UNITED STATES EXPERIMENTAL MODEL BASIN

NAVY YARD, WASHINGTON, D.C.

THE CALCULATION AND MEASUREMENT OF ELASTIC NATURAL FREQUENCIES OF SHIP HULLS

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EXPERIMENTAL MODEL BASIN
ERECTED 1898
BUREAU OF
CONSTRUCTION AND REPAIR
NAVY DEPARTMENT

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See Report 547

JUNE 1934

REPORT NO. 382
TRANS. NO. 7

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Schiffbautechnische Gesellschaft
Vol. 33, 1932

Translated by R. T. McGoldrick and M. C. Roemer

U.S. Experimental Model Basin
Navy Yard, Washington, D.C.
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I. Previous attempts at a solution of vibration problems of ship hulls.

Knowledge of the natural frequency of ship hulls in advance of the construction is at the present time of increasing practical importance. Particularly in motor ships and in ships of special construction, cases have increased in which, as a result of resonance between periodic forces and the natural frequency of the hull, annoying and dangerous vibrations have been observed. In most cases it is a question of vertical vibrations, especially of the fundamental type, which in themselves also cause the most violent shocks. Because of the predominating importance of vertical vibration this type will be dealt with exclusively in this paper.

The researches of O. Schlick (1)* have furnished us with basic information concerning the nature and significance of the elastic natural frequencies of ship forms. The well-known Schlick formulas give even today useful values for the types of ships investigated by him under loaded conditions. In the case of unusual ship types and of ships under ballast the empirical formulas of Schlick are no longer applicable.

Therefore, it has been attempted to solve the problem theoretically. As far as the calculation of the natural frequencies of ship forms based on the free-free rod with non-uniform distribution of mass and moment of inertia in undamped vibration is concerned this part of the problem can be considered theoretically solved. We might mention in this connection the exact analytical method of A. Kriloff (2) and C. E. Inglis (3) as well as the exact graphical method of L. Guembel (4). The methods of Kriloff and Inglis presuppose an extensive mathematical knowledge which the practical operating engineer usually does not have at his disposal, whereas Gumbel's method is quite tedious. Therefore numerous methods have been proposed for shortening the computations by such men as Akimoff (5) Pawlenko (6) Cole (7) Tobin (8) and Nicholls (9). The analytical methods of Akimoff and Pawlenko are likewise tedious and in part depend upon the assumption of a parabolic form for the mass and moment of inertia curves especially in cases where the Schlick formulas are inadequate and not applicable. The graphical methods of Cole, Tobin, and Nicholls give questionable results since the end conditions for the free-free bar, namely resulting inertia force and resulting mass moment equal zero, are not simultaneously satisfied.

The graphical methods of J. L. Taylor (10) and F. Horn (11) are based on the familiar method of Stodola (12) for the determination of the flexural critical speeds of rotation for shafts with two bearings. Whereas in the case of a shaft with two bearings the nodal points are known at the outset, in the case of a ship they must first be determined by fulfilling the end conditions. F. Horn satisfies the end conditions in the assumption that the neutral axis of vibration must be the

* Numbers refer to bibliography on page 48

principal axis of inertia of the vibrating system and makes its previous determination prerequisite. It is precisely in this, however, that a practical difficulty in the rapid execution of the method is evident. Although J. L. Taylor gives several directions regarding methods for meeting the end conditions nevertheless the application of the formula offered by him for fulfilling the second end condition might not lead to the desired result.

Of the other common methods for the determination of flexural critical speeds in machine construction such as those of O. Föppl (13), S. Dunkerly (14), A. Morley (15) and G. Kull (16) hitherto no application to ship hulls has been made with the exception of Morley's method. The methods of O. Föppl and S. Dunkerly are scarcely to be considered here since they give incorrect values for beams with considerably overhanging ends which are under heavy loads. W. Dahlman (17) has attempted to apply to ship hulls Morley's method which by the way is identical with that of O. Kull. Since Dahlman's solution is based on various false assumptions (load equals difference between weight and buoyancy; points of support of the ship form at the ends) it is superfluous to go into it in greater detail. Fundamentally Morley's method is probably less accurate than Stodola's when we are dealing with projecting ends in some cases under heavier loads.

In spite of the works already mentioned, it has not yet been possible to predetermine the natural frequencies of any desired type of ship hulls as they occur in practice, with an accuracy essentially greater than that of the well known Schlick formula. As reasons for this we must consider the imperfect knowledge of the elastic behavior which manifests itself in actual flexure, and the ignorance of the damping resistance to which the ship is subject during vibration in water.

As far as the elastic behavior of ship forms in vibration has been considered the calculation has been confined to the determination of the shear deflection under the assumption that only the web takes the shearing stresses (Tobin, Nicholls, Taylor) unless it was considered preferable to introduce a modulus of elasticity precisely suited to the individual case in order to bring into agreement calculation and tests.

Thus far little has been actually known about the magnitude of the damping resistance. Whereas L. Gumbel regards this effect as of only secondary importance, H. W. Nicholls has proved by model experiment with rectangular and triangular beams that the natural frequency decreases in water by about 10 to 20 per cent. Because of the limited scope of these experiments and their restriction to straight-sided forms no conclusion as to the magnitude of the damping resistance of ship hulls can be drawn. To the mathematical methods of F. M. Lewis (18) and J. L. Taylor (19) is to be attributed no more than theoretical significance.

In view of this state of our knowledge the chief task is to provide a basis for clearing up mathematically the hitherto observed disagreement between calculation and experiment. We attempt to achieve this end by application of the knowledge of the elastic theory of thin-walled box girders to vibration processes and by experimental determination of the laws governing damping resistance. Besides this, there is the problem of setting up a method for computing the natural frequency of undamped vibration of non-uniform bars which can be carried out so quickly and simply under rigid fulfillment of the end conditions that its practical application offers no difficulty. Since the calculation of the theoretical frequency forms the foundation, we take this for the starting point of our investigation.

II. Calculation of the natural frequency of undamped vibrating rods.

1. Influence of mass and moment of inertia distribution in a free-free vibrating rod.

The fundamental differential equation for the motion of translation of an undamped vibrating rod:

$$\frac{\partial^2}{\partial x^2}\left[EJ(x)\frac{\partial^2 y}{\partial x^2}\right] + m(x)\frac{\partial^2 y}{\partial t^2} = 0$$

yields by the substitution $y = y_1 \cdot \sin \omega t$ where y_1 is a function of x only, the total differential equation:

$$\frac{d^2}{dx^2}\left[EJ(x)\frac{d^2 y_1}{dx^2}\right] - m(x)\,\omega^2 y_1 = 0 \quad \ldots\ldots\ldots \quad 1)$$

Where:

- E = modulus of elasticity of the material of the rod.
- $J(x)$ = distribution of moments of inertia as a function of the length.
- y = amplitude of vibration at any point on the rod at a distance x from the origin of coordinates.
- $m(x)$ = distribution of mass as a function of the length.
- t = time.
- ω = circular frequency.

Since the ship represents a free-free rod the solutions of this differential equation must satisfy the following end conditions:

$$\left.\begin{aligned} E\int_0^L \frac{d^2}{dx^2}\left[J(x)\frac{d^2 y_1}{dx^2}\right]dx &= 0 \\ E\iint_0^L \frac{d^2}{dx^2}\left[J(x)\frac{d^2 y_1}{dx^2}\right]dx\,dx &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad 2)$$

Equation (1) becomes by a fourfold integration

$$y_1 = \frac{\omega^2}{E}\iint \frac{\iint m(x)\,y_1\,dx\,dx}{J(x)}\,dx\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3)$$

Equation (3) is satisfied when the assumed amplitudes y_1 of the vibration curve under the integrals are identical with the calculated amplitudes y_1 on the left side of the equation and ω is properly chosen at the start. Assuming this agreement for the time being the end conditions in equation (2) become:

$$\left.\begin{aligned} E\int_0^L \frac{d^2}{dx^2}\left[J(x)\frac{d^2y_1}{dx^2}\right]dx &= \int_0^L m(x)\,\omega^2 y_1\,dx = 0 \\ E\iint_0^L \frac{d^2}{dx^2}\left[J(x)\frac{d^2y_1}{dx^2}\right]dx\,dx &= \iint_0^L m(x)\,\omega^2 y_1\,dx\,dx = 0 \end{aligned}\right\} \ldots\ldots 4)$$

For the general solution of the vibration calculation we make use of the method of Stodola with the change suggested by J. L. Taylor that the integrations be carried out as in the usual longitudinal strength calculations of ships.

The satisfying of the end conditions is of the greatest influence on the form of the amplitude curve to be assumed. In what follows we will presuppose that the assumed frequency $\omega_1^2 \equiv g = 9.81\ sec^{-1}$.

If we further set $m(x) = \frac{\Delta G}{\Delta L \cdot g}$ the first end condition in equation (4) reads

$$\int_0^L \frac{\Delta G}{\Delta L}\, y_1\, dx = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4a)$$

As a first approximation for the amplitude curve y_1 we may take that of a uniform bar. Equation (4a) means that there can remain no resultant inertia force, that is, that the center of gravity of all accelerated parts must always remain at rest. From this it follows that the axis of vibration must be the gravity axis. We must, therefore, first find the center of gravity of the vibrating system and shift the axis of vibration to it. The magnitude of the shifting is given by:

$$y_s = \frac{\int \frac{\Delta G}{\Delta L}\, y_1\, dx}{\int \frac{\Delta G}{\Delta L}\, dx} = \frac{\text{Residual Dynamic Shear Force}}{\text{Total Weight}}$$

In order to permit a comparison of the change in form of the amplitude curve the ordinates of the first assumed curve (but taken from the shifted base) are divided by the forward end ordinates $y_{1\,V.P.} + y_s$ whereby the amplitude curve satisfying the first end condition is obtained.

The second end condition:

$$\iint_0^L m(x)\,\omega^2 y_1\,dx\,dx = \iint_0^L \frac{\Delta G}{\Delta L}\, y_1\,dx\,dx = 0 \ldots\ldots\ldots 4b)$$

means that the mass moment at the end of the rod must vanish. Since according to the first end condition the axis must pass through the center of gravity, the only possible way to satisfy the second end condition is to rotate the base. The magnitude of this rotation is to be calculated.

According to a familiar law of mechanics the axis of vibration must coincide with the principal axis of inertia. This as is known is determined by the fact that it passes through the center of gravity of the vibrating system and that the centrifugal moments of the masses referred to this axis must vanish. If it is assumed that the transfer of the base has already been made, the centrifugal moment C_{xy} is given by reference to Fig. 1:

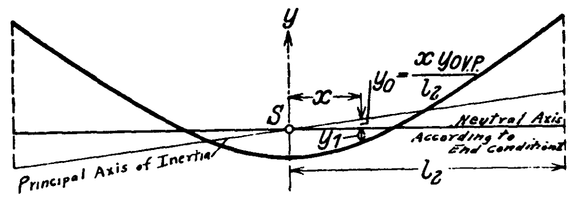


Fig. 1- Diagram of Computation

$$C_{xy} = \Sigma m x y = \int_{-\frac{1}{2}L}^{+\frac{1}{2}L} \frac{\Delta G}{\Delta L}\, x y \, dx = \int_{-\frac{1}{2}L}^{+\frac{1}{2}L} \frac{\Delta G}{\Delta L}\, x\left(y_1 - \frac{x\, y_{0\,V.P.}}{l_2}\right) dx = 0$$

From which it follows that

$$\int_{-\frac{1}{2}L}^{+\frac{1}{2}L} \frac{\Delta G}{\Delta L}\, y_1 x \, dx = \frac{y_{0\,V.P}}{l_2} \int_{-\frac{1}{2}L}^{+\frac{1}{2}L} \frac{\Delta G}{\Delta L}\, x^2 \, dx$$

Since now

$$\int_{-\frac{1}{2}L}^{+\frac{1}{2}L} \frac{\Delta G}{\Delta L}\, y_1 x \, dx = \iint_{-\frac{1}{2}L}^{+\frac{1}{2}L} \frac{\Delta G}{\Delta L}\, y_1 \, dx \, dx = RM_{V.P.} = \text{Residual Mass Moment}$$

$$\text{and} \int_{-\frac{1}{2}L}^{+\frac{1}{2}L} \frac{\Delta G}{\Delta L}\, x^2 \, dx = J_G = \text{Moment of Inertia of Weight Curve}$$

the rotation of the base at the forward perpendicular (V.P.) is found to be·

$$y_{0\,V.P.} = \frac{l_2\, RM_{V.P}}{J_G} \quad \dots\dots\dots\dots\dots\dots \quad 6)$$

In order now to be able to make a comparison of the change in form of the amplitude curve according to end condition II, the amplitudes already corrected by end condition I are measured from the rotated base and divided by the end ordinate at the forward perpendicular.

The preceding method requires the construction of the amplitude curve twice, fulfilling both end conditions. There remains the problem of setting up a formula by which it is possible to obtain the amplitude curve in a single step without intermediate calculations.

The residual moment contained in equation (6) can be written

$$RM_{V.P.} = \iint_0^L \frac{\Delta G}{\Delta L} \left(\frac{y_1 - y_s}{y_{1V.P.} - y_s} \right) dx\,dx$$

Here y_1 gives the ordinate of the free-free uniform bar. Equation (6) then becomes:

$$y_{0\,V.P.} = \frac{1_2}{J_G (y_{1V.P.} - y_s)} \left[\iint_0^L \frac{\Delta G}{\Delta L} y_1\, dx\,dx - y_s \iint_0^L \frac{\Delta G}{\Delta L} dx\,dx \right]$$

The second double integral of the latter formula is the static moment of the weight curve referred to V.P. which can be written

$$\iint_0^L \frac{\Delta G}{\Delta L} dx\,dx = G l_2$$

(G = total weight) and with the use of equation (5) we obtain

$$y_{0\,V.P.} = \frac{1_2}{J_G (y_{1\,V.P.} - y_s)} \left[\iint_0^L \frac{\Delta G}{\Delta L} y_1\, dx\,dx - l_2 \int_0^L \frac{\Delta G}{\Delta L} y_1\, dx \right]$$

For simplicity we designate:

$RM_1^{V.P.}$ = Residual moment of the inertia forces referred to the forward perpendicular which are obtained by multiplication of the ordinates of the weight curve by the amplitudes of the vibration curve of the uniform bar.

$RS_1^{V.P.}$ = Residual shear force of the same inertia forces.

The desired rotation can then be written:

$$y_{0\,V.P.} = \frac{1_2}{J_G (y_{1\,V.P.} - y_s)} \left[RM_1^{V.P.} - l_2 RS_1^{V.P.} \right] \quad \ldots\ldots\ldots \quad 7)$$

The rule of signs to be observed is:

1. The area below the base is negative.
2. The shifting of the base y_s is given the negative sign when it is downward.
3. The residual shear force is to be made negative when the lower parts of the dynamic load curve $\frac{\Delta G}{\Delta L} y_1$ are greater than the upper.
4. The residual moment will be negative when the dynamic mass moment curve ends below the base assuming that its ordinates at the aft perpendicular take an upward course.

5. The rotation of the base must be made upward (counter clockwise) at the forward perpendicular when the residual moment is negative.

After making both base corrections the vibration calculation can be carried out as already indicated. This gives for the final integration curve the computed deflection line multiplied by the modulus E:

$$Ey_2 = \int_0^L\int \frac{\int_0^L\int \frac{\Delta G}{\Delta L} y_1\, dx\, dx}{J(x)}\, dx\, dx \qquad \text{8)}$$

In order to be able to compare the form of the computed bending curve y_2 with the assumed values y_1, the curves are superimposed and the line joining the end points of the assumed y_1 curve is chosen as a basis of comparison. If the peak value of the Ey_2 curve $= x\ t/m$, and that of the y_1 curve $= 1cm$ in the graph the scale of superposition is found to be:

$$1cm = y_2/z = x/E\ z\ m.$$

The ordinate of the superimposed y_2 curve is measured from the base at the forward perpendicular and we will let it be $z'cm$ on the graph; then the comparative deflection value at the forward perpendicular will be found to be $y_{2\,V.P.} = z'x/Ez$ in meters.

Since the vibration frequencies are inversely proportional to the square root of the deflections, we get:

$$\omega_2 = \sqrt{\frac{y_{1\,V.P.}\,\omega_1}{y_{2\,V.P.}}} = \sqrt{\frac{1 \cdot 9.81}{y_{2\,V.P.}}}$$

The theoretical frequency per minute is then:

$$N = \frac{30}{\pi}\sqrt{\frac{9.81}{y_{2\,V.P.}}}$$

The application of the method described makes it possible to carry out a vibration calculation for general cases in a short time and in a simple and practical way. The use of the integraph is especially recommended for carrying out the graphic integration by means of which it is possible, as soon as the weight and moment of inertia curves are known, to complete the vibration calculation in a few hours.

2. Influence of the position of the nodes investigated in the case of uniform bars supported at two points and with overhanging ends.

The changes in form of the amplitude curve are especially characterized by the shifting of the nodal points. The effect of this shifting will be examined in the case of a uniform beam with two supports and overhanging ends of any desired length.

We will start with the fundamental differential equation for a uniform bar:

$$EJ\frac{d^4z}{dx^4} = m\omega^2 z \quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 9a)$$

with the solution:

$$z = A\sin\alpha x + B\cos\alpha x + C\sinh\alpha x + D\cosh\alpha x \quad\ldots\ldots\ldots \quad 9b)$$

Substituting z from equation (9a) into equation (9b) gives:

$$EJ\alpha^4 = m\omega^2$$

$$\alpha = \sqrt[4]{\frac{m\omega^2}{EJ}}\,; \quad \omega = \alpha^2\sqrt{\frac{EJ}{m}}$$

By setting up the differential equation (9b) for each section of the beam

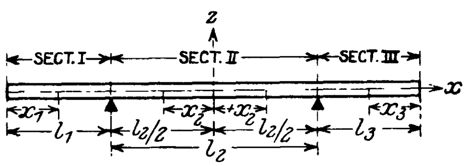


Fig. 2 – Diagram for Computation

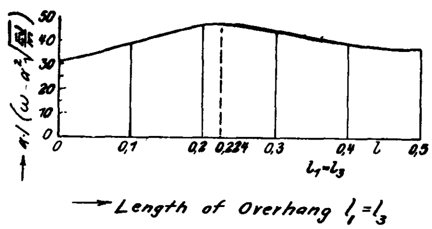


Fig. 3 – Effect of the Distance between supports on th natural frequency of uniform beams with two supports and equal overhanging ends

and determining the constants from the end conditions and after several intermediate computations, we obtain the periodic equation in simplified form for rods with equal overhanging ends. This reads:

$$2(1+\cos\alpha l_1\cdot\cosh\alpha l_1)+(\sinh\alpha l_1\cdot\cos\alpha l_1-\sin\alpha l_1\cdot\cosh\alpha l_1)\left(\tan\alpha\tfrac{l_2}{2}+\tanh\alpha\tfrac{l_2}{2}\right)=0$$

The analysis of this equation gives the values for αl shown in Fig. 3 as a function of the position of the points of support.

Fig. 3 shows that the natural frequency has a maximum value when the distance of the points of support from the ends is $l_1 = .224\,l$; αl in this case equals 4.730.

From the comparison with the corresponding $l\alpha$ values for the free-free bar it follows that the free-free bar has a maximum frequency above all others, which agrees with the general principal that: "The natural periods of a system fulfill the maximum or minimum conditions and the greatest of the natural periods exceeds any that can be obtained by a variation of type." (Rayleigh, Theory of

Sound, Vol. I, p. 287).

If the points of support of the bar with overhanging ends are considered as the nodes we come to the conclusion that every approximate method that fails to consider or considers only in part the end conditions always gives too low values of natural frequency, since the nodal points do not correspond with their position in a free-free bar. Since on the other hand the elastic behavior of a ship hull considered as a box girder as well as the influence of the vibrating water always lowers the natural frequency, the peculiar fact is found in the method of T. C. Tobin for example that in certain cases it yields results approximately in agreement with actual conditions although neither the end conditions are fulfilled nor the influence of the water vibrating with the ship is taken into account. The validity of the above derivation has already been established by Mallock (20) by purely experimental means.

