-3-10\nu^2) - \delta_1(9-10\nu) + \delta_2(6+10\nu) + 150\delta_1\delta_2, \\
\alpha_8 &= 6[(6\nu-5\nu^2+3) + (\delta_1+\delta_2)(9+5\nu) + 75\delta_1\delta_2], \\
\alpha_9 &= (1-\nu)(3+\nu) - (5+\nu)(\delta_1+\delta_2) + 3\delta_1\delta_2, \\
\alpha_{10} &= (1-\nu)(3+\nu) + (9+\nu)(\delta_1+\delta_2) + 15\delta_1\delta_2, \\
\delta_1 &= \frac{A}{\rho_0 D}, \quad \delta_2 = \frac{C}{\rho_0 D}, \quad \delta_3 = \frac{B}{\rho_0 D}, \quad \delta_4 = \frac{E_1 F \rho_0}{D},
\end{aligned} \tag{4}$$

r = Poisson's ratio; D = flexural rigidity of the cylinder; h = shell thickness.

If in Equations [3] and [4] we let $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$, we obtain the solution of the case considered by A. I. Lurye;¹ that is, when the reinforcing ring is absent. (In the solution¹ there was an inaccuracy in the equation (4.9.22) and formula (4.9.15).) If $\delta_1 = \delta_2 = \delta_3 = \delta_4 = \infty$, we have a limiting case of an absolutely rigid ring or disk.

We have calculated the complete normal stresses σ_ρ and σ_λ at the points $P(\rho_0, 0)$ and $N(\rho_0, \frac{\pi}{2})$ for the case of the rectangular ring with the sides b, h_1 as a function of the ratio of the Young's moduli of the ring and the shell $\frac{E_1}{E} = \frac{\delta}{1-\nu^2}$ and when $b = 0,1\rho_0, \nu = 0,3, \rho_0 = 15h, h_1 = 3h, \rho_0 = 0,5\sqrt{Rh}$.

The curves of the most significant quantities $\frac{1}{q} \sigma_\lambda^P$ (curve 1), $\frac{1}{q} \sigma_\rho^N$ (curve 2) and $\frac{1}{q} \sigma_\rho^P$ (curve 3) for the exterior surface of the shell are given in Figure 1. We have used $\kappa = \frac{\delta}{\delta + 20}$. The curves show that for $\kappa = 0.14$, that is, when $\delta = 3.25$, then $\sigma_\rho^P = \sigma_\lambda^P \approx 1.45$; and the stress concentration factor² at this point is decreased in the order of $\frac{3.31}{1.45} = 2.28$ as a consequence of reinforcing the edge of the opening by elastic ring.

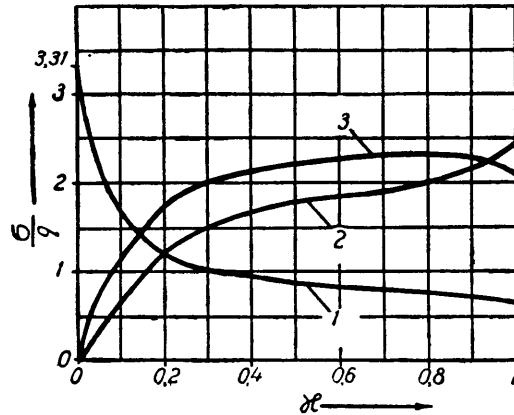


Figure 1

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*Translator's Note: See also Lurye, A. I., "Concentration of Stresses in the Vicinity of an Aperture in the Surface of a Circular Cylinder," IMM-NYU280, Translated from Prik. Mat. Mech. 10, 397-406 (1946).

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