3. Examples of calculations and experimental check of results.

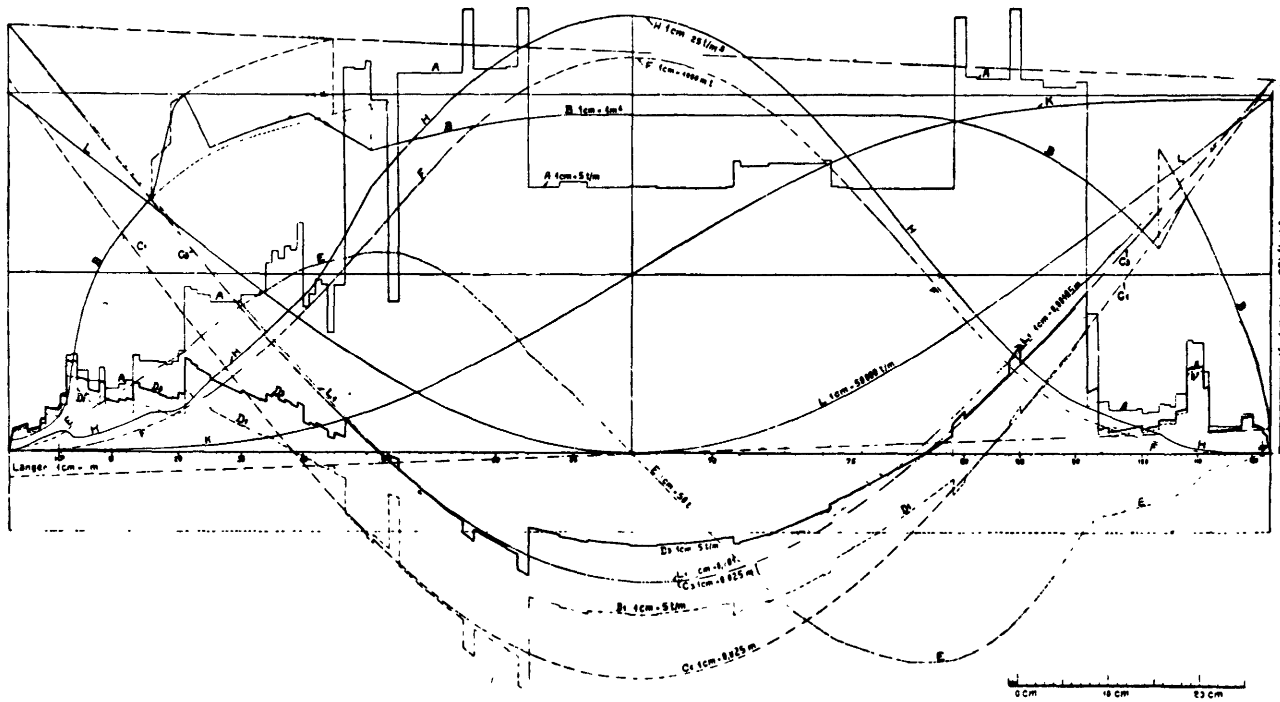
In the following are given the results of various vibration calculations which have served as a theoretical basis for comparison between tests on models and measurements on board ship. The calculations apply to the following special ships:

(a) Motor tanker of about 11,000 tons light displacement with cylindrical tanks for transporting lubricating oil. Type – Isherwood system with gusset plates

Ship Data

Overall length	140.65 m
Length at water line	136.53 m
Length between perpendiculars	134.11 m
Moulded breadth	19.51 m
Moulded depth from main deck	10.744 m
Loaded draft	8.230 m
Displacement loaded in sea water (about)	16,900 tons
Coefficient of fineness δ	0.762
Moment of inertia of midship section	
including longitudinals	36.159 m^4
without longitudinals	29.965 m^4
Distance of the center of gravity	
from the after perpendicular	72.76 m

The calculation for the fully loaded condition (fig. 4) follows. First an amplitude curve is assumed having the ordinates of a uniform bar, the amplitude at the forward perpendicular $y_{v.P.}$ being set at 1 m. (curve C) The ordinates of the weight curve (A) are multiplied by the assumed amplitude taking $\omega_1^2 = 9.81/sec^{-2}$



- A — WEIGHT CURVE (1 CM. = 5.0 T/M
- B — MOMENT OF INERTIA CURVE (1 CM. = 1.0 M^4)
- C_1 — CURVE OF VALUES OF Y_1 WITHOUT FULFILLMENT OF END CONDITIONS (1 CM. = 0.025 M.)
- C_3 — CURVE OF VALUES OF Y_1 WITH FULFILLMENT OF END CONDITIONS (1 CM. = 0.025 M. ASSUMED DEFLECTION)
- D_1 — CURVE OF INERTIA FORCES WITHOUT FULFILLMENT OF END CONDITIONS (1 CM. = 5 T/M)
- D_3 — CURVE OF INERTIA FORCES WITH FULFILLMENT OF END CONDITIONS (1 CM. = 5 T/M)
- E — CURVE OF MASS SHEARING FORCE (1 CM. = 50 T.)
- F — CURVE OF MASS BENDING MOMENT (1 CM. = 1000 M.T.)
- H — CURVE OF M/J VALUES CM. · 25 T/M^3)
- K — CURVE OF INTEGRAL OF M/J · DX VALUES (1 CM. - 2000 T/M^2)
- L — CURVE OF COMPUTED 1/2 · E VALUES (1 CM = 50,000 T/M)
- L_1 — CURVE OF COMPUTED DEFLECTION SUPERIMPOSED (1 CM. = 0.00165 M, CALCULATED DEFLECTION)

FIG. 4 VIBRATION CALCULATION FOR A TANKER WITH CAPACITY OF 11,000 T.

which gives the dynamic load curve D . By integration we obtain the residual dynamic shear force at the forward perpendicular $RS_{V.P.}$ = −3586.20 t and by integration of the dynamic shear force a residual dynamic moment $RM^{V.P.}$ = −223,900 tm. With these values the shifting of the base is found directly to be: $y_s = RS^{V.P.}/G$ = −0.212 m and with J_G = 17,044,500 tm² and l_2 = 67.89 m. the rotation of the base according to equation (7) is $y_{o\,V.P.}$ = 0.0652 m. After dividing the amplitudes measured from the shifted and rotated base by the ordinate at V.P. the vibration curve (C_3) is obtained the end conditions being fulfilled. The further computation is carried out in the familiar manner by four-fold integration. For the moment of inertia the computed value including the longitudinals was inserted. Finally we get for the end ordinate of the superimposed curve (L_1) y_2 = 0.06445 m. From this we obtain the theoretical number of vibrations per minute:

$$N = \frac{30}{\pi}\sqrt{\frac{9.81}{0.06445}} = 117.9 \text{ min}^{-1}.$$

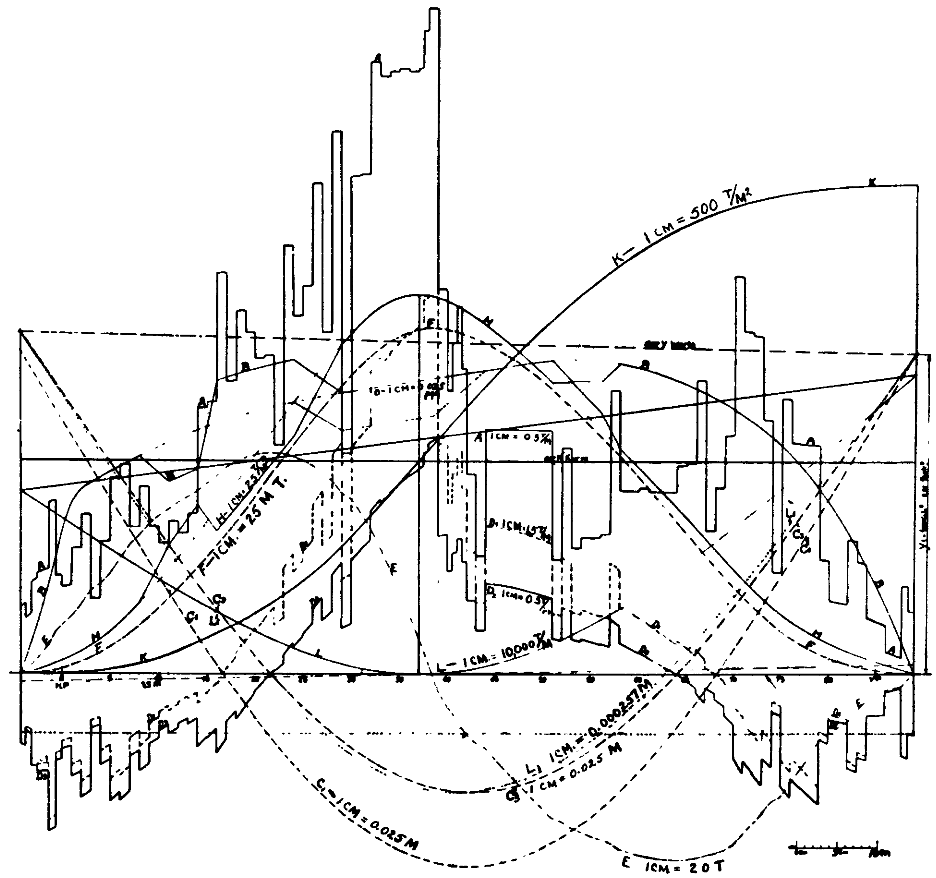
(b) Suction dredge of 1000 m³ capacity with diesel electric drive.

Ship Data

Length over all	85.78 m
Length between perpendiculars	82.70 m
Moulded breadth	15.20 m
Moulded depth from the main deck	6.00 m
Draft loaded with 1900 t dredging equipment	4.15 m
Displacement loaded	4,250 t
Displacement without dredging equipment	2,350 t
Moment of inertia of midship section	3.64 m⁴
Distance of center of gravity from after perpendicular loaded	40.87 m
Distance of center of gravity from after perpendicular unloaded	38.30 m

Since the dredge seems especially adapted as an example of the effect of weight distribution on the natural frequency because of the concentrated heavy load in a relatively short length of ship (see Fig. 9), the computation was carried out for both the loaded and unloaded conditions. Simultaneously it was intended to investigate whether under extreme loading conditions a repetition of the vibration calculation is necessary. For the fully loaded condition a theoretical frequency of 138.20 min.$^{-1}$ was obtained by a single computation.

Since the comparison of the assumed and computed amplitude curves shows a certain discrepancy in the after part of the ship, the whole computation was



A WEIGHT CURVE (1 CM. = 0.5 T/M)
B MOMENT OF INERTIA CURVE (1 CM. = 0.025 M^4)
C_1 CURVE OF VALUES OF Y_1 WITHOUT FULFILLMENT OF END CONDITIONS (1 CM. = 0.025 M.)
C_3 CURVE OF VALUES OF Y_1 WITH FULFILLMENT OF END CONDITIONS (1 CM. = 0.025 M., ASSUMED DEFLECTION)
D_1 CURVE OF INERTIA FORCES WITHOUT FULFILLMENT OF END CONDITIONS (1 CM. = 1.5 T/M)
D_2 CURVE OF INERTIA FORCES WITH FULFILLMENT OF END CONDITIONS (1 CM. = 0.5 T/M)
E CURVE OF MASS SHEARING FORCE (1 CM. = 2.0 T)
F CURVE OF MASS BENDING MOMENT (1 CM. = 25 MT)
H CURVE OF M/J VALUES (1 CM = 25 T/M^3)
K CURVE OF INTEGRAL OF M/J · DX VALUES (1 CM. = 500 T/M^2)
L CURVE OF COMPUTED 1/2·E VALUES (1 CM. = 10,000 T/M)
L_1 CURVE OF COMPUTED DEFLECTIONS SUPERIMPOSED (1 CM. = 0.000257 M CALCULATED DEFLECTION)

LENGTH BETWEEN PERPENDICULARS = 50.15 METERS
MOULDED BREADTH = 9.50 METERS
DEPTH = 4.15 METERS

FIG. 5 VIBRATION CALCULATION FOR A CABLE-LAYER repeated by starting from the vibration curve obtained in computation I. There was obtained with good agreement between assumed and computed amplitude curves a frequency of 138 $min.^{-1}$. Therefore the frequency showed practically no alteration after the first calculation.

The vibration calculation for the unloaded condition gives N_{th} = 164.3 $min.^{-1}$ If we now wish to represent both conditions of loading by Schlick's constant, we get for the loaded condition C = 3,750,000 and the frequency for the unloaded condition would be 186 $min.^{-1}$ in contrast with the exactly computed value 164.3 $min.^{-1}$ It is evident therefore how dependent these constants are upon the load distribution disregarding for the moment the elastic behavior and the virtual mass. In certain cases, therefore, Schlick's formulas are not applicable. Even with ships of normal weight distribution they can only give useful values for the loaded condition.

(c) Cable layer of 481 Br.-Reg. tons with Diesel-electric drive.

Ship Data

Length over all	55.05 m
Length between perpendiculars	50.15 m
Moulded breadth	9.50 m
Moulded depth from main deck	4.15 m
Mean draft without keel	2.88 m
Loaded displacement in sea water	846.50 t
Coefficient of fineness	.59
Moment of inertia of midship section . . .	.936 m^4
Distance of center of gravity from after perpendicular	23.41 m

With E = 2.1 x 10^6 kg/cm² and the moment of inertia calculated with no deductions we get after carrying out the base corrections (Fig. 5), N_{th} = 299 $min.^{-1}$

The results of the computations have been checked by model experiments. The experimental arrangement is shown in Fig. 6. The models were set on knife-edges at the mathematically computed nodes, and set in vibration by electromagnetic impulses on the principle of the self-interrupter and these vibrations transmitted through a system of rods were measured by means of a Geiger vibrograph.

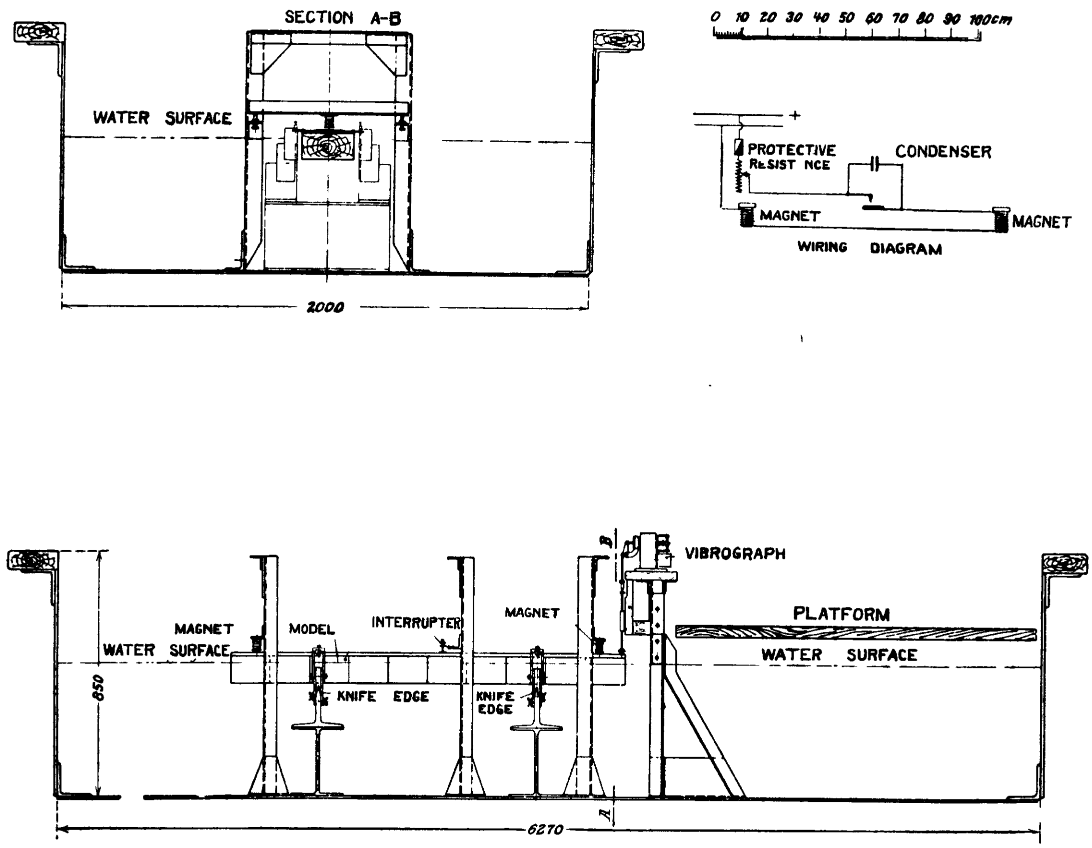


FIG. 6 EXPERIMENTAL SET-UP

First an experiment was undertaken to test the experimental reliability. The model hull consisted of a machined piece of iron bar measuring 1000 x 75.1 x 13.1 mm. The weight including the 80 gram interrupter disc amounted to 7.800 kg. The position of the supports was varied from 0.1 l to 0.35 l from the ends and the accompanying natural frequencies measured. The corresponding theoretical natural frequencies are found by means of the formula:

$$n = \frac{\alpha^2}{2\pi}\sqrt{EJ/m}$$

where the value of αl can be determined from Fig. 3. E was assumed to be 2.1 x 10^6 kg/cm². Figure 7 gives the computed and measured natural frequencies. We perceive the rapid falling off of the natural frequency beyond the position of the nodes for a free–free bar (0.224 l) and in general a very good agreement between theory and experiment.

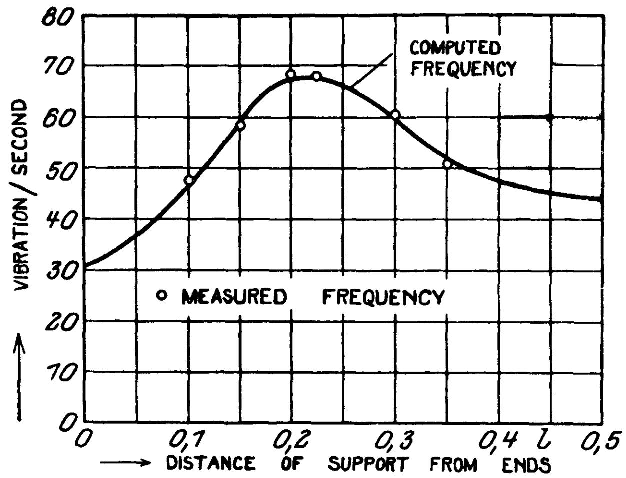


FIG. 7 COMPARISON OF MEASURED AND CALCULATED FREQUENCIES FOR A UNIFORM BAR SUPPORTED AT TWO POINTS, AS A FUNCTION OF DISTANCE OF NODES FROM THE ENDS.

No additional determination of the modulus E by tensile test was carried out since with E = 2.1 x 10^6 kg/cm² good agreement was found between theory and experiment in all cases.

The value of E = 2.1 x 10^6 kg/cm² was therefore retained as a basis for all experiments.

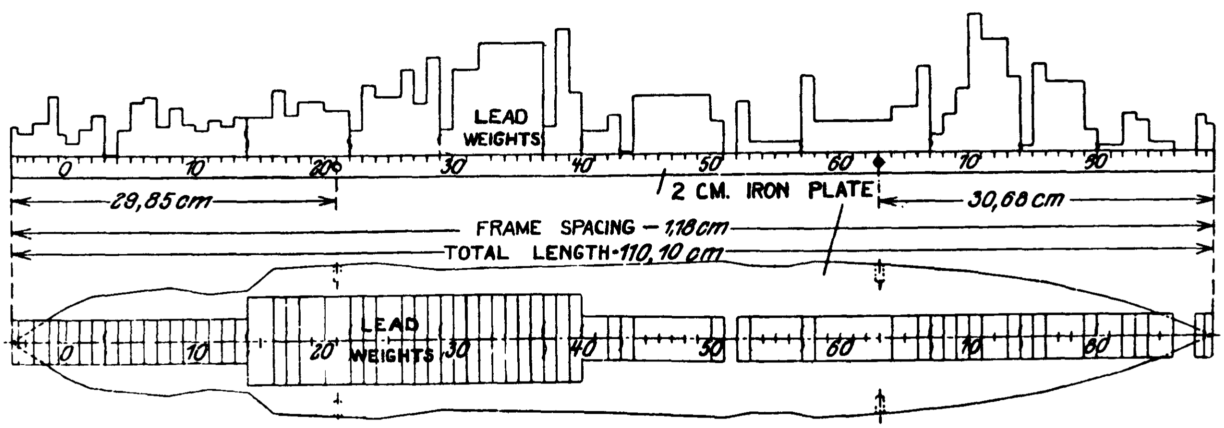


FIG. 8 VIBRATION MODEL FOR A CABLE LAYER

In order to test the accuracy of the method of computing non-uniform bars, the vibration models of the cable layer and the suction dredge were set up with steel plates and lead weights in the manner employed by C. Henderson (21). The method of designing the models and the computation of the natural frequencies to be expected are given in the appendix. In table I the model data are tabulated. (See Figs. 8 and 9.)

In carrying out the experiment the knife-edge bearings were placed at the computed nodes. The measured frequencies for the cable layer model fluctuated between 70 and 71.5 sec^{-1}. The average value was 70.6 vibrations per second. Comparison with the computed value of 71.1 shows very good agreement. For the dredge model the measured frequencies with loading amounted to 37.7 sec^{-1} and without loading 44.7 sec^{-1}. Compared with the computed values of 38.1 and 45.1 the difference resulting in these cases also is very small.

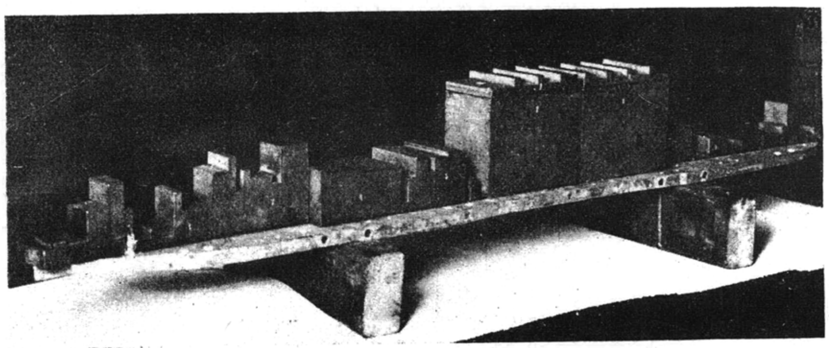


FIG. 9 VIBRATION MODEL FOR A SUCTION DREDGE

Table I. Model Dimensions

	Cable Layer	Suction Dredge	
		loaded	light
Length overall in cm.	110.10	114.37	114.37
Greatest Moment of Inertia cm^4	9.48	3.00	3.00
Weight in kg.	52.00	66.00	37.00
Form constant $C = N/\sqrt{J/GL^3}$	3,650,000	4,150,000	3,675,000
Frequency per second computed	71.1	38.1	45.1

Next the change in the natural frequency of the loaded dredge model due to shifting of the supports (nodal points) was determined in order to get an idea of the magnitude of the error which would result from partial or entire neglect of

First, keeping the after nodal point fixed at the computed position the forward nodal point was shifted forward one cm. (model length) corresponding to .75 m. ship length and then was shifted the same distance amidships and the accompanying frequencies measured. The after nodal point was then shifted the same distance with the forward nodal point in its calculated position. The results are given in Fig. 10.

It is evident in both diagrams that the maximum natural frequency occurs in the case of the computed nodal position; moreover we see how sensitive the natural frequency is to a shifting of the nodal points and what significance is attached to the rigorous fulfillment of the end conditions.

Comparison of the computed and measured natural frequencies shows that the proposed method of computation, notwithstanding its simplicity, gives correct values. Therefore, it is unnecessary to use more complicated methods which require a much greater expenditure of time.

Let us briefly take up the experiments of C. Henderson (21). Henderson attempted without previous vibration calculations to determine the modulus of elasticity of the ship's structure by comparing the measured model frequencies with the natural frequencies observed on board ship. Since the nodal points were determined, obviously without a knowledge of the law, merely by strewing sand which appears impracticable at least for two-noded vibration because of the identity of the points of support with the nodes, it is to be assumed that as a result of the incorrect placing of the supports the measured model frequencies were too low. This

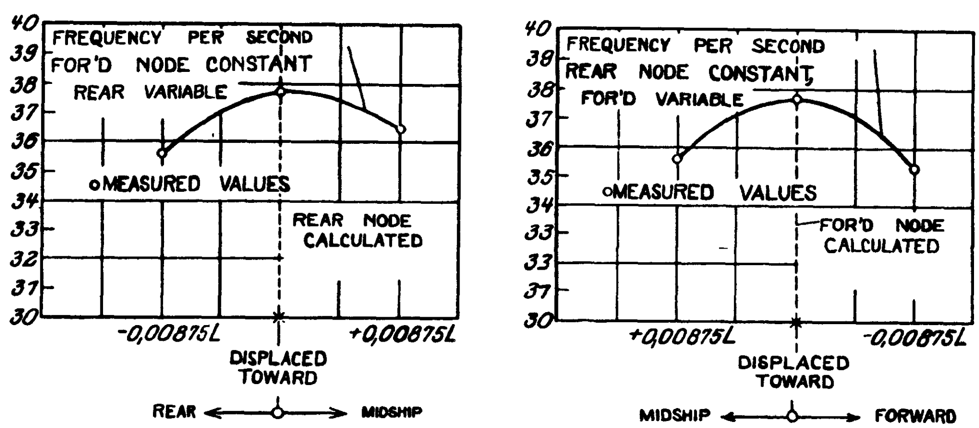


FIG. 10 EFFECT OF POSITION OF NODAL POINT ON FREQUENCY OF THE DREDGE MODEL

assumption is confirmed by the relatively slight difference between the measured model and ship natural frequencies (for the Pathfinder 127/106; for the Lusitania

74.6/65). For this reason and particularly in consideration of the following investigations of the elastic behavior of ship hulls as box girders, and the influence of the virtual mass-inertia, the conclusions drawn by Henderson as to the apparent modulus E appear too high.

III The Elastic Behavior of Ship Forms in Vibration.

4. Influence of effective width.

From the investigations of G. Schnadel (22) it was found that the decrease in stress in the flanges of girders under static load contributes to the increase in the deflection. In what follows an effort is made to apply Schnadel's theory to vibration phenomena. In the case of vibration the dynamic "mass-moment curve" logically takes the place of the static bending moment curve. For a box girder of length 2a, width 2b, with an xy coordinate system in the middle of the flange surface this was developed into a Fourier series according to Herman's method. Assuming the maximum moment equal to unity the following series results:

$$M_x = 0.8870 \cos \frac{m\pi}{2a} x + 0.1416 \cos \frac{m\pi}{2a} x - 0.0386 \cos \frac{m\pi}{2a} x$$

$$+ 0.00822 \cos \frac{m\pi}{2a} x - 0.00533 \cos \frac{m\pi}{2a} x + 0.0040 \cos \frac{m\pi}{2a} x + \ldots$$

$$m = 1, 3, 5, 7 \ldots$$

The computation of the effective width b'_m was first carried out according to the formula given by Schnadel in WRH, Mar. 7, 1931, p. 92 for a box girder with a/b = 2 and h/b = 0.754, where h represents the depth of the box girder, for six different sections distant 0.2a apart. We obtain for the effective width b'_m and the ratio of the effective moment of inertia J_w to the total moment of inertia of the entire cross-section J_{full} the values given in Table II. Since the nodal points lie at 0.552a it is evident that only between the nodal points, from x = 0 to $x = \pm 0.5a$, the effective width decreases in approximately constant ratio and amounts to about 94% of the entire section of the flange. Outside the nodal points the flanges are fully effective so that our investigation remains valid also for ships (pointed ends).

Table II. Decrease in effective width of flanges of a box girder during vertical vibrations of the fundamental type.

$a/b = 2\pi$; $h/b = 0.754$

Station a = L/2	b'_m	J_w/J_{full}
0	0.939b	0.944
±0.2a	0.939b	0.944
±0.4a	0.939b	0.944
±0.6a	1.000b	1.000
±1.0a	1.000b	1.000

For establishing the numerical dependency of the effective width on the ratio of the length-to-width and length-to-depth of the box girder, the effective widths and the corresponding effective moments of inertia for the L/B ratios arising in ship construction were calculated each with a series of appropriate L/H ratios. Here the computation was required to include only the determination of the effective moment of inertia of the midship section, since in the case of similar moment curves the effective moment of inertia found for the midship section remains the same up to the nodal points as has been proved. The results are as shown in Fig. 11.

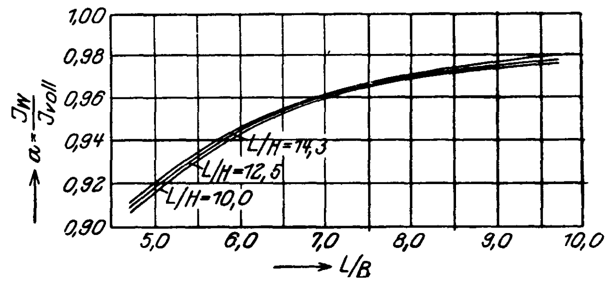


Fig. 11 Influence of L/B and L/H on Moment of Inertia of Box Girders Considering Effective Width.

From the diagram it follows that the L/B ratio has an important influence on the effective moment of inertia. The effect of L/H is, on the contrary, of very little significance. In practical cases the effective moment of inertia fluctuates between 90 and 98% of the moment of inertia of the full cross-section according to the L/B ratio which corresponds to a lowering of the frequency from 5.5 to 1%.

5. Influence of the shear deflection.

In recent years numerous writers have made an exhaustive investigation of the influence of the shear deflection on the total deflection under static load. Whereas W. Dahlmann (23) assumes that mainly the webs alone take up the shearing forces, and G. Wrobbel (24) regards as a value for deflection due to shear $y_s/y_b = 25\ (H/L)^2$ as sufficing for practical cases, the investigations of O. Lienau (25), G. Schnadel (22), and J. L. Taylor (26) probably approximate reality more closely since they consider the distribution of shearing stress in the flanges of built-up girders.

The known investigations do not give, however, numerically complete data on this point, namely as to what extent the shear deflection is influenced by the principal dimensions, and especially by the dimensions of and the number of flanges. The following investigation has as its object the solution of this question.

We start with the familiar formula for shear deflection obtained by comparison of the external and internal work of deformation

$$y_{s_x} = \frac{1}{G} \int_0^x \int_0^u \frac{\tau^2 s\, du}{Q}\, dx \qquad \ldots\ldots\ldots\ldots\ 10)$$

in which G is shear modulus of elasticity
Q is shearing force
t is shearing stress
s is plate thickness
u is total differential*

For the exact calculation of the shear deflection from equation (10) the accurate distribution of the shear stresses over the entire area of each cross-section is required. In the case of a decrease in bending stresses in the flanges varying for the individual frame sections it would be necessary for an exact solution to have recourse to Schnadel's formula for the distribution of shearing stresses. As has been demonstrated in section 4 the effective width between the nodes is constant. Therefore in our case the distribution of shear stresses throughout the section can be computed by the familiar formula:

$$\tau = \frac{Q\,St}{J\,s} \quad \ldots\ldots\ldots\ldots\ldots \quad 11)$$

where St is the static moment of (area of) the adjoining section, and
J is the moment of inertia.

By inserting equation (11) in equation (10) and under the assumption that the moment of inertia for the individual cross sections is constant equation (10) reduces to:

$$y_{s_x} = \frac{1}{G} M_x \, {}^1\!/_{J^2} \int_0^u \frac{St^2}{s}\, du \quad \ldots\ldots\ldots\ldots \quad 12)$$

Since computation of the frequency was carried out graphically on the basis of the deflection at the forward perpendicular the shear deflection at this point must logically be determined. For the uniform bar the ratio of the deflection at the ends to that in the middle referred to the base line is 1:1:608. On introducing the maximum dynamic bending moment which for a uniform rod of weight P and length 2a with two nodes was determined as 0.0704 Pa, when we assume $y_{V.P.} = 1m$ and $\omega_1^2 = 9.81\ sec^{-2}$ we get from equation (12)

$$y_{s_{V.P.}} = 0.137 \frac{Pa}{EJ^2} \int_0^u \frac{St^2}{s}\, du \quad \ldots\ldots\ldots\ldots \quad 13)$$

For the moment of inertia we introduce J_{full}; likewise we compute the static moment under the assumption that the flanges are fully loaded. This assumption can be made without adversely affecting the accuracy because in the vibration under consideration the effective moment of inertia amounts to only 90% to 98% of the completely loaded section.

The bending deflection is found by the vibration formula for uniform bars and for vibrations of the fundamental order

$$\omega = 2\pi \cdot 3.57 \sqrt{\frac{EJg}{PL^3}} \quad \ldots\ldots\ldots\ldots \quad 14)$$

---

*This formula is identical with the formulas of Lienau, Föppl, Hovgaard and Taylor.

Since the frequencies vary inversely as the square root of the deflections we get under the same assumptions as above for y_{1vp} and ω_1^2:

$$\omega = \sqrt{\frac{9.81}{y_{2\,V.P.}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 15)$$

where $y_{2_{vp}}$ represents the computed deflection at the forward perpendicular. By equating 14 and 15 and inserting L = 2a, g = 981 cm/sec², and $J_w = \alpha J_{full}$ we obtain:

$$y_{2\,V.P.} = 0.0001589 \frac{Pa^3}{qEJ} \quad \ldots\ldots\ldots\ldots \quad 16)$$

and by division of equation (13) by (16)

$$(y_s/y_b)_{V.P.} = 7.16 \frac{\alpha}{J_{roll}} a^2 \int_0^v \frac{St^2}{s} dv \quad \ldots\ldots\ldots \quad 17)$$

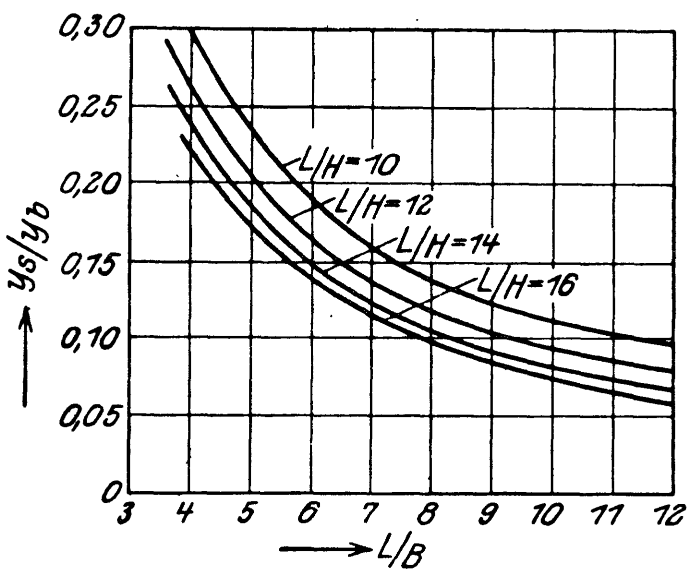


FIG. 12 INFLUENCE OF L/B AND L/H ON THE RATIO OF SHEAR TO BENDING DEFLECTION FOR SIMPLE BOX GIRDER

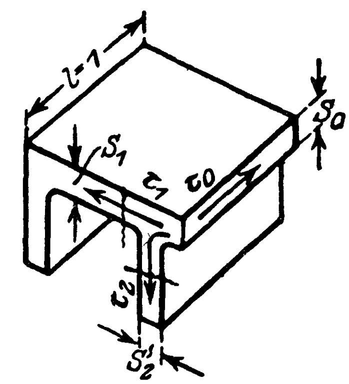


FIG. 13 DISTRIBUTION OF SHEARING STRESS AT A JUNCTION-POINT

On the basis of this formula, the ratio of shear to bending deflection was computed for the practical L/B and L/H ratios occurring in ship construction. The diagram gives remarkable data in several respects. First it is evident that for ships the ratio L/B is of markedly greater influence on Y_s/Y_b than L/H, in contrast with the widely accepted belief that the shear deflection becomes of importance only when the ratio L/H decreases considerably. Thus it is shown for example that in a relatively very low ship of L/H = 14, with a normal L/B of about 6.3, the shear amounts to 14%; L/B = 4.7, shear = 19.5%, proof of the fact that in no case is the shear deflection to be neglected.

Moreover it is plainly evident that the ratio y_s/y_b cannot be assumed as even approximately a constant function of $(H/L)^2$. Because although with L/B = 4.72 and L/H = 10 we still get $y_s/y_b = 24.78\,(H/L)^2$, with L/B = 9.42 and L/H = 10 we get $y_s/y_b = 11.90\,(H/L)^2$, a difference of over 100% with the same L/H.

The influence of the flanges is accordingly of transcending importance. By considering the webs alone we get errors which even for single deck ships without double bottoms, with the ratios L/B and L/H under consideration, lie between a minimum of 21% and a maximum of 83%. In general it can be said that even for a box girder with only one upper and one lower flange the shear deflection amounts to about 6 ÷ 25% of the bending deflection. We extend our investigations to sections with intermediate bracing such as inner decks, double bottom, longitudinal bulkheads, etc. In this case the distribution of shear stresses at the junction points must be determined according to Taylor (27) from the equilibrium conditions that for every enclosed space there be no shifting in the longitudinal direction and no local torsion about a longitudinal axis. With reference to Fig. 13 the equilibrium conditions for the junction points are:

$$\tau_0 s_0 = \tau_1 s_1 + \tau_2 s_2 \;;\quad \int_0^u \tau\, du = 0$$

By substituting for $\tau = Q \cdot St/J\,s$ we get for sections with uniform wall thickness:

$$St_0 = St_1 + St_2 \;;\quad \int_0^U \frac{St}{s}\, du = 0$$

for every enclosed section.

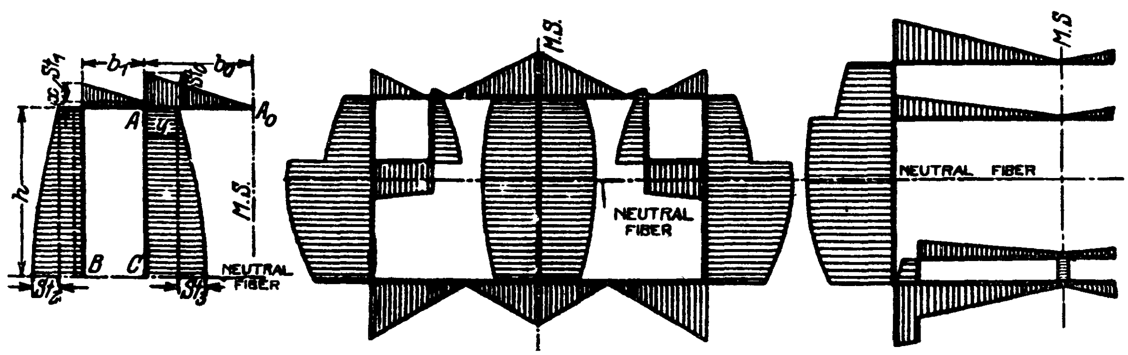


FIG. 14 DISTRIBUTION OF SHEARING STRESS IN CROSS-SECTIONS WITH STIFFENERS

In the simplest case of a ship with two longitudinal bulkheads (Fig. 14) the components x and y can be directly computed by the corresponding equations:

$$St_0 = x + y$$

$$\int_B^A \frac{St}{s} du = \int_C^A \frac{St}{s} du = s b_1 + St_1 \frac{b_1}{2} + (x + St_1) h + \tfrac{2}{3} St_2 h = yh + \tfrac{2}{3} St_3 h$$

With non-uniform plate thicknesses we first take up the integration of the static moment without considering the parts x and y. On introducing $St' = St - x$ in conjunction with $St_o = x + y$ in

$$\int_B^A \frac{St'}{s} dv - \int_C^A \frac{St'}{s} dv = y \int_C^A \frac{1}{s} dv - x \int_B^A \frac{1}{s} du$$

we obtain also in this case the shear force component without difficulty.

By plotting the St^2/s curve and integrating the same over the circumference of the section the value for the coefficient of the section $\int St^2/s \, du$ can be determined and therefore according to equation (17) the ratio y_s/y_b.

By this method y_s/y_b was computed for various ship sections with double bottoms and multiple decks and compared with the corresponding values for single deck ships without double bottoms with equal L/B and L/H ratios. The proportional values C_1 are tabulated in Table III with the other calculated results.

Table III. Influence of Double Bottoms and Decks on the Shear Deflection.

Type of Ship	L/B	L/H	$\frac{\int \frac{St^2}{s} dv}{sb^5}$	$\frac{J_{full}}{sb^3}$	J_w/J_{full}	y_s/y_b	y_s/y_b for single deck	C_1
A	7.63	12.86	2.620	2.000	0.963	0.155	0.118	1.315
B	7.18	10.80	4.863	2.775	0.962	0.234	0.144	1.625
C	7.57	10.87	5.960	3.183	0.965	0.226	0.138	1.633
D	7.58	9.90	9.659	4.789	0.966	0.243	0.150	1.620

A. Single-decker with double bottom 122.0 x 16.0 x 9.5 m D.B. 13.5 x 1.5 m

B. Two decker with double bottom 111.3 x 15.5 x 10.3 m D.B. 13.2 x 1.1 m
Height between decks 2.6 m

C. Three decker with double bottom 119.5 x 15.8 x 11.0 m D.B. 13.8 x 1.1 m
Height between decks 2.5 m

D. Five decker with double bottom 151.5 x 20.0 x 15.3 m D.B. 18.0 x 1.25 m
Height between decks 2.5 m

We perceive that the shear deflection through the influence of the double bottom as against single deckers without double bottom is increased about 30% and according to the number of decks up to 63%.

The value of y_s/y_b determined on the assumption of constant plate thickness and a uniform mid-ship section over the entire ship's length may be useful for actual ship forms with fined ends and variable plate thickness as a mean value. It must not be overlooked that in case of extreme loading the shape of the shearing force curve may also exert a great influence on the shearing deflection.

6. Influence of the complex nature of the ship members.

There can be no question that the actual deflections of a ship hull to be expected as a result of the complex nature of the framing (superposition of stresses through hatches and rivets, reinforcing of weaker members by stronger, etc.) will be greater than they would be considering only the effective width and the shearing deflection. Effort was made therefore to determine the difference mathematically by comparison of the computed values on the basis of sections 4 and 5 and the deflection measurements of numerous other investigators. For this purpose the deflection measurements of T. C. Read and G. Stanbury (28), W. Dahlmann (22), J. L. Taylor (27), J. H. Biles (30) on ship hulls and of O. Lienau (25) on model hulls were referred to.

1. Measurements of T. C. Read and G. Stanbury: (a) Two decker with double bottom 105.80 x 13.90 x 9.12 m; L/B = 7.61; L/H = 11.60; capacity 5000 tons (estimated), load 5000 tons. Measured deflection 5.870 cm. Computed deflection with $E = 2.1 \times 10^6$ kg/cm² and J_{full} without deduction for rivets according to the data of R. and St. 4.975 cm. Addition for the shear deflection taking into consideration the number of decks according to Fig. 12 and Table III = 1.050 cm. The effective width was not added because J_{full} was computed disregarding the decks between the hatches. Accordingly the total computed deflection amounts to 6.025 cm. If in the following y_{th} = theoretical total deflection and y_w = measured deflection we get $y_w/y_{th} = 0.975$. (b) Ship with platform decks with double bottom 91.50 x 12.68 x 7.14; L/B = 7.22; L/H = 12.82; capacity 4000 tons (estimated); load 1800 tons. Measured deflection 1.575 cm. Computed deflection under the same assumptions as in (a) 1.098 cm., added for shear deflection (Fig. 12 and Table III) = 0.228 cm. Computed total deflection accordingly 1.326 cm. $y_w/y_{th} = 1.190$.

In order to make possible a comparison of the measurements with one another the loads should be expressed as fractions of the capacity. We get accordingly for a load of 45% of the capacity $y_w/y_{th} = 1.190$ and for a load equal to the full capacity $y_w/y_{th} = 0.975$.

2. Measurements of W. Dahlmann: Single decker with double bottom and very large hatchways (ore steamer) 121.92 x 16.50 x 9.42 m. L/B = 7.39; L/H = 12.94; capacity = 8300 t. To the bending deflections computed by D. are added the shear deflections according to Fig. 12 and Table III ($y_s/y_b = 0.12 \times 1.315 = 0.159$) and the deflection resulting from the effective width according to Fig. 11 ($J_w/J_{full} = 0.963$). The deduction of 10% for rivets made by D. was equalized by a corresponding increase in J and instead of $E = 2.15 \times 10$, $E = 2.1 \times 10^6$ was introduced in order to obtain a uniform basis of comparison. The inclusion of the longitudinal hatch coamings in the moment of inertia should be let stand since such coamings, according to the measurements of J. L. Taylor (27) are very nearly fully effective. The deflections obtained under these assumptions are tabulated in table IV.

Table IV. Comparison of measured and computed deflections for an ore steamer from the measurements of W. Dahlmann

Condition	Load * ton	y_b computed according to Dahlmann cm.	y_b according to calculation cm.	$y_b \times \frac{1.159}{0.963}$ cm.	Difference y_{th} cm.	Measurement y_w cm.	y_w/y_{th}
0		-2 410	-2.242	-2.700			
1	1640	-0.165	-0.154	-0.185	2.515	3.500	1.392
2	3710	2.800	2.610	3.140	3.325	3.900	1.172
3	5570	4.060	3.760	4.530	1.390	0.900	0.648
4	5800	2.660	2.475	2.980	1.550	1.000	0.643
5	9130	2.800	2.610	3.140	0.160	0.300	

First it must be said of the measurements themselves that they refer only to one station (forward edge of bridge house). We obtain from this somewhat different y_w/y_{th} ratios than if the mean were taken of several stations distributed over the ent re length of the ship. Further the values of y_w/y_{th} appear unreliable in transition from condit on of loading 2 to 3 and 3 to 4 since the readings of y_w refer to relatively small deflections (0.9 to 1 cm.). The final measurement (0.3 cm.) might be dropped for the same reason. According to Dahlman's data the possible error amounted in all cases to 1 mm.

From Dahlmann's investigations it is unmistakable that as the load increases y_w/y_{th} decreases, that is to say more and more parts contribute to taking up the stresses, which agrees with the experiments of Read and Stanbury; quantitatively only the values of y_w/y_{th} = 1.392 and likewise 1 172 appear to be useful. When again the loads are expressed as a fraction of the capacity, we get for a load f 0.1975 of the capacity y_w/y_{th} = 1.392 and for a load of 0.446 of the capacity y_w/y_{th} = 1.172.

3. Measurements of J. L. Taylor: Tanker with transverse frames 121.92 x 16.15 x 10.67 m· L/B = 7. 5· L/H = 11.46; E = 2.0744 x 10^6 ; J computed in the usual manner.

The shear deflections were likewise computed by Taylor. Since by taking E = 2.1 x 10^6 and J_w/J_{ful} = 0.965, y_{th} would be only about 1% greater, the values given by Taylor for the theoretical total deflection may be adopted as in agreement with the hitherto applied basis of comparison.

* Approximate weights calculated according to Dahlmann's directions.

Table V. Comparison of the measured and computed deflections for a tanker from the measurements of J. L. Taylor

Condi-tion	Loading ts	y_{th} according to calc.* cm	y_w measured* cm	y_w/y_{th}	y_w/y_{th} mean value	Loading / capacity mean value
1	70					
2	1502	-1.295	0.794	0.613		
2a	1282	-1.994	0.477	0.239		
3	2498	1.577	2.382	1.510		
3a	2428	1.918	4.525	2.360	1.635	0.245
3b	2498					
4	2722	2.298	2.382	1.037		
5	3682	5.205	5.951	1.144		
5a	3722	2.032	3.335	1.640	1.514	0.503
6	4792	6.109	8.732	1.429		
6a	4942	2.338	4.290	1.844		
7	6266	10.278	12.467	1.213		
7a	6106	3.875	5.397	1.392	1.235	0.772
8	7284	5.057	5.560	1.099		
8a	7324	12.604	14.285	1.133		
9	8488	13.898	16.672	1.200	1.150	0.958
9a	8578	13.438	15.082	1.122		

The deflections obtained are given in the order of increasing load in Table V. When the measurements for conditions 2 and 2a are stricken out on account of too great inaccuracy of measurement ($y_w < 0.8$ cm) a glance at the two last columns of the table shows that y_w/y_{th} with increasing load becomes smaller, in agreement with the experimental results of Read and Dahlmann. That the absolute values of y_w/y_{th} are relatively high must be attributed to the type of ship being a tanker with its many weaknesses due to riveting at the transverse bulkheads and in part also to the buckling of the deck plating observed by Taylor.

4. Measurements of J. H. Biles: Biles computed the deflections for individual test conditions disregarding the shear deflection but considering the effect of riveting and with E = 2.21 x 10^6, of which only those for "sagging" will be noted because only in this case does the position of the supports correspond to the nodes in two-noded vibration. Biles derived the apparent E modulus by comparison of measured and computed values (average values over the ship's length). Biles' data must first be reduced to our basis of comparison. The "Wolf" had an

* Sum of the values at 3 different points.

L/B of 10.88 and L/H of 16.80 from which we obtain according to Fig. 11 J_w/J_{full} = 0.98 and according to Fig. 12 y_s/y_b = 0.064. Since the destroyer had platforms fore and aft the addition of approximately 30% to y_s/y_b based upon table III appears justified. Accordingly the shear deflection should be about 8%. For equalizing the deductions made for riveting the apparent E modulus must be multiplied by 15/16. (See A. Robb (31)). Finally we take the computed deflection on the basis of E = 2.1 x 10 kg/cm². Then the y_w/y_{th} values in Table VI are obtained.

Table VI. Comparison of measured and computed deflections for the destroyer "Wolf" according to the measurements of J. H. Biles.

Draft '	E modulus (Biles) ts/in.²	y_w/y_{th}
6	11813	1.148
5	11950	1.135
4	11390	1.192
3	11500	1.180
2	11110	1.222
1	10550	1.286
Dry	10340	1.312

With decreasing draft, that is with increasing load, according to the experimental results of the other investigators, y_w/y_{th} must become smaller; actually, this phenomenon does occur just at from 6 to 5 ft. draft. With further increase of load, which for lightly built ship hulls must be regarded as extreme, the opposite case begins to apply; y_w/y_{th} increases. This phenomenon, however, is satisfactorily explained according to Schnadel's investigations (22) by the occurrence of buckling phenomena, which naturally lead to constantly increasing values of y_w with increasing load.

It can, therefore, be established from the comparison of Biles investigations with the results of Read, Dahlmann, and Taylor that buckling phenomena occurred at about 4 ft. draft in the case of the "Wolf."

Since the freely floating condition of the "Wolf" at a draft of 6 ft. must correspond to the displacement in fully loaded condition of a freighter, the derived ratio y_w/y_{th} = 1.148 is comparable with the values measured by the remaining investigators.

5. Measurements of O. Lienau: The experiment was carried out with a box girder of dimensions 6400 x 1100 x 400 x 0.5 mm under the action of a concentrated load at mid-length of the girder the magnitude of the load having been so chosen that the buckling of the flange members was barely avoided. Comparison of the theoretical deflection, taking into account the effective width according to

Schnadel, and the shear deflection, with the measured values yields with E = 2.1 x 10^6 (as compared to the calculated values of E = 2.15 x 10^6) the y_w/y_{th} values of Table VII. (See also Fig. 19 of the paper of Lienau). As a mean over the length of the girder we obtain from Table VII, y_w/y_{th} = 1.121.

Table VII. Comparison of the measured and computed deflections for a box girder from the measurements of O. Lienau.

Station (a = L/2)	y_{th} computed mm.	y_w measured mm.	y_w/y_{th}
0 = ℄	9.34	10.27	1.100
0.2a	8.61	9.48	1.100
0.4a	7.19	7.99	1.111
0.6a	5.06	5.94	1.174

Unfortunately deflection measurements for smaller loads were not made; however, we can probably compare the values obtained for the fully loaded condition below the buckling limit with the other tests

The results of the topics under 1 to 5 are summarized in Fig. 15.

In consideration of the widely divergent types of ships investigated and the improvement in the course of time of design strength, as well as in consideration of the well known difficulties in obtaining the actual loading conditions on board, and of greater accuracy required for measurement for small deflections, it is not surprising that the measured points are scattered over a certain range. Undoubtedly it follows from the results already obtained that the ratio of y_w/y_{th} is dependent upon the load as well as the type of ship. This knowledge is of especial importance in vibration processes since it gives an explanation of the familiar phenomenon that the frequencies of ships in the lightly loaded and fully loaded conditions do not differ from one another nearly as much as is to be expected on the basis of weight difference and weight distribution alone. Here we purposely exclude the effect of the water vibrating with the ship.

7. Summarizing by Formulas.

The effect of the "elastic behavior" on the frequency may be expressed by the following formula in which at the same time the corresponding factors for the effect of the rotation of the beam section and the effect of the damping resistances are introduced:

$$\omega = \omega_2 \Big/ \sqrt{K_1 \cdot K_2 (1+K_3)(1+K_4)(1+K_5)} \quad \ldots\ldots\ 18)$$

In this formula ω_2 represents the theoretical frequency derived from the

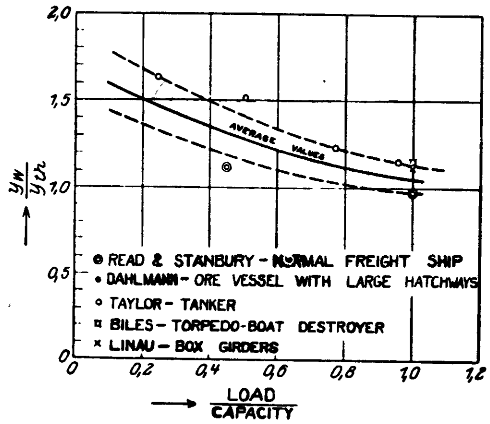


FIG. 15 VARIATION WITH LOAD OF RATIOS OF MEASURED TO CALCULATED TOTAL DEFLECTION FROM DEFLECTION MEASUREMENTS OF VARIOUS RESEARCHERS

vibration calculation. $K_1 = J_{full}/J_w$ (Fig. 11); $K_2 = y_w/y_{th}$ (Fig. 15); $K_3 = C_1 \cdot y_s/y_b$ (Fig. 12 and Table III); $K_4 = 7.26\ (H/L)^2$ according to Taylor (10) for vibrations of the fundamental type, K_5 = damping resistance/ship weight (See section 12).

8. Model experiments with a box girder.

The results obtained from the investigation of elastic behavior were checked experimentally. A box girder built up of standard structural materials, whose

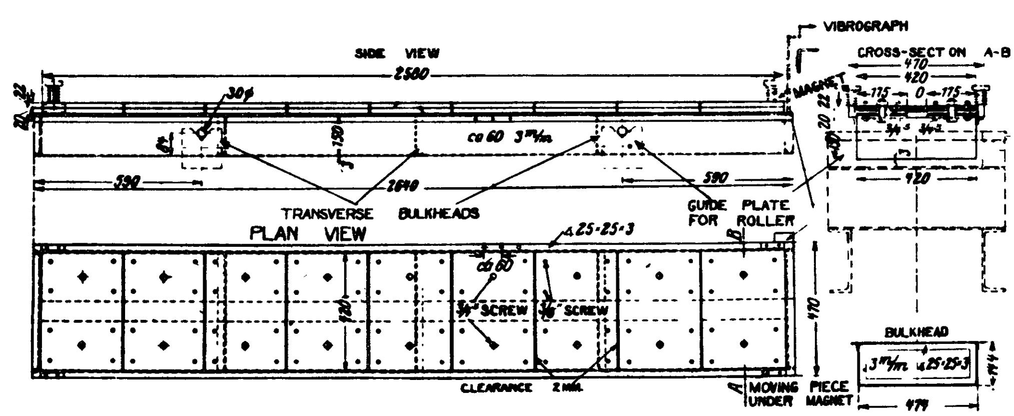


FIG. 16 TEST SPECIMEN BOX GIRDER

dimensions and details of construction are illustrated in Fig. 16, was set in vibration by electromagnetic impulses and the vibrations were measured by means of a vibrograph. Figure 17 shows the experimental set up. For preventing local

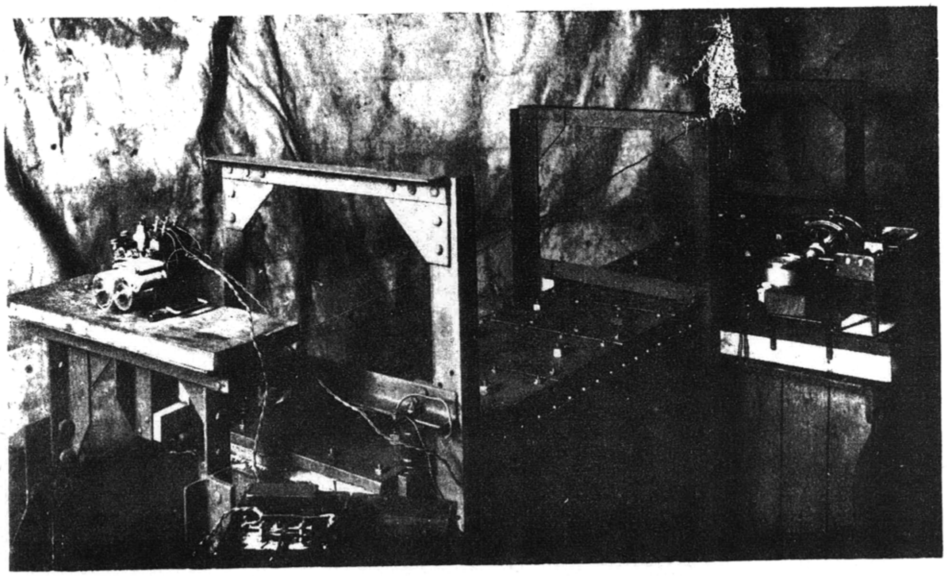


Fig. 17 Experimental Set Up for Vibration Test of Box Girder.

vibrations the model was divided into four parts by the two ends and three transverse bulkheads. By the loading plates screwed to the deck (321 kg) and by filling with dry sand (179 kg) the total weight of the box girder which itself was only 105 kg. was brought to 605 kg. in order to obtain as low a natural frequency as possible to facilitate measurement. The moment of inertia computed without deductions amounted to 1766 cm.^{4} With E = 2.1 x 10^6 kg/cm² we get for the theoretical natural frequency:

$$n = 3.57\sqrt{\frac{EJg}{GL^3}} = 3.57\sqrt{\frac{2.1 \times 10^6 \times 1766 \times 981}{605 \times 264^3}} = 64.6 \ \mathrm{sec}^{-1}$$

The constants K_1 to K_4 characterizing the elastic behavior were determined from Figs. 11, 12, and 15 for the present ratios, namely L/B = 6.28 and L/H = 17.25 as K_1 = 1.052; K_2 = 1.13; K_3 = 0.13; K_4 = 0.0245. With these values according to equation (18) the natural frequency to be expected is 55 sec^{-1}.

The number of electromagnetic impulses per unit time could be controlled by a motor with an interrupter disc which was connected in the circuit of the electromagnet in series with a variometer. As a check on the frequency the number of impulses was recorded on a drum. The model was supported on iron rods which were inserted through the web at the neutral axis at distances of .244 L from the ends. The natural frequency of the iron rods loaded with the weight of the experimental model lay outside the range of vibration of the girder to be measured as was determined by special experiment.

The frequency of the impulses was varied by changing the terminal voltage of the motor. The maximum amplitude occurred at 53.7 vibrations per second (Fig. 18). By reference to the resonance curves for the logarithmic decrement, Δ = 0.3, 0.4, 0.5 in the familiar formula for the amplification: (32)

$$V_z = 1 \Big/ \sqrt{(1-z^2)^2 + \frac{4\Delta^2 z^2}{\pi^2 + \Delta^2}}$$

with z = "out of resonance" we find that the measured amplitudes lie very closely on the curve Δ = 0.4. If it is assumed that the internal damping is proportional to the frequency, we get in this case for the damping factor

$$b = \frac{2\Delta}{\sqrt{\pi^2 + \Delta^2}} \sqrt{mc} = \frac{1}{4} \sqrt{mc}$$

m = mass,

c = "elasticity"

Under the influence of this damping the period of vibration is increased by 0.7% so that the natural frequency reduced to the undamped condition will be 53.7 x 1.007 = 54.2 sec.$^{-1}$.

The agreement with the value 55 sec.$^{-1}$ computed by considering the elastic behavior is almost perfect. The trivial difference may be assigned to the friction at the supports.

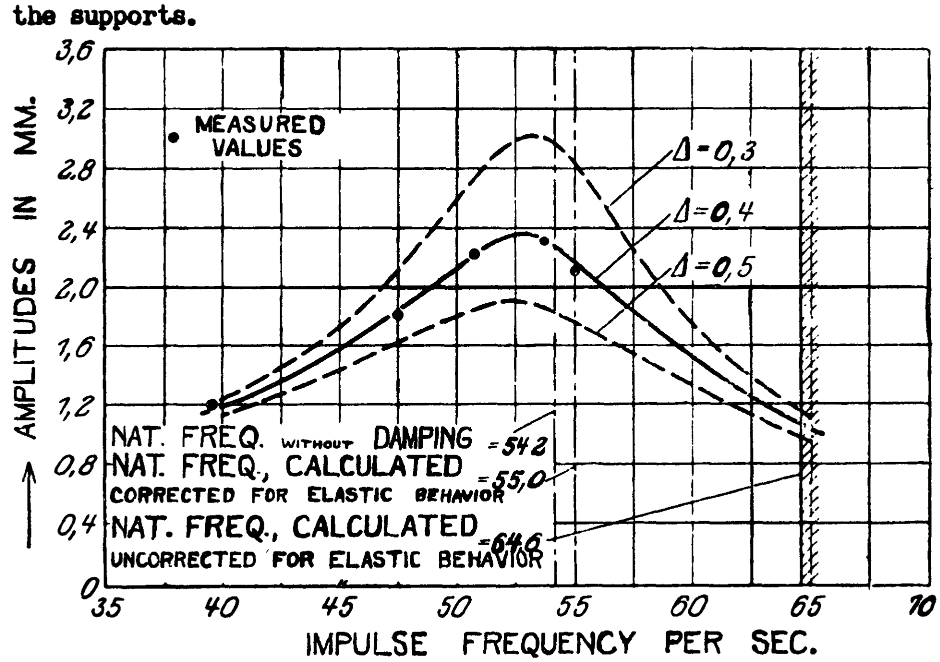


FIG. 18 INVESTIGATION OF VIBRATION OF BOX GIRDERS
COMPARISON OF CALCULATED AND MEASURED NATURAL FREQUENCIES

Comparison with the uncorrected theoretical frequency which equals 64.6 sec.$^{-1}$ illustrates with great effect the importance of the consideration of the elastic behavior in the calculation of the natural frequency of ship hulls and shows moreover that the derived values for K_1 to K_4 can be regarded as agreeing with actual conditions.

IV Damping resistance.

9. Determination of the model laws.

It is well known that all damping manifests itself in a lowering of the natural frequency. The magnitude of the damping resistance may be expressed by the virtual increase of mass which corresponds to the lowering of the natural frequency. Since the frequency varies inversely as the square root of the load we find the damping resistance to be

$$W = G\left(\frac{n_L^2}{n_w^2} - 1\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 19)$$

In this equation

- W = damping resistance in kg.
- G = weight of model in kg.
- n_L = frequency in air.
- n_w = frequency in water.

For determining the laws of similitude we set down according to M Weber (33), the original relation of measured quantities:

$$F(l, t, \rho, W, \gamma, \eta, a, \epsilon) = 0$$

Here:
- l = any linear dimension of the model.
- t = time of one complete vibration.
- ρ = mass density of the medium.
- γ = weight density of the medium.
- η = viscosity of the medium.
- a = maximum amplitude.
- ϵ = roughness of the surface.

The problem is stated in dimensionless terms by converting the relation of measured quantities $F = 0$ into the relation of known quantities $\psi = 0$. Since the process is a dynamical one we choose as fundamental units l, t, and ρ. There remain as parameters $W, \gamma, \eta, a, \epsilon$ which with l, t, ρ are to be changed into dimensionless power products π_1 to π_5. We write therefore:

1. For W: $\pi_1 = W\, l^x\, t^y\, \rho^z$ and with the appropriate unit equation $[1] = [\text{kg}\ m^x\ \text{sec.}^y\ m^{-4z}\ \text{sec}^{2z}\ \text{kg}^z]$ where the three fundamental units correspond to the law of similarity of dimensions, we get three independent equations: $0 = x - 4z$; $0 = y + 2z$; $0 = 1 + z$. The solution of these equations gives $\pi_1 = W\, l^{-4}\, t^2\, \rho^{-1}$ and by introducing the frequency per second $n = t^{-1}$, we obtain Newton's number: $N = W/l^4\, n^2\, \rho$.

2. For γ: $\pi_2 = \gamma / t^x g^y \rho^z$, a relation similar to that for W gives $\pi_2 = \gamma / t^{-4} t^2 \rho^{-1}$, and with n = t^{-1} and g = γ/ρ we obtain Froude's number $\mathcal{F}^{-1} = g/l n^2$

3. In a similar manner by introducing the kinematic viscosity in the case of η where $\nu = \eta/\rho$ we get Reynold's number: $R^{-1} = \nu / l^2 n$

4. a and ϵ lead directly to the parameter numbers $\pi_4 = a/l$ and $\pi_5 = \epsilon/l$ likewise the ratio values for determining the under water form: L/B, B/T, and coefficient of fineness δ go directly into the numerical relation ψ = 0 as parameter numbers. If a/l in π_4 is replaced by $y_{⊗}/y_{vp}$ (= maximum amplitude amidships/maximum amplitude at forward perpendicular) the numerical relation reads $\psi(\pi_1, \pi_2, \pi_3 \ldots\ldots \pi_n) = 0$, whereby the characteristic value for the damping resistance becomes:

$$K_w = W/l^4 n^2 \rho = \phi\left(g/l n^2,\ \nu/l^2 n,\ y_{⊗}/y_{v.p.},\ \epsilon/l,\ L/B,\ B/T,\ \delta\right)$$

The damping resistance obeys therefore the laws of Newton, Froude, and Reynolds. On account of the familiar conflict of the two latter laws complete dynamic similarity cannot be realized. The separate determination of frictional coefficients was not undertaken. The error arising therefrom must be of minor significance since the frictional resistance amounts only to a fraction of the total resistance and the latter is only a fraction of the total load affecting the natural frequency.

10. Experiments with rectangular hulls.

The models as in Nicholl's experiments consisted of a vibrating strip of wood or steel on the under side of which individual wooden blocks were fastened forming a displacement body. Rubber strips were glued over the spaces between the blocks which had been made water tight by varnishing in order to prevent eddy formation. In order to check up any possible change of weight the models were weighed before and after each test. They were supported on pins screwed into the strips at the calculated nodes. The experimental set up is shown in Fig. 6.

The first series of experiments was carried out with four models which being of similar form (1500 x 200 x 100 mm) possessed various frequencies in air due to suitable choice of vibrating strips; this series served for determining the dependence of the damping resistance $W = K_W \cdot l^4 n^2 \rho$ upon Froude's number $\mathcal{F}^{-1} = g/l n^2$ and the ratio B/T.

Every model was measured at five different drafts and four different amplitudes. Measurements in air were made before beginning the immersed tests as well as after in order to counteract the deviation due to imperfect elasticity of the rubber. It was evident that the natural frequencies in air were higher than those computed and decreased with increasing amplitude, which obviously is to be attributed to the effect of the rubber strips. Therefore the frequencies in water and in air are only to be compared at equal amplitudes. Since it appeared, however, that the effect of the absolute value of the amplitude lay within the range of accuracy of the measurements the calculation of W by equation 19 is made with the mean of the frequencies measured at each depth for four different amplitudes. Each measurement is the mean of four different values. The test data and the analyses are shown in Table VIII (see Appendix).

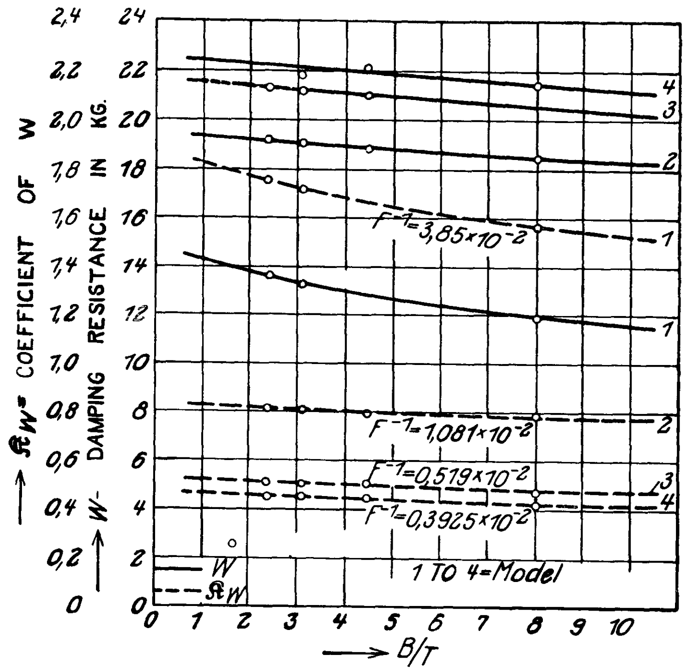


FIG. 19 DAMPING RESISTANCE FOR RECTANGULAR HULLS (L/B = 7.5) AS FUNCTIONS OF DRAFT (TEST SERIES I)

In Fig. 19 the damping resistances as well as their index numbers F^{-1} for the four fundamental vibration frequencies investigated are shown as functions of B/T. We see first that the draft in the case of rectangular bodies has very little effect on the damping resistance. Since the bottom of the models is the same in all cases and the frictional surface of the four side walls increases with increasing draft, we can conclude from the slight increase in W that there is only a small frictional component of the measured total resistance. At the same time it follows that W is by no means proportional to the displacement.

However, the damping resistance is quite dependent on the frequency in air. It increases with increasing frequency first rapidly, then more slowly. Beginning with a definite frequency whose corresponding magnitude for ships needs no longer be considered it even appears to be approximately independent of frequency. Further it should be recalled that it is not permissible to regard the ratio of the frequency in air to that in water in the case of similarly formed bodies as constant. Constancy of this ratio is only to be expected when similarity of body form accompanies similarity of mass of both bodies. This is true because the damping resistances, (although they will be of equal magnitude when the forms are similar) act in lowering the natural frequency in water only in proportion to the ratio which they bear to the dead weight of the vibrating body. To establish this fact and to prove whether the calculation of W from the formula $W = G\,(n^2_L/n^2_W - 1)$ is permissible as a basis for the comparison of damping resistances with one another an experiment with a model of exceptionally great dead weight was carried out.

Model No. 5 had the same body form as models Nos. 1 to 4 but, by attaching uniformly distributed lead plates to the layers, they were brought to a weight of 82 kg.

The natural frequencies measured in water of model No. 5 were compared with the natural frequencies computed by means of equation 19) wherein W was taken from Fig. 19 for the vibrations in air of model No. 5 and for equal values of B/T ($n = 26.85$ sec.$^{-1}$) (see Table IX).

Table IX. Comparison of the natural frequencies measured with model No. 5 with the frequencies computed according to the equation: $n_W = n_L\sqrt{\frac{G}{W+G}}$ on the basis of series I:

B/T	n_W according to tests with model 5	W according to Fig. 19 for n_L = 26.85 sec.$^{-1}$	$n_W = n_L\sqrt{\frac{G}{W+G}}$	Deviation
2	23.80 sec.$^{-1}$	20.0 kg.	24.00 sec.$^{-1}$	+0.8%
3	24.20 "	19.8 "	24.12 "	-0.3%
4	24.40 "	19.6 "	24.15 "	+1.0%

The comparison shows that the deviation for n_W lies within the range of accuracy of measurements. The evaluation from equation 19 is therefore to be regarded as permissible. Moreover Table IX shows that for 82 kg. dead weight n_W amounts to only 90% of n_L whereas in the case of model 2 for example (see Table VIII) of 17.90 kg. weight n_W is about 70% of n_L.

Table X. Model Measurements - test series II.

Model No.	Dimensions in mm. Form body l x b x h	Dimensions in mm. strips l x b x d	Weight in kg.	J cm^4	n. per sec. in air calculated
6	1500 x 100 x 200	1500 x 100 x 15 (each)	33.00	2.81	about 24.4/sec.
7	1500 x 300 x 66.6	1500 x 100 x 15	35.70	2.81	24.4/sec.
8	900 x 200 x 100	900 x 72 x 7.5	12.50	0.253	24.4/sec.
9	2100 x 200 x 100	2100 x 174 x 24	88.30	20.05	24.4/sec.

A second test series was carried out in order to determine the dependence of damping resistance upon the ratio L/B. The dimensions of the model were chosen (see Table X) so that two models having the same length had different widths, and two others having different lengths had the same width, and the computed natural frequencies in air were the same in order to permit testing simultaneously the validity of the model laws and to make possible a direct comparison of measured results with one another. The results of the measurements are shown in Table XI (see appendix).

A direct comparison of the results is possible for models 6 and 7 since both have almost the same Froude's number. By supplementing by series I at corresponding F^{-1} values we get for the increase of damping resistance with diminishing L/B the results given in Table XII, where the damping constant was determined on the one hand as previously, by means of the equation $K_w = W/l^4 n^2 \varrho$ and on the other hand by means of the equation $K_w' = W/f l^2 n^2 \varrho$ with f = damping surface.

Table XII. Evaluation of the results for rectangular shaped bodies according to various characteristics

F^{-1}	B/T	Model No. 7			Model of Series I			Model No. 6		
		L/B = 5			L/B = 7.5			L/B = 15		
		L x B = 1500 x 300 mm, f = 0.45 m²			L x B = 1500 x 200 mm, f = 0.30 m²			L x B = 1500 x 100 mm, f = 0.15 m²		
		W in Kg.	K_w	K_w'	W in Kg.	K_w	K_w'	W in Kg	K_w	K_w'
0.635×10^{-2}	2	39.80	$0.746 \cdot 10^{-4}$	$3.740 \cdot 10^{-4}$	20.75	$0.390 \cdot 10^{-4}$	$2.930 \cdot 10^{-4}$	7.40	$0.139 \cdot 10^{-4}$	$2.085 \cdot 10^{-4}$
	3	39.75	$0.745 \cdot 10^{-4}$	$3.735 \cdot 10^{-4}$	20.70	$0.389 \cdot 10^{-4}$	$2.925 \cdot 10^{-4}$	6.90	$0.130 \cdot 10^{-4}$	$1.945 \cdot 10^{-4}$
	4	39.70	$0.744 \cdot 10^{-4}$	$3.730 \cdot 10^{4}$	20.65	$0.388 \cdot 10^{4}$	$2.920 \cdot 10^{-4}$	6.45	$0.120 \cdot 10^{-4}$	$1.818 \cdot 10^{-4}$

Table XIII. Index of damping resistances for rectangular shapes.

$F^{-1} = g/ln^2$	$K_w = W/l^4 n^2 \varrho$ for B/T = 2					
	L/B					
	5	7	9	11	13	15
$0.5 \ 10^{-2}$	$0.530 \ 10^{-4}$	$0.325 \ 10^{-4}$	$0.220 \ 10^{-4}$	$0.140 \ 10^{-4}$	$0.115 \ 10^{-4}$	$0.100 \ 10^{-4}$
1 "	1.000 "	0.625 "	0.410 "	0.275 "	0.225 "	0.190 "
2 "	1.730 "	1.120 "	0.720 "	0.500 "	0.400 "	0.355 "
3 "	2.300 "	1.505 "	0.965 "	0.660 "	0.540 "	0.480 "
4 "	2.760 "	1.830 "	1.180 "	0.800 "	0.640 "	0.580 "
5 "	3.160 "	2.080 "	1.370 "	0.915 "	0.730 "	0.660 "
6 "	3.540 "	2.310 "	1.550 "	1.020 "	0.810 "	0.740 "

It is evident that the side ratio has a strong influence on W; even when the damping surface is taken into account by means of K_W' the characteristic term does not remain constant but increases considerably with decreasing L/B (in the foregoing cases W is approximately proportional to $f^{1.5}$). In view of these circumstances, and in order to avoid further extensive test series the resistance constant $K_W = W / l^4 n^2 \rho$ previously used, in which the damping surface was not taken into account, was likewise used as a basis for evaluating further experiments.

In Fig. 20 and Table XIII the values of K_W determined from test series II which are converted for various F^{-1} values based on the results of series I are given as a **function of L/B.**

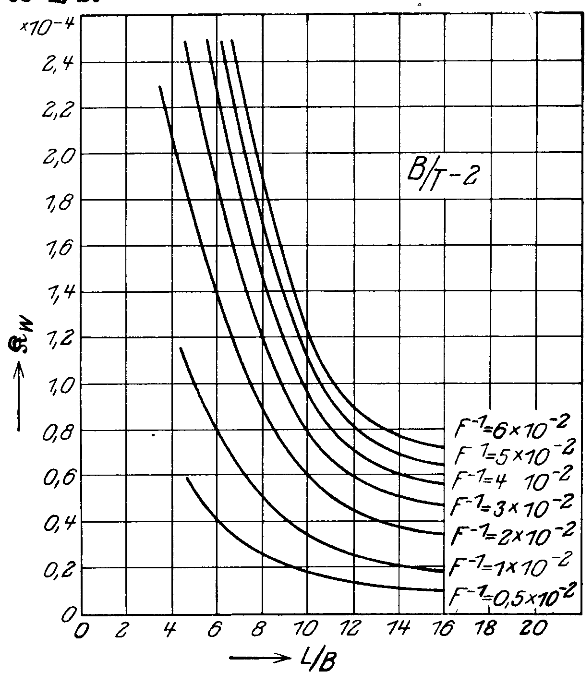


FIG. 20 COEFFICIENT OF DAMPING RESISTANCE FOR RECTANGULAR BODIES AS A FUNCTION OF L/B AND F^{-1}

The values **determined by experiments with models 8 and 9 are fairing** points of the plotted curves so that the validity of the model law is established at least within the lengths in question.

A criticism of our experiments and of the method of analysis used is furnished by comparison with the experiments of H. W. Nicholls (9).

Nicholls carried out experiments with two rectangular models. In analyzing Nicholls tests for characteristics for L/B = 15 and B/T = 2 we get for model A: $K_W = 0.1064 \times 10^{-4}$ for $F^{-1} = 0.526 \times 10^{-2}$ and for model B: $K_W = 0.0336 \times 10^{-4}$ for $F^{-1} = 0.1621 \times 10^{-2}$. The corresponding values of our experiments are found by interpolation from Table XIII and are plotted together with the values of Nicholls in Fig. 21. It is evident that the present investigations are completely confirmed by the measurements of Nicholls.

11. Experiments with bodies of ship form.

The exact determination of damping resistance for hulls of any form would require many systematic series of experiments, a task which would far surpass the scope of the work discussed. We have therefore confined ourselves to the investigation of 3 models with different degrees of fullness and constant L/B. The conversion of resistance coefficients for any F^{-1} and L/B values can be carried out on the basis of the results with rectangular shaped bodies, a method which at least would permit an estimate of the damping resistance for any desired ship form to an extent adequate for practical purposes.

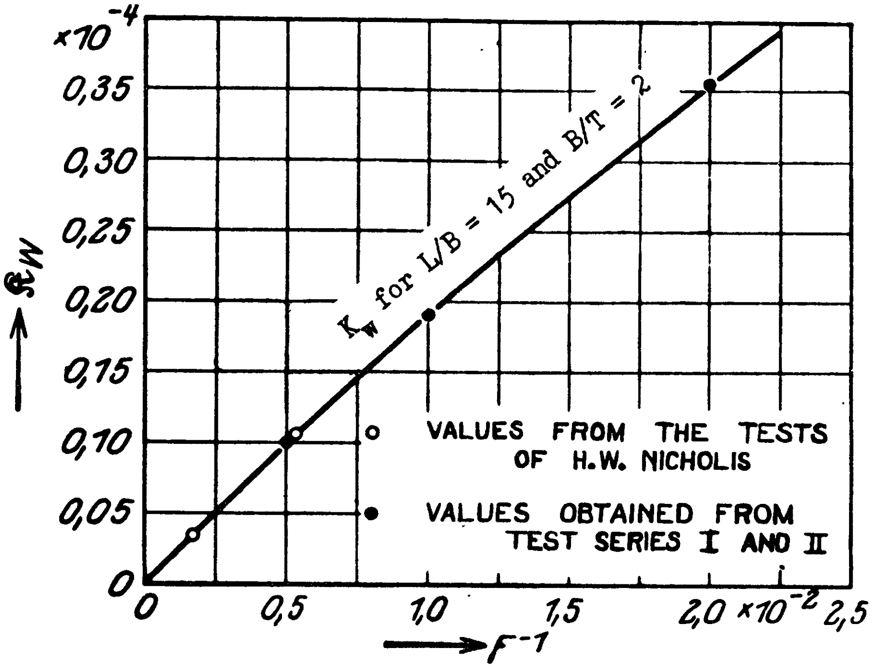


FIG. 21 COMPARISON OF COEFFICIENTS OF DAMPING RESISTANCE FOR RECTANGULAR-SHAPED HULLS WITH THE COEFFICIENTS FROM THE RESEARCHES OF H. W. NICHOLIS

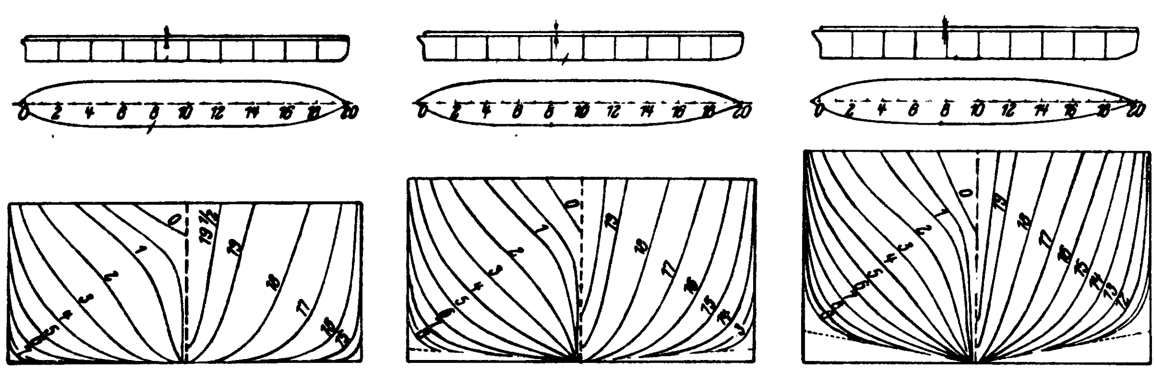


FIG. 22 MODELS FOR TESTS ON SHIP FORMS.

In Fig. 22 the models are reproduced by means of the body plans. The position of the nodal points for vibrations in the fundamental frequency and free-free condition were determined by special computation and are tabulated together with the remaining model data in Table XIV. The results are shown in Fig. 23 (see Table XV in appendix).

Table XIV. Model Dimensions – Series III

Model No.	Dimensions			Dis-place-ment cm^3	δ	L/B	B/T_c	Nodal Point Distance		y_{ϕ}/y_{vp}
	Length cm	Width cm	Draft cm					from AP	from FP	
10	150.0	21.83	9.20	22900	0.76	6.87	2.375	0.269L	0.267L	0.380
11	150.0	21.83	9.20	18975	0.63	6.87	2.375	0.280L	0.280L	0.334
12	150.0	21.83	9.20	15090	0.50	6.87	2.375	0.285L	0.283L	0.314

We see that in contrast with rectangular shapes the draft has considerable effect on the damping resistance contrary to the opinion of several English authors (Cole, Nicholls). Even when the damping surfaces (in this case the area of the momentary planes of flotation) are introduced, we find that the damping resistances increase with a higher power than one of the damping surface, in harmony with the results established with rectangular shapes.

Further Fig. 23 shows that the fullness of the ship form has a very marked influence on W. Since the three models investigated possess practically the same Froude index their K_W curves can be compared directly with one another. So for example when B/T = 2.375 for $\delta = 0.76$ K_W is 2.92 times greater than for $\delta = 0.50$. We have undertaken the conversion of measurements on three tested models for the remaining F^{-1} values in question under the assumption that the dependence of the damping resistance on F^{-1} is the same as for rectangular shapes and have represented them together with the results for rectangular bodies of equal L/B and B/T ratios in Fig. 24. The dependence of the K_W value on the ship form and the frequency of vibration is clearly set forth in Fig. 24.

In the method used for determining the damping resistance from the frequencies it is assumed that the damping resistance has the same distribution over the length of the ship as the dead weight, that is, that the position of the nodes in the prototype is the same as in the models. To establish the general effect of the position of the nodes on the magnitude of the damping resistance, tests were completed with models 10 and 11 having their nodes successively shifted amidships and towards the ends.

The K_W values obtained from these tests and converted for $F^{-1} = 1$ are plotted against y_{ϕ}/y_{vp} together with those from test series III in Fig. 25. It is evident that on shifting the model points towards the ends the damping resistance increases considerably since simultaneously the effective damping surface increases whereas on shifting the nodal points toward the middle the damping resistance

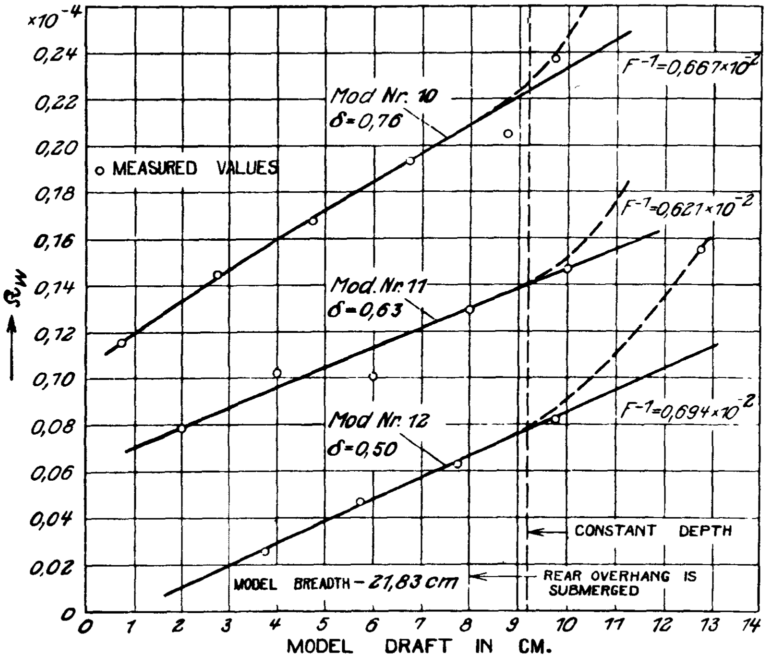


FIG. 23 COEFFICIENT OF DAMPING RESISTANCE FOR THE SHIP HULLS INVESTIGATED

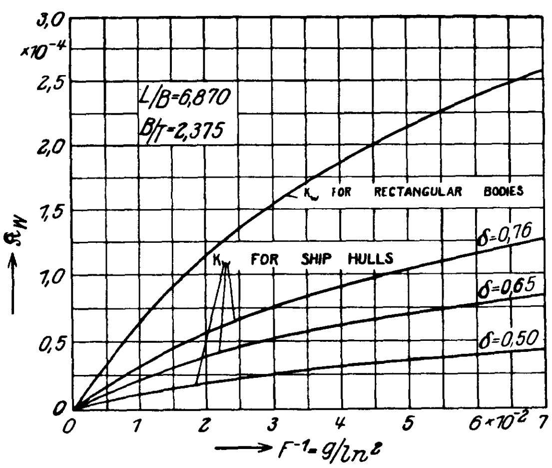


FIG. 24 COMPARISON OF COEFFICIENT OF DAMPING RESISTANCE FOR RECTANGULAR AND SHIP BODIES

increases to a lesser degree. For the position of the nodes, therefore, which corresponds to their position in a free-free bar the damping resistance is a minimum.

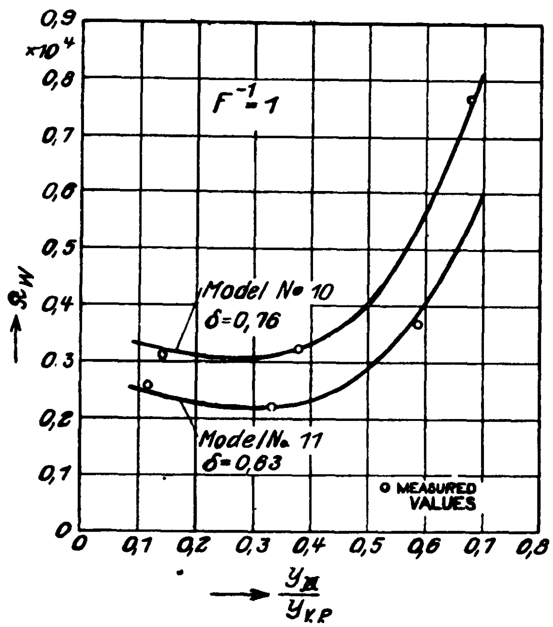


FIG. 25 DEPENDENCE OF DAMPING RESISTANCE ON THE POSITION OF NODAL POINTS.

In order to permit general use of model results the K_W values for various positions of the nodes were divided by those corresponding to the free-free position for the models under investigation. The correction curves are given in Fig. 26 with the help of which it is possible to make use of the resistance coefficients hitherto found even in those cases in which the position of the nodes does not agree with those of the models investigated.

12. Application of experimental results to the computation of damping resistances.

The indices of damping resistance obtained from model experiments strictly speaking may be applied only in the case of geometrically similar ship forms and of similar distribution of weight. However, a sufficient estimate of the damping resistance appears possible for any case whatever if the following procedure is carried out.

(a) Since to every B/T value of models 10, 11 and 12 there corresponds a definite δ (see Fig. 27) we first determine the corresponding δ value for the given B/T for the three models and take from Fig. 23 for the same B/T the corresponding K_W value.

(b) By means of the K_W values for rectangular shaped hulls of L/B = 6.87 which can be obtained from Fig. 24 the conversion to any desired F^{-1} value is undertaken. We get for the given B/T and the three corresponding degrees of fullness 3 K_W curves from which the indices for the given F^{-1} may be found and are to be plotted as a function of δ. We then find for a given δ a definite value of K_W.

(c) In order to give due consideration to the L/B ratio it is necessary to convert the last obtained K_W value by means of the K_W values for rectangular shapes (Table XIII) for the given L/B.

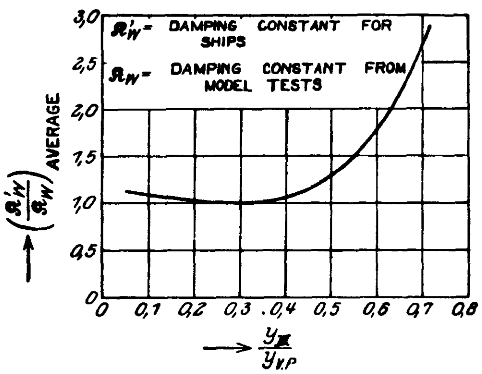


Fig. 26 Correction Curve Accounting for the influence of nodal position on magnitude of damping resistance

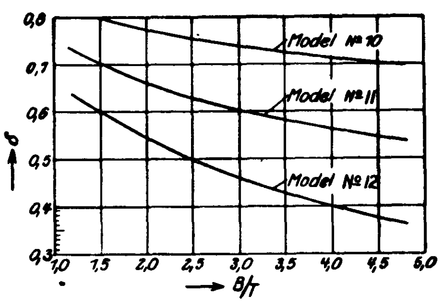


Fig. 27 Degree of Fulness as a function of B/T for ship hulls investigated

(d) In case the nodal points derived from the vibration calculation do not check with those of the model a correction according to Fig. 26 must be made.

The damping resistance determined in the foregoing manner must be introduced into the vibration calculation as an additional load corresponding to the weight distribution of the models which would involve a repetition of the entire calculation. In order to avoid this the additional deflections which arise from the damping resistance alone can be computed separately in the following manner: The effect of the weight distribution on the frequency may be expressed by the constant C' in $\omega = 2\pi C' \sqrt{EJg/PL^3}$ If we consider the vibrating system to be loaded only by the weight W corresponding to the damping resistance we obtain a deflection y_{2W} at the forward perpendicular which corresponds to the computed frequency $\omega_2 = \sqrt{9.81/y_{2v.p.}}$ under the same assumption as in the case of the actual vibration calculation ($y_{1\,v.p.}$ = 1 m; ω_1^2 = 9.81 sec.$^{-2}$).

Setting ω equal to ω_2 we get:

$$(y_{2w})_{V.P.} = \frac{0.01}{(2\pi C')^2} \frac{WL^3}{EJ}$$

For C' the corresponding model constants are to be introduced which were determined for model 10 as 4.42, model 11 as 4.59 and model 12 as 4.61. For intermediate values of δ the model constants must be interpolated. W is set down in kg, L in cm., E in kg/cm² and J in cm^4. The constant K_S = damping resistance/ships weight in equation 18 is then = $(y_{2w} / y_{2\,th})_{V.P.}$ Under y_{2th}, the end amplitude computed from the vibration calculation at the forward perpendicular without any correction is to be understood. For E is set down the same value (2.1 x 10^6) as in the theoretical vibration calculation whereas for L the length between perpendiculars is used. A numerical example can be found in the appendix.

V. Frequencies of higher order.

Since the theoretical computation of natural frequencies of higher order involves certain difficulties because of the uncertainty as to the magnitude of the damping resistances, an estimate made from similar ships is preferable for the present.

Results on the determination of natural frequencies of higher order have as yet been given little publicity. The following data may therefore contribute to filling out the deficiency.

First a survey of the influence of mass distribution is obtained from the comparison of the natural frequency of uniform rods with that for rods with pointed ends which H.W. Nicholls (9) computed. The ratio of natural frequencies of higher order to the fundamental are:

for uniform bars . . . . 1 : 2.755 : 5.400 : 8.940
for bars with pointed ends . . . . 1 : 2.260 : 3.700 : 5.700

In contrast to these, the corresponding values for ship forms vibrating in water according to measurement carried out are:

for fast steamers . . . . .1 : 2.130 : 3.365 : 4.755
for cargo and passenger ships . . . . .1 : 1.870 : 2.670
for cargo ships (according to Schlick) . . . . .1 : 1.850

From this tabulation we find that the relative positions of frequencies of higher order is strongly dependent upon the mass distribution but that also the damping resistance appears to increase with increasing number of nodes. This phenomenon probably must be attributed to the simultaneous increase of frequency with which is associated an increase in damping resistance according to the present experiments. Also the fact that for fuller ships the numerical ratios lie closer together serves to explain the increase of damping resistance with increasing fullness.

If we compare the natural frequencies especially of the fast steamer type with one another we find that the critical zones lie closer to one another the greater the number of nodes.

The coincidence of the impulse frequency with a natural critical of higher order for this reason and also because of strong damping will no longer manifest itself in the appearance of pronounced nodes and loops, but rather there must be expected a general indeterminate vibration of the hull which cannot be decisively affected even by altering the impulse frequency. Heavy and pronounced vibrations, therefore, especially in the case of fast ships, whose engine speed in general lies so high that only resonance with criticals of higher order (about 4 – 6 nodes) is to be expected, are mainly of a local nature.

Table XVI – Comparison of Computed and Measured Frequencies

No.		1	2	3	4	5
Type of Ship		Tanker of 11,000 tons	Cable Layer 481 Br. Reg. T	Torpedo-boat Destroyer	Tanker 5700 t s.D.	Freighter 5300 t s.D.
		Single Deck Double bottom	Single Deck	Single Deck	Single Deck	Double deck Double bottom
		Longitudinal frames	Transverse frames	Transverse frames	Transvers. frames	Transverse frames
Length over all		140.65 m	55.05 m	94.49 m	111.50 m	113.00 m
" between perpendic.		134.11 "	50.15 "	91.44 "	106.68 "	108.20 "
L/B		6.88	5.28	10.15	7.50	6.97
L/H		12.48	12.00	16.42	12.40	10.70
B/T		2.375	3.290	3.100	2.085	2.325
Displacement		16,900 t	846.5 t	1400 t	8280 t	8500 t
δ		0.762	0.590	0.570	0.769	0.735
Moment of Inertia Φ		36.159 m^4	0.936 m^4	1.442 m^4	14.000 m^4	15.80 m^4
Theoretical frequency		117.9 min^{-1}	299 min^{-1}	158 min^{-1}	156 min^{-1}	141.5 min^{-1}
Value for effect of elastic behavior	K_1	1.042	1.082	1.020	1.036	1.000
	K_2	1.150	1.150	1.080	1.080	1.050
	$1+K_3$	1.179	1.192	1.073	1.130	1.237
	$1+K_4$	1.050	1.050	1.027	1.047	1.063
Frequency in air		96.6 min^{-1}	239.6 min^{-1}	141 min^{-1}	135.4 min^{-1}	120.2 min^{-1}
Value for effect of damping resistance	W	6200 t	317 t	566 t	3140 t	3110 t
	W/D	0.367	0.375	0.404	0.379	0.366
	$1+K_5$	1.400	1.300	1.470	1.458	1.333
Frequency in water		81.5 min^{-1}	210 min^{-1}	118.2 min^{-1}	112 min^{-1}	104.1 min^{-1}
Measured frequency		81 "	207.5 "	120 "	112 "	105 "
Frequency Deviation		+.05%	+1%	1.5%	0%	−0.9%
$c = N_{meas} / \sqrt{J/DL^3}$		2.922×10^6	2.550×10^6	3.435×10^6	3.210×10^6	2.925×10^6
$C_{Schlick}$		2.802 "	———	3.437 "	2.802 "	2.802 "
" Deviation		−4%	———	0%	−14.5%	−4.5%

VI Comparison of measured results on a full sized ship with computed results.

On the basis of the previous investigations the natural frequencies of the vertical vibrations of the first class which might be expected in propulsion, were computed for five different types of ships, and compared with the values measured on board. A summary of the measured and computed values is given in Table XVI. Several observations and explanations of the results are here appended.

1. Tank motor ship of 11,000 tons light displacement.

The measurements were carried out with a displacement of 15,300 tons although computations were made on a basis of 16,900 t. Fig. 28 shows the calculated and measured vibration curves. The agreement between the two curves, considering the difference in displacement, appears to be satisfactory.

In Fig. 29 the measured amplitudes for stations I and III are plotted against propeller revolutions. Vibration becomes noticeable at 77 RPM, increases to a maximum at 84 RPM and disappears at 90 RPM. The total range amounts to about 8 per cent below and 7 per cent above the natural frequency.

From the sharp rise and fall of the resonance curve we can conclude that the internal damping is only slight. Therefore there can be no question in practice of an influence of internal damping on the frequency. Comparison with the resonance curve determined for the box girder shows in addition that internal damping is dependent in large measure upon the frequency since the range of resonance in the case of the box girder investigated is considerably greater when $N_{cr.}$ = 3250 $min.^{-1}$

From the measured maximum amplitude we get moreover an inference as to the bending stresses produced during resonant vibration. Since from the vibration calculation (Fig. 4) it is known that for a maximum bending moment of 42,250 mt there must be an end amplitude of 0.06445 m we find for the measured end amplitude of 6 mm the corresponding maximum moment of 3950 mt. The moment of resistance (I/c) amounts to 6.10 m^3, so that $\sigma_{max.}$ = about 65 kg/cm^2. Even though the absolute value of the stress is found to be insignificant it may, as a rapidly varying superimposed stress which generally represents an increase of about 7 to 10 per cent of the total stress, lead to fatigue phenomena at especially critical places.

In order to be able to compare the measured frequency of 84 $min.^{-1}$ measured at 15,300 t with that computed at 16,900 t a conversion to the calculated displacement is necessary. Under the assumption that the natural frequency varies as the square root of the displacement we get N = 80. Since, however, it is known from measurements obtained elsewhere that the natural frequency decreases at a somewhat smaller ratio we shall have to regard N = 81 as corresponding to a displacement of 16,900 t.

The natural frequency determined from the vibration calculation amounts to 117.9. According to the factors given in Table XVI for the elastic behavior, the

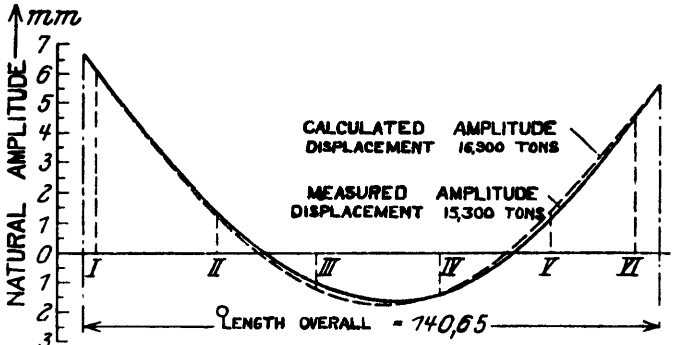


FIG. 28 MOTOR TANKER OF 11,000 TONS D.W.
COMPARISON OF MEASURED AND CALCULATED VIBRATION CURVES

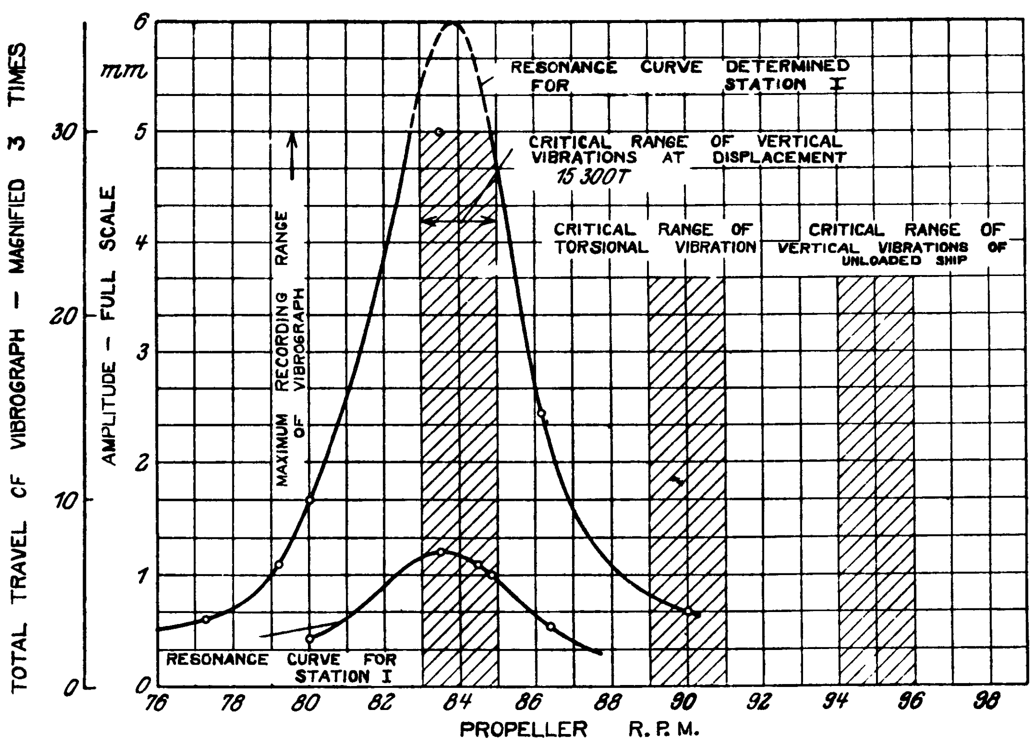


FIG. 29 MOTOR-DRIVEN TANKER OF 11,000 TON DISPLACEMENT
RESONANCE

theoretical natural frequency decreases to 96.6 = 82% of N_{th}. With K_5 = 0.4 we get N = 81.5 = 69% of N_{th}. The deviation from the corresponding measured value N = 81 is negligible.

2. Cable layer of 481 Br.-Reg. T.

For this ship vibrograph measurements were not undertaken. However, according to the report of the ship's officer the fundamental natural frequency lies between the limits N = 205 and 210. The elastic behavior lowers the theoretical frequency from 299 to 239.6 = 80.3%. Considering the damping resistance we get N = 210 = 70% of N_{th}.

3. Torpedo boat destroyer of 1400 tons displacement.

The principle dimensions given in Table XVI are taken from the article of H.W. Nicholls (9) who computed the theoretical fundamental natural frequency as N_{th} = 135 min.$^{-1}$ with E = 1.575 x 10^6 kg/cm². In carrying out the calculation according to Tobin's method only the first end condition was considered by Nicholl's. As pointed out we get too low natural frequencies unless both end conditions are strictly regarded. Therefore to compensate for the non-observance of the second condition the theoretical natural frequency must be assumed higher and was set at 137 min.$^{-1}$ in the calculation. If we convert this value to our basis of comparison with E = 2.1 x 10^6 we get $N_{th} = 137\sqrt{2.1/1.575} = 158$. By the influence of the elastic behavior this value is lowered to 143.3 = 89% of N_{th} and because of the damping resistance (see appendix) still further to 118.2 = 75% of N_{th}.

The observed natural frequency given in the discussion of the foregoing paper amounted to 120 min.$^{-1}$ More detailed data as to whether the displacement used in this study agrees with that in the calculation are not given.

4. Tanker of 5800 tons displacement.

The principal measurements are taken from the papers of A. P. Cole (7). Cole determined the natural frequency by computation to be 112 min.$^{-1}$ Here it is assumed that E = 1.81 x 10^6 kg/cm², y_s/y_b = 0.16 and the damping resistance distributed approximately in parabolic form amounts to 3590 tons. With E = 2.1 x 10^6 and discarding the remaining assumptions we get:

$$N_{th} = 112\sqrt{\frac{2.1}{1.81} \times 1.16\left(1 + \frac{3590}{8151}\right)} = 156 \text{ min}^{-1}$$

Upon introducing the factors K_1 to K_4 this value falls to 135.4 = 86.6% of N_{th} and with K_5 = 0.458 we get N = 112 = 72% of N_{th}.

In the discussion of the papers of T.C. Tobin the measured natural frequency for the fully loaded condition is given by P. A. Hillhouse as 112 min.$^{-1}$ This agreement with the computed value is perfect.

5. Freight motor ship of about 5300 tons displacement.

The displacement in the fully loaded condition amounted to 9,302 tons at a draft of 7.22 m. The vibration calculation is based on a displacement of 8500 tons according to the data of F. Horn (34). The corresponding theoretical frequency is given by Horn as 138 where E was taken as 2 x 10^6 kg/cm² and in which, in the calculation of the moment of inertia, the deck plating between hatches is neglected. Converted to E = 2.1 x 10^6, we get N = 141.5. The factor K_1 designating the effective width was set to equal 1 accepting the given computation of the moment of inertia. $K_2 = y_w/y_{th}$ was determined for a ratio of load to capacity = 4500/5300 = 0.85. The shear deflection determined from Fig. 12 and Table III is considerable for the two deck type with double bottom (y_s/y_b = 1.625 x 0.146 = 0.237). With the constants given in Table XVI the frequency in air = 120.2 and with K_σ = 0.333 that in water = 104.1 min.$^{-1}$ The frequency measured with a displacement of 8500 tons according to the data of F. Horn was N = 105.

In regard to the K factors it should in general be noted that the factor K_2 which accounts for the effect of the complex nature of the ship's members will generally be that factor which must be estimated from Fig. 15 with special care. So for the tanker with longitudinal frames (No. 1) we have chosen K_2 = 1.15 since it is known that at the interruptions of the longitudinals by the transverse bulkheads a weakening of the longitudinal frames is produced (the longitudinals were fully included in the calculation). For the cable layer (No. 2) the same K_2 value was used since this ship is very considerably reenforced in its longitudinal structure above classification requirements and therefore its behavior with regard to deflection will be similar to that of a ship not fully loaded whose members are not under high stress and therefore are not fully utilized. In the remaining ships with transverse frames we have taken the mean values given in Fig. 15 (K_2 = 1.08 – 1.05). However, the possible error in the computation of natural frequency when the factor K_2 is carefully estimated is in all cases slight.

The final comparison of computation and experiment gives a mean error of –0.18% with limits of plus 0.5% and –1.5%. On the other hand comparison of the C values according to Schlick with those determined by measurement gives a mean deviation of –4.6% with limits of –14.5% and 0%, a proof that in the case of special types of ships Schlick's estimated values are to be replaced by more exact methods.

Of further interest is the part played by the individual factors which tend to lower the theoretical frequency. In the five cases investigated, the frequency in water amounted to 69 – 75% of the theoretical. The difference, therefore, amounts to 25 – 31%. This is made up of from 11 – 18% on the basis of elastic behavior and from 13 – 14% on the basis of damping resistance. Damping resistance and elastic behavior, therefore, lower the theoretical natural frequency according to estimate approximately by the same amount. The damping resistance fluctuates approximately between 30 and 45% of the displacement, that is within relatively narrow limits.

The papers of F. M. Lewis (18) and J. L. Taylor (19) which were published only after the conclusion of the present investigation, on the other hand, report a damping resistance in a distribution similar to that in the case investigated of more than 100% of the displacement in certain instances. These data are not confirmed by our experiments; even though a different distribution of damping resistance may have considerable influence on its absolute magnitude the resulting difference is still inexplicable. It is worthy of note that the dependence of the damping resistance upon the frequency was not considered by the authors mentioned. The satisfactory agreement between computation and measurement reported by J. L. Taylor, moreover, probably must be attributed to an underestimate of the elastic behavior in favor of damping resistance.

VII Summary.

An attempt is made to give a numerical explanation of the difference between calculated and observed natural frequencies for two noded vertical vibration a problem as yet not cleared up in detail on the basis of theoretical and experimental research. The theoretical computation of the natural frequency of a ship hull on the basis of a non-uniform beam is simplified by a readily executed method which exactly fulfills the end conditions. The factors which embody the influence of the effective width, the shear deflection, the complex nature of the ship's structure, and the damping resistance are investigated. As a result of this investigation, they can be derived for any case of computation from dimensionless diagrams.

As a result of the good agreement shown in five cases between computations and measurements on board, the investigations carried out probably will form a suitable basis for computing the vertical natural frequency of ship hulls as they are to be expected under operating conditions.

In conclusion it is the pleasant duty of the author to express his sincerest appreciation to the management of Deutsche Schiffs-und Maschinenbau-Aktiensgesellschaft for their constant cooperation, through whose especial efforts it was possible for the author to carry out investigations on a large scale. Further the author wishes to thank Prof. F. Horn, of the Technische Hochschule of Berlin for several reports of test results obligingly given concerning the results and data on cargo ships, as well as Mr. C. Cole, for valuable aid in the comprehensive graphic and numerical evaluations.

VIII Appendix.

1. Design of the model (for model dimensions see Table I as well as Figs. 8 and 9).

For determining the weight scale of the model we proceed from the point at which the largest moment of inertia corresponds to the smallest weight, since the weight of the plate serves as the basis of weight. In the case of the cable layer this point lies between frames 51 and 52 (see Fig. 8) for which the weight of the ship amounts to 4,180 tons and the weight of the model, with a plate thickness of 2 cm. and a plate width of 14.20 cm., assumed at this point amounts to 0.258 kg. Accordingly 1 kg. for the model corresponds to 16.21 tons for the ship and one ton for the ship corresponds to .0617 kg. for the model. Since the width of plate is fixed by the assumption of maximum width and the thickness by the moment of inertia curve it follows that its weight is known for each individual section. The difference between the plate weight and the weight per frame space according to the weight curve gives therefore the required weight of lead.

The calculation of the natural frequency of the model is obtained most simply from the determination of the form constant $C = N/\sqrt{J/GL^3}$ for the ship and transferring the same to the model. For the cable layer with $E = 2.1 \times 10^6$ kg/cm², the theoretical natural frequency amounted to N = 299. With $J = 0.945$ m⁴, G = 846.5 t, $L_{o.a.} = 55.05$ m, C is therefore 3,650,000, and the model natural frequency with $i = 9.48$ cm⁴, g = 52 kg, l = 110.10 cm: $n = C\sqrt{i/gl^3} = 4268$ min^{-1} = 71.1 sec.$^{-1}$ In a similar way we get the model natural frequency for the dredge model. It is self-evident that the moment of inertia to be introduced into the formulas both in the prototype and in the model is always to be measured at the same position. For the dredge model the relative moment of inertia of the ship amounted to 2.96 m⁴. The remaining data are given in Table I.

2. Computation of the damping resistance.

Torpedo boat destroyer of 1400 t displacement
(91.44 x 9 x 2.9 m draft)

Frequency in air according to Table XVI 143.3 min.$^{-1}$ = 2,390 sec.$^{-1}$

Vibration velocity factor $F^{-1} = g/ln^2 = 1.880 \times 10^{-2}$

$L/B = 10.15$; $B/T = 3.10$: $\delta = 0.57$

For B/T = 3.10 we get from Fig. 27 the following δ and F^{-1} values of the model under investigation (L/B = 6.87)

F^{-1}	$= 0.694 \times 10^{-2}$	0.621×10^{-2}	0.667×10^{-2}
δ	$= 0.448$	0.591	0.736

From Fig. 23 we find the accompanying damping coefficients for the corresponding model draft of 21.83/3.10 = 7.05 cm as

F^{-1}	$= 0.694 \times 10^{-2}$	0.621×10^{-2}	0.667×10^{-2}
K_w	$= 0.057 \times 10^{-4}$	0.121×10^{-4}	0.197×10^{-4}

The K_W values for any F^{-1} value are given in the following table:

F^{-1}	For Rect. Hulls L/B = 6.87 See Fig. 24	$\delta = 0.736$	$\delta = 0.591$	$\delta = 0.448$
0.500×10^{-2}	0.350×10^{-4}	0.1498×10^{-4}	0.0985×10^{-4}	0.0422×10^{-4}
0.621×10^{-2}	0.430×10^{-4}	–	0.1210×10^{-4}	–
0.667×10^{-2}	0.460×10^{-4}	0.1970×10^{-4}	–	-
0.694×10^{-2}	0.473×10^{-4}	–	–	0.0570×10^{-4}
1.000×10^{-2}	0.654×10^{-4}	0.2800×10^{-4}	0.1840×10^{-4}	0.0788×10^{-4}
2.000×10^{-2}	1.153×10^{-4}	0.4940×10^{-4}	0.3245×10^{-4}	0.1390×10^{-4}
3.000×10^{-2}	1.550×10^{-4}	0.6640×10^{-4}	0.4361×10^{-4}	0.1868×10^{-4}

By interpolation we find for $F^{-1} = 1{,}880 \times 10^{-2}$

$\delta = 0.736$ 0.591   0.448

$K_W = 0.465 \times 10^{-4}$ 0.310×10^{-4} 0.130×10^{-4}

and for $\delta = 0.57$ $K_W = 0.285 \times 10^{-4}$

For rectangular shaped bodies $F^{-1} = 1.880 \times 10^{-2}$

L/B = 6.87   10.15

$K_W = 1.13 \times 10^{-4}$ 0.552×10^{-4}

Therefore in the present case

$$K_W = \frac{0.285 \times 10^{-4} \times 0.552}{1.13} = 0.139 \times 10^{-4}$$

and the damping resistance

$$W = K_W l^4 n^2 \varrho = 0.139 \times 10^{-4} \times 91.44^4 \times 2.390^2 \times 102 = 566 \text{ t}$$

The corresponding model constant for the distribution of W is $C' = 4.61$.

For determining the constants K_δ the deflection must first be computed which corresponds to $N_{th} = 158 \text{ min}^{-1}$ with $\omega_1^2 = 9.81 \text{ sec}^{-2}$ and $y_{1_{V.P.}} = 1$ m

We find

$$(Y_{2_{th}})_{V.P.} = \frac{9.81}{\left(2\pi \cdot \frac{158}{60}\right)^2} = 0.0360 \text{ m}$$

Under the same assumption for ω_1 and $Y_{1\,V.P.}$ we get the additional deflection resulting from the damping resistance as

$$(Y_{2_W})_{V.P.} = \frac{0.01}{(2\pi C')^2} \times \frac{WL^3}{EJ} = \frac{0.01}{840} \times \frac{566 \times 91.44^3 \times 10}{2.10 \times 10^6 \times 1.442} = 0.0169 \text{ m}$$

Therefore:

$$K_\delta = \frac{0.0169}{0.0360} = 0.470$$

Table VIII. Measured Results of Test Series I with Rectangular Shaped Hulls.

L x B = 1500 x 200 m/m
L/B = 7.5
$y_{x}/y_{v.p.}$ = 0.608

Model No.	Model wt. kg.	I = before immersion; II = after	n_L = frequency per sec. in air: Amplitudes 1 (Mean of 4 Values)	2	3	4	Mean	Mean of I and II	Draft cm.	B/T	n_W = frequency per sec. water: Amplitudes 1 (Mean of 4 Values)	2	3	4	Mean	$\frac{n_L^2}{n_W^2} - 1$	W kg.	$K_W = W/l^4 h^2 g \times 10^4$	F^{-1} = g/ln²
									0.5	40	10.10	11.50	10.75	9.35	10.42	0.562	9.44	1.080	
1		I	13.48	12.78	12.35	12.05	12.67		2.5	8	9.56	10.06	9.46	10.00	9.95	0.712	11.96	1.368	
	16.40							13 02	4.5	4.45	9 50	9.51	9.19	8.41	9.16	—	—	—	3.85 x 10⁻²
		II	14.78	13.84	12.52	12.39	13.37		6.5	3.08	9 82	9.40	9.46	8.87	9.73	0.790	13.28	1.518	
									8.5	2.35	9 40	9.50	9.14	10.17	9.67	0.812	13.64	1.560	
									0.5	40	18.57	17.80	17.60	17.40	17.84	0.902	16.15	0.518	
2	17.90	I	24.54	23.47	23.19	22.75	23.50		2.5	8	18.30	17.04	17.20	16.99	16.85	1.030	18.43	0.592	1.081 x 10⁻²
								24.60	4 5	4.45	18.22	17 00	16.50	—	17.24	1.038	18 60	0.596	
		II	27.04	25.52	24.56	—	25.70		6.5	3.08	18.22	17.75	17.11	16.28	17.12	1.065	19.05	0.610	
									8.5	2.35	17.92	17.24	16.85	16.36	7.08	1.070	19.15	0.614	
									0 5	40	26.30	25.67	25.31	25.60	25.75	0.905	17.72	0.260	
3		I	36.00	—	34.90	—	35.45		2.5	8	26.91	25.96	24.97	24.56	25.60	0.925	18.00	0.267	0.519 x 10⁻²
	19.50							35.50	4.5	4.45	25.60	24.56	24 21	24.22	24.65	1.075	21.00	0.315	
		II	37.40	35.70	34.90	33.85	35.50		6.5	3.08	24.92	24.86	24.65	23.82	24.60	1.085	21.20	0.314	
									8.5	2.35	25 51	24.75	24 21	23.71	24.55	1.090	21.30	0.315	
									0.5	40	34.55	34.90	34 45	33.90	34.50	0.400	15.35	0.1785	
4		I	41.80	40.70	—	40.30	40.93		2.5	8	33 00	32.70	33.40	33.55	33.25	0.510	19.55	0.227	0.3925 x 10⁻²
	38.35							40.80	4.5	4.45	32.20	32.15	32.60	32 67	32.50	0.578	22.18	0.258	
		II	—	40.85	40.70	40.60	40.67		6.5	3.08	32.70	32.50	32.50	32.60	32.60	0.568	21.80	0.254	
									8.5	2.35	32.60	32.45	32.25	32.40	32.45	0.578	21.18	0.246	

TABLE XI. Measured Results of Test Series II with Rectangular Shaped Hulls.

Model measurements from Table X – $x/y_{v.p.}$ = 0.608

Model No.	Model wt. kg.	I = before immersion, II = after	n_L Amplitudes 1	2	3	4	Mean	Mean of I and II	Draft cm.	B/T	n_w Amplitudes 1	2	3	4	Mean	$\frac{n_L^2}{n_w^2} - 1$	W Kg.	$K_W = W/\frac{1}{4}h^2\rho \times 10^4$	F^{-1} = g/In²
			Mean of 4 Values								Mean of 4 Values								
6									0.5	20.00	30.60	30.55	29.75	29.55	30.12	0.134	4.42	0.0831	
L/B = 15	33.00	I	33.30	32.30	31.60	31.15	32.10		2.5	4.00	30.40	29.10	29.00	28.85	29.32	0.197	6.50	0.1221	0.636 x 10^{-2}
								32.09	4.5	2.22		29.96	29.69	28.83	29.50	0.186	6.14	0.1153	
		II	33.56	32.27	31.24	31.15	32.08		6.5	1.54	29.68	29.11	29.18	28.62	29.15	0.211	6.95	0.1305	
									8.5	1.18	27.50	28.78	29.15	29.12	28.65	0.254	8.38	0.1575	
7									0.66	45.00	22.75	22.67	22.22	—	22.55	1.030	36.80	0.690	
L/B = 5	35.70	I	32.96	32.55	31.75	31.89	32.28		2.66	11.30	22.39	22.25	21.92	21.82	22.10	1.110	39.60	0.743	0.634 x 10^{-2}
								32.12	4.66	6.43	22.32	22.15	21.96	21.81	22.08	1.110	39.60	0.743	
		II	32.52	32.00	31.72	31.50	31.95		5.66	5.30	21.96	22.50	22.35	21.70	22.11	1.110	39.60	0.743	
									—	—	—	—	—	—	—	—	—	—	
8									0.5	40.00	25.75	25.62	25.00	24.52	25.22	0.438	5.48	0.895	
L/B = 4.5	12.50	I	30.99	30.00	30.02	29.80	30.20		2.5	8.00	25.69	25.05	24.64	24.10	24.85	0.480	6.00	0.980	1.192 x 10^{-2}
								30.22	4.5	4.45	25.69	25.05	24.50	24.00	24.81	0.485	6.06	0.990	
		II	30.75	30.45	30.06	29.80	30.24		6.5	3.08	25.15	24.55	24.02	23.71	24.40	0.536	6.70	1.095	
									8 5	2 35	25.35	24.76	24.02	23.60	24.42	0.535	6.70	1.095	
9									0.5	40 00	23.25	23.60	23.20	23.01	23.30	0.245	21.62	0.161	
L/B = 10.5	88.30	I	25.80	26.06	25.95	25.85	25.90		2.5	8.00	22.60	22.52	22.76	22.50	22.60	0.322	28.41	0.212	0.691 x 10^{-2}
								26.00	4.5	4.45	22.58	22.48	22.36	22.28	22.45	0.341	30.10	0.225	
		II	26.20	26.10	26.10	26.06	26.10		6.5	3.08	22.51	22.45	22.50	22.22	22.41	0.345	30.45	0.227	
									8.5	2.35	22.62	22.50	22.42	22.40	22.49	0.338	29.85	0.223	

Table XV. Measured Results of Test Series III with Ship Shaped Hulls.

Model Measurements from Table XIV $L/B = 6.87$ $B/T_c = 2.375$

Model No.	Model Wt. kg.	I = before immersion, II = after	n_L = Frequency per sec. in air: Amplitudes 1 (Mean of 4 Values)	2	3	4	Mean	Mean of I and II	Draft cm.	T/T_c	n_w = Frequency per sec. water: Amplitudes 1 (Mean of 4 Values)	2	3	4	Mean	$\frac{n_L^2}{n_w^2} - 1$	W Kg.	$K_W = W/\ell^4 n_s^2 \times 10^4$	$f^{-1} = g/\ell n^2$
									0.75	0.081	27.39	28.10	26.79	26.40	27.20	0.324	5.84	0.1153	
		I	32.36	31.52	31.01	30.15	31.25		2.75	0.299	26.50	26.68	25.75	25.50	26.40	0.405	7.30	0.1441	0.667
10	18.00								4.75	0.516	26.30	26.15	25.45	25.40	25.80	0.472	8.50	0.1678	x
δ = 0.76								31.32	6.75	0.734	—	25.60	25.35	24.75	25.20	0.543	9.78	0.1930	10^{-2}
		II	31.72	32.00	31.50	30.35	31.39		8.75	0.951	26.12	25.10	24.20	24.40	24.95	0.575	10.35	0.2045	
									9.75	1.059	24.80	24.40	24.20	23.66	24.26	0.664	11.95	0.2360	
									2.00	0.217	30.04	29.30	28.70	28.08	29.05	0.248	4.282	0.0787	
		I	33.43	32.50	32.18	31.78	32.50		4.00	0.435	29.01	28.45	28.08	27.38	28.22	0.322	5.570	0.1023	0.621
11	17.30								6.00	0.652	29.30	28.28	28.00	27.45	28.28	0.316	5.465	0.1004	x
δ = 0.63								32.47	8.00	0.870	28.35	27.60	27.12	26.36	27.36	0.406	7.020	0.1290	10^{-2}
		II	33.42	32.52	32.24	31.50	32.45		10.00	1.086	28.00	26.98	26.42	25.85	26.82	0.464	8.030	0.1475	
									11.00	[illegible]	27.50	[illegible]	[illegible]	25.88	26.50	0.499	8.640	0.1587	
									3.75	0.408	30.40	29.85	29.55	28.90	29.70	0.070	1.252	0.0257	
		I	31.80	31.05	30.50	30.00	30.82		5.75	0.625	29.70	29.05	28.40	28.60	28.95	0.126	2.256	0.0463	0.694
12	17.90								7.75	0.842	29.30	28.82	28.10	27.95	28.55	0.169	3.050	0.0625	x
δ = 0.50								30.71	9.75	1.059	28.40	28.30	27.65	26.80	27.80	0.221	3.960	0.0814	10^{-2}
		II	30.85	31.10	30.40	30.10	30.60		11.75	1.277	26.66	26.50	25.70	25.60	26.10	0.384	6.875*	0.1413	
									12.75	1.386	26.50	25.85	25.66	25.08	25.80	0.419	7.500*	0.1541	

* Aft overhang begins at 8 cm draft

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TRANSLATION

Discussion of Paper entitled "Über Rechnung und Messung der elastischen Eigenschwingungen von Schiffskörpern"

By Dr. Ing. E. Schadlofsky

Schiffbautechnische Gesellschaft
Vol. 33, 1932.

U.S. Experimental Model Basin
Navy Yard, Washington, D.C.
November, 1935. Supplement to Report No. 382.

Prof. Dr.-Ing. Horn, Berlin

Since in latter years the problem of the most exact possible preliminary calculation of the elastic vibrations of ships has found increasing attention and especially in England and America a group of very capable scientists and engineers has undertaken this task with visible results, it is exceedingly gratifying that at this time there has also been found in Germany a man who has undertaken to give the especially important topic of vertical flexural vibration a thorough in estigation. In doing so he adopts the correct and only means possible for such an exceedingly complex problem of a synthesis of theory and experiment, and we can only be grateful to the author for the abundance of important and new material which his fine experiments were chiefly instrumental in supplying and which the interested experts certainly will use to great advantage.

The author probably understands better than anyone else that this synthesis of theory and experiment is not altogether flawless and in many respects had to be replaced by a sort of working hypothesis. Insofar as such an hypothesis is not based on false premises we can use it freely and in case of necessity which may arise on account of the effect of elastic behavior it will be possible to extend and improve it. I have however certain fundamental objections to an important point in the working hypothesis adopted by the author. I have in mind his statements about "damping resistance." In principle this expression, "damping resistance," must first be characterised as incorrect and misleading in view of the concept the author claims actually to have formed regarding this resistance. In all vibration phenomena we understand by damping a process in which energy of vibration is lost, whether this be converted into heat-, wave-, or some other form of energy. Damping always causes a decrease of the amplitude of free vibration with time but in addition as a rule a very slight decrease in the frequency. The author, however, does not have in mind such a process; he has in mind expressly the inertia resistance which is produced by the inertia force of the water vibrating with the hull and whose effect amounts to an increase in the virtual mass of the hull. It would not be a serious matter if only the child were called by the wrong name, but my objection is not merely a matter of form. Herr Schadlofsky discredits the two methods which have become well known in recent years, namely Taylor's and Lewis' which should give this inertia resistance and which from my point of view indicate a decided advance in this direction; in any case he denies their practical significance and expresses the opinion that they lead to essentially incorrect results. He bases his objection on the actually noteworthy fact that in his experiments with rectangular hulls the coefficient of water resistance, other conditions being the same, were found to be quite dependent on the frequency whereas the methods of Lewis and Taylor stipulated that the inertia resistance is independent of frequency. To my mind, this is not in order and requires explanation. Either the additional resistance arising during vibration in water as compared with vibration in air is entirely or predominantly inertia resistance, in which case the coefficient of this additional resistance would not be materially dependent upon the velocity and in the results of the experiments with rectangular hulls at various frequencies there would have to be some erroneous conclusion, and a calculation carried out according to Lewis and Taylor could not lead to erroneous results, or if the results of experiments with rectangular hulls are reliable and contain no erroneous conclusions, it would indicate that the additional resistance is not a pure inertia resistance but contains a very considerable proportion of actual damping resistance.

I do not profess to be able to decide this question offhand; I can only offer certain conjectures and propose a method for clearing it up. Beforehand, I wish to remark that in several vibration computations which I recently made on my own initiative, Lewis' method appeared to give correct results, and for simple types of ships for which one can safely compute by Schlick's method gave quantitatively satisfactory agreement with the Schlick computation. In these calculations the effect of shear deflection was allowed for according to Taylor; further corrections for elastic behavior had not been made. The good agreement in these calculations naturally is not definite proof. To permit appraisal of the state of affairs, I wish to make the following remarks:

Should strong dependence of the coefficient of water resistance on the vibration frequency actually exist then there would have to be a very strong actual damping. For the instantaneous motion of the water produced by the rapid vibrations we may assume, however, that it follows approximately the laws of flow of potential, and that therefore frictional and extraneous resistances play no significant role. On the other hand, a damping can occur theoretically as a result of the free surface, since then energy of vibration is changed into wave energy— it may be stated here that only this circumstance would justify the application of Froude's law of similarity used by Schadlofsky; if it were a case of pure inertia resistance this law of similarity would be of no value. Apparently it may now be determined by a simple experiment whether the damping is actually sufficient to explain such a close dependence of the coefficient of the resistance on the velocity. It would only be necessary to determine the decay of amplitude of free vibration in water. From the so-called logarithmic decrement we can then conclude as to the magnitude of the damping resistance in the familiar manner.

I am inclined to surmise that such an experiment would have negative results, that is, that the degree of damping thus obtained, at least for the greater drafts, will not be nearly sufficient to explain the close dependence of the value of resistance on the velocity. Then there must be some error in the test data of Schadlofsky at this point. I would, for example, suggest investigating whether in the experiments with rectangular hulls the nodal points were not displaced outwards in water as compared with air. Such an investigation has been carried out only with ship forms and not with rectangular hulls; however, it was in the case of the latter that the experiments for determining the dependence of the damping resistance on the frequency were made. Any possible displacement of the nodal points outward as has been proved results in a considerable change in the frequency in water that is to say in the sense of a lessening of the discrepancy now existing. Another possibility which might be followed up is that the increase in the coefficient of resistance in the case of low frequencies is in some manner connected with the limited dimensions of the tank.

Whether the foregoing suggestions will clear things up or not, I consider it important to go thoroughly into this question because it has considerable practical value. If the methods of Lewis and Taylor are confirmed in practice, then certain elastic effects have been overestimated by the author and must be corrected.

In any case much praise is due the author for having supplied a quantity of positive and valuable material for solving the most complicated problem of elastic vibration and thus having brought nearer to a solution the task of predetermining the natural frequency with sufficient accuracy for practical purposes.

Chief Naval Constructor Burkhardt- Berlin:

I would like to amplify the unusually valuable work of the author with some data on vibration measurements on new ships of the German navy. Whereas formerly vibration measurements could not be made before the actual trial tests, the Spaeth-Losenhausen system vibration machine renders it possible to obtain the desired information in advance.

The first such experiment was carried out on the artillery training ship "Bremse" before the engines had been installed. The tests are to be repeated later after completion of the engine installation.

In Fig. 1 are shown the experimental results. The vibration machine was installed on one of the main engine foundations approximately amidships. The amplitudes were measured at ten stations with suitable vibrographs but chiefly with a three-component seismograph of the Askania Works. The vibration machine covered frequencies from 2 to 10 Hertz. As you see from the diagrams, not only can the deflection be definitely determined for the fundamental mode of vibration, but also for the second and third harmonics.

When the frequency was raised to 30 Hertz the nodal points could no longer be satisfactorily determined; it was possible, however, to determine the natural frequency up to the seventh harmonic. The results are given in the upper left hand corner where the natural frequencies are expressed in logarithmic coordinates.

Of especial interest is the rectilinear course of the curve from the third harmonic on. From these curves it is evident that at the higher natural frequencies the ship exhibits the same law of proportionality as a homogeneous elastic beam.

A few weeks ago the same tests were carried out on the cruiser "Leipzig." In this case the ship was fully constructed but without fuel and equipment.

In this case it was possible to determine the natural frequencies of the first and second harmonics (1.3 and 2.1 Hertz) but not the forms of flexure because the capacity of the vibration machine is not large enough to give sufficient amplitude to the larger ships at these low frequencies. On the other hand, it was possible to measure the form of flexural vibrations for the third, fourth, and fifth order vibrations satisfactorily.

It was possible to determine the higher natural frequencies even up to the thirteenth order as you see in Fig. 3. Here again is evident the rectilinear course of the curve for the higher natural frequencies from the seventh harmonic on.

Besides the vertical vibrations the horizontal vibrations were also determined for the cruiser. These results may be seen in Fig. 4. As was to be expected, the horizontal vibrations show loops where the vertical vibrations show nodes and vice versa. The planes of the frames must therefore execute a rocking motion about a transverse axis which was also confirmed by measurements on the upper deck and inner bottom. In the lower part of Fig. 4 is shown a measurement for one frame. As you see the location of the neutral axis can therefore be determined.

Thus, I have indicated in what way such measurements may be profitable. Naturally it has been impossible to conclude the analyses in the short time since the experiments were undertaken. I thought, however, that in connection with the author's statements it was not fitting to withhold from you the results of these measurements carried out for the first time with such precision. The further development of such experiments and the conclusions to be derived from them will have to be undertaken along the following lines:

1. Determination of vibration relationships
   (elimination of disturbing engine speeds, choice of location for setting up instruments on war ships)
2. Determination of important static relationships by vibration tests
   (determination of effective moment of inertia and the neutral axis of the hull; dependence of the moment of inertia on the amplitude; as well as the effect of various stiffeners on the pressure zone.)
3. Observation of structural conditions of ships by determination of the natural frequencies.

The experiments described were carried out at the Wilhelmshaven Yard with the cooperation of the mechanical division of the Heinrich Hertz Institute for vibration research.

Prof. Dr. Ing. Schnadel- Berlin:

As the hour is late, I will be as brief as possible. The address which represents a monumental piece of work has evoked general interest as one can see from participation in the discussion. I am joining in the discussion in an effort to eradicate certain obscurities or errors.

In the first place, I might point out as did Prof Horn that I cannot agree with the authors opinion of the work of Lewis and Taylor. On the contrary, these investigators are materially more advanced in the realm of computing natural frequencies of ships than the author recognizes. Thus Dr. Taylor has considered the influence of the shear stresses in the flanges and webs of ships with sufficient accuracy. He showed as early as 1924 that a uniform distribution of tensile or compressive stresses in deck and bottom can no longer occur when shear stresses are present in addition to normal stresses. From equilibrium conditions alone it would follow that shear stresses would increase linearly from the middle of the girder to the webs if it is assumed that the whole width of the flange is uniformly effective. The shearing stresses cause a change in the uniform stress distribution in such a way that the normal stresses are non-uniformly distributed over the width or in other words the effective width is reduced. The deflection due to shearing stress in the flanges considered by the author is therefore identical with the change in deflection as a result of the reduction of effective width. This follows also from my precise calculation in which the total energy of deformation of the flanges is applied to the computation of effective width. For the case of flexural vibrations of lower order my method gives only slightly different values. The ends of the ship, as the author correctly remarks, are unimportant. This interdependence between shear stresses and effective width escaped the notice of the author and thus a mistake in his method of computation is explained. He has considered the influence of effective width twice in the coefficients K_1 and K_3. Hence it would follow that the computed frequencies would not agree with the measured. Either the influence of effective width K_1 must be set equal to unity or we must so compute K_3 that only the shear stresses in the web are considered. The second method is appropriate for frequencies of higher order since in this case the effective width decreases rapidly.

On the other hand the author has not considered the influence of water pressure on the reduction of effective width. This, however, can reach a magnitude of 10 per cent or more. It seems to me important to point this out because the author, in connection with the "complex nature of the ship's frame" introduces an empirical coefficient which he deduces from the average of his experiments on large ships. A check of the reports on the experiments of Read, Stanbury, and Dahlmann shows that Dr. Schadlofsky has not computed the shear deflection correctly. The magnitude of the shear deflection is in fact not only dependent on the ratios L/B and L/H but also on the shape of the moment curve. This latter decisive point, however, is not considered although this materially alters the numerical results. In a small experimental girder we must remember that the allowable tolerances in thickness can cause a variation of several per cent. With regard to the deflection of the destroyer "Wolf", I refer to my publications on this subject. The coefficient K_2 appears to me therefore not well established.

I might amplify somewhat the remarks of Prof. Horn. He has already shown that the extremely large change in the frequency cannot be ascribed to damping but must be due to the vibrating water. There is now a simple means of determining the effective magnitude of the damping. We need only consider the free vibration of the hull after an impulse. Dr. Taylor, already mentioned, made such measurements during a launching. On leaving the ways the ship had been set in vibration. The seismograph which was set up amidships recorded the amplitudes which decayed according to the equation $y = f_0 e^{-0.66t}$. The damping due to wave resistance was so small that it could have only a slight influence on the frequency. Nevertheless a large decrease in frequency was observed in the case of this ship in agreement with the theory of Lewis, Moullin, and Taylor. Thus it appears to be proved that the opinion of the author as to the influence of damping is incorrect, that the large decrease in frequency can be explained by the vibrating mass of water, and that this induced vibration is practically frictionless. If the author has made contrary observations in his experiments, which does not appear to be probable, this might be due to experimental errors. Whereas the vibration amplitudes of ships are very small compared to the draft-1/1000 to 1/10,000-so that here the loss due to wave formation is vanishingly small, the experiments of the author were made with small models and relatively great amplitudes. Therefore, they are not with certainty applicable to large ships. The author arbitrarily introduces the damping in the calculation as a mass which is proportional to the mass of the ship by introducing a damping coefficient K_5.* In spite of the fact that he has an incorrect principle of computation he obtains by this method results which in many cases do not differ greatly from the experimental results and the more accurate calculations of Lewis and Taylor. This is inherent in the nature of the calculation. Thus as far as the results are concerned it is unimportant how the individual masses are distributed over the

*Damping would have to appear as a special term in the differential equation, while the mass vibrating with the body belongs with the mass distribution.

length but only the mean values over the elements of length are significant. The case is similar to the computation of the deflection by means of the trapezoidal rule where a continuous load can be subdivided into individual loads. The quadruple integration equalizes the uneven units if the mean values over not too great distances are of equal magnitude. As long as the distribution of the vibrating water mass over the length does not differ on the average essentially from the distribution of the ship's mass, the results according to Dr. Schadlofsky's calculation also yield a useful value. On the other hand if a markedly non uniform mass distribution exists as in the case of a battleship with armored turrets the results in general will not agree. The calculation of the English investigators is therefore to be preferred.

The author in his investigation also tested the influence of ship's fineness δ on the frequency of vibration. He assigned to a fineness coefficient δ , independent of the remaining form of the ship a definite damping (really mass of water vibrating with the ship, "mitschwingende Wassermasse.") Even this method is questionable. Actually the form plays an important part. Thus the effective water mass will be quite differently distributed if we consider first a cruiser stern on the one hand and on the other an ordinary cargo vessel stern. The method of Schadlofsky is only permissible when the ships are of such forms as those investigated by the author. The process here is similar to that of a hydroplane which goes through the water at a high velocity. In this case an entirely different mass of water is set in motion according as the portion of the ship has a flat bottom or curved keel. This error was eliminated in the authors method.

With reference to the work of Dr. Dahlmann, I wish merely to bring out the basic difference between our points of view. Dr. Dahlmann attempts to include in the modulus of elasticity all effects such as shear, effective width, and vibrating water mass. This however produces no great difference as compared with the old empirical formula of Schlick since an empirical coefficient is used. We are investigating on the other hand the various causes which influence the frequency in order to get sound results.

The results which chief naval constructor Burkhardt has here presented appear to me especially interesting. He has given the results of measurements of vibrations of very high frequencies on board war ships. Thus he finds that the moment of inertia decreases sharply with the frequency but appears gradually to approach a constant value. This agrees very well with my theory of box girders. This shows that in the higher harmonics of vibration only the web and a small strip of deck contribute to the moment of inertia. The shear deflection moreover is greater than the deflection due to moments. The phenomenon observed by Herr Burkhardt, namely the great decrease in computed moment of inertia and the gradual approach to a limiting value shows the combined action of these two effects.

In conclusion, I wish to call attention to several errors which occur in the author's calculation. In Fig. 14 the shearing stresses for the tankers with longitudinal bulkheads are not given correctly. On account of the fastening to the side walls through the transverse bulkheads and because of the deformation the longitudinal bulkheads can take up only essentially smaller shearing stresses and therefore also transverse forces than the author has given. Also that part of the flange included with the longitudinal bulkhead is therefore essentially small so that a quite different distribution of shearing stress exists in the flanges and webs.

The statement of the author that double bottoms and decks greatly increase the shear deflection is not correct in this form. The actually state of affairs is that the moment of inertia is essentially increased due to the inclusion of double bottoms and decks, and therefore the deflection due to bending moment is greatly diminished. The actual shear deflection is on the other hand only slightly increased. Moreover the effective width is decreased in the usual manner when there are decks and double bottoms. From the combination of these circumstances it follows that the ratio of shear deflection to deflection due to moments is greater, while the absolute magnitude of the shear deflection is only slightly increased.

I must not fail for my part to thank Herr Schadlofsky for stimulating this exhaustive discussion by his work and contributing essentially to clearing up this interesting subject.

Dr. Dahlmann- Hamburg:

As the author has mentioned that part of my work in question may I be granted a few remarks. The problem is exceedingly difficult. I should like therefore to confine my remarks to the elastic behavior since the second involved question, that of damping resistance, is of so complicated a nature that in my opinion it is not yet ready for public discussion. Personally, it is my opinion that this resistance is not as important as represented.

The analysis of the factors which influence the frequency, that is, the segregation of the Schlick factors is to be commended and therefore the work of the author is to be marked as progressive. In my opinion, the question of effective width cannot be solved by the use of an ideal box girder but only by experiment on an actual ship. The ideal box girder has, however, actually done its duty but now we should deal with actual ship proportions. Likewise the effect of shear deflection on the elastic line of the complete hull cannot be solved mathematically. The reason for this as is well known is the ignorance of the stress distribution. The expression chosen by the author is to be considered only an approximation. Actually the shearing stress is a function of the deflection. An attempt must be made to introduce the shear deflection term as an auxiliary term in the basic vibration equation $EJ\frac{\partial^4 y}{\partial x^4} = \mu\frac{\partial^2 y}{\partial t^2}$

The author obtains for the effect of shear deflection relatively high values which at least with respect to the influence of double bottoms must be doubted.

We require further extension of the theoretical expressions by full scale experiments. I indicated the correct line of investigation some time ago in the journal Werft und Reederei. With regard to the influence of the complex nature of the hull, I believe that an additional differentiation of the factors which influence the bending factor is possible particularly with regard to the modulus of elasticity. I repeat the question already asked of the author whether he is of the opinion that two otherwise similar ships one of which is welded and the other riveted have the same modulus of elasticity. For the complete longitudinal frame of the ship, I am of the opinion that it is considerably lower than that of the material as used in the author's computation.

A still further separable factor is the influence of the transverse contraction of the section between bulkheads. Its analysis is only possible on the basis of systematic experiments.

The author deals with only a part of the important practical vibration problem which still offers many difficulties from the standpoint of measuring technique. In order to make clear the deformation processes a number of vibrographs must be distributed over the ship inasmuch as in practice superpositions occur due to horizontal and torsional vibrations. This complication of the actual vibration process due to which local vibration phenomena are also of considerable practical importance places difficult requirements on the technique of measurement. It is therefore gratifying that the STG will attack the problem of elasticity in a separate committee. In this connection the wish must be expressed that in this time of necessity the scant means at our disposal be distributed among all investigators in a more equitable manner in order that all may be able to contribute to further clarifying so important a practical problem in elasticity.

Dr. Ing. Weinblum, Berlin:

First of all several formal observations on the interesting work of Dr. Schadlofsky which is based on the fundamental equation of the vibrating bar. It is appropriate first to consider all terms involved in the differential equation even though a solution cannot be found in this general form. Besides the potential energy due to normal stress the shearing stress is to be considered; in the kinetic energy there appears in addition to the mass of the bar the additional mass of the water; for the damping a dispersion function can be written. If we now undertake an estimate of the influences of the individual terms (e.g. there exists for prismatical bars a solution by Timoshenko which takes into consideration the shear stresses) it is possible to make simplifications without encountering the danger of overlooking essential points. Then it would also have been possible to do without a number of experiments.

The concept of virtual mass is of fundamental importance in this investigation. The theoretically determined values of the same agree very well for bodies on the surface so long as it is a case of vertical motion – buoyancy vibration; e.g. we undertook such experiments on a sphere in the Versuchsanstalt fur Wasserbau und Schiffbau, Berlin in charge of Dr. Erbach (see Schiffbau 1931 p 490) which confirmed with good accuracy the value of effective mass of 50% of the original volume even for relatively large amplitudes. Moullin (Transactions for the Third International Congress of Applied Mechanics, Stockholm 1930) gives for prismatical bars an experimentally determined value of about 90% of the theoretical; at the same time he points out that for vibration with nodes the percentage appears to be still lower. On the basis of the present data we must assume that, even in complicated forms of motion, theory gives in the main results for determining virtual mass which are useful both qualitatively and quantitatively. The effect of the damping on the period seldom exceeds 1%.

Dr. Ing. Gustav Wrobbel – Hamburg:

May I also express my acknowledgment of the unusually interesting statements of Dr. Schadlofsky as well as the support given him by the Deschimag I share the opinion of Dr. Dahlmann that the shear effect of double bottoms cannot be as great as shown by the calculations in this paper. Contrary to Dr. Dahlmann and in agreement with Prof. Schnadel, thanks to whose statements I am able to express myself comparatively briefly, I should like to point out that the modulus of elasticity of the material has been used as a basis for all calculations as opposed to the apparent modulus of elasticity constantly proposed by Biles and Dahlman. When Dr. Dahlman states that the elastician must consider that model technique has been gnawed to the bone may I in contrast express my view that there is still a quantity of flesh thereon and, as emphasized by me on various occasions, model research and research on full size ships must go hand in hand and supplement one another if we wish to obtain a scientific knowledge of the processes actually taking place and the existing laws. For brevity I refer to an article of mine on this topic in the journal WRH Vol. 12, No. 7, Sept. 1, 1931 which treats these questions exhaustively. The fact that the present work of Dr. Schadlofsky is based on model research as well as full scale research gives the work special value.

Closing remarks of the author:

Prof. Horn objects in the first place to the expression chosen by me "damping resistance" on the grounds that by damping we understand a process by which energy of vibration is dissipated. In this connection I might refer to the well known book of Prof. Hort "Technische Schwingungslehre" in which in section six is stated: "Under damping we understand all general resistance forces which oppose the motion of a vibrating body."

I therefore chose the expression damping resistance as suitable chiefly so that the other remaining resistances such as waves, friction, and turbulence resistances are included.

To the contradiction which exists between my measured results and the theory of Lewis and Taylor I should like to state first that in no sense do I underestimate the great scientific significance of the work of Lewis and Taylor. My experimental results agree with the results of this theory except for the case of relatively low frequencies. At high frequencies they probably would agree fully with the theory mentioned as shown by the agreement with Nicholl's experiments. An answer to the question whether the observed difference at low frequencies is due to any source of error in my experiments or to the actual presence of a true damping resistance can only be given by new experiments. I will only point out in this respect that with models of very low natural frequencies considerable difficulties of measurement arise since these models are very soft and light. The effect of the pressure of the vibrograph rods and the effect of the rubber strips are thus somewhat greater. Moreover the light models float at a certain draft and must be held in their original position by special precautions. For these reasons the series of measurements with test model 1 was repeated not less than three times. It was necessary to reject one series because the frequencies before and after immersion differed widely. The two other repetitions gave results in agreement.

The proposal of Prof. Horn to alter the test conditions in repeating the experiments appears very valuable as a means of finally clarifying the relation between damping resistance and frequency. Thus I might point out that the absolute magnitude and distribution of the damping resistance cannot be derived from the foregoing experiments. The values determined are relative and are bound by a fixed nodal distance and an assumed distribution. Clarification of the absolute magnitude can only be given by experiments with entirely freely vibrating models.

I will briefly touch upon the remarks of the other speakers in the discussion. The statements of chief naval constructor Burkhardt which bring out what attention the navy is paying to the vibration problem are unusually interesting. The experiments with the familiar vibration generator of Losenhausen are naturally best adapted for determining vibration constants according to the Schlick formula especially for vibrations of the higher harmonics. The application of such constants however is possible only for similar ships. For different ship types the execution of a vibration computation is unavoidable. A comprehensive program evidently has been set up by the naval authorities. It would be highly gratifying if further particulars of the results could be reported at future meetings.

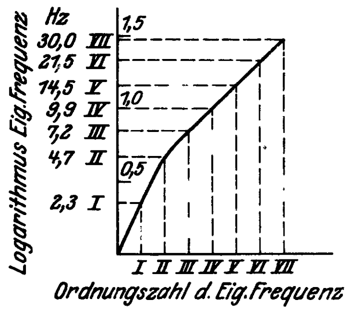
I would like to thank especially Prof. Schnadel for his very interesting remarks and encouragement. I know very well that Dr. Taylor had gone thoroughly into the problem of shear deflection and in this connection have scarcely anything to add to the statements of Prof. Schnadel. With regard to the conclusions drawn by him from Taylor's measurements on board ship after launching regarding the slight magnitude of "real damping resistance" it must be admitted that it corroborates the view of Prof. Horn to the effect that the experimental conditions have not been without effect on the dependence of total resistance on frequency observed by me. Therefore, in the interest of a final clarification, it is absolutely necessary that such experiments be repeated. As I have heard further experiments in this direction will be made.

To the remarks of Dr. Weinblum I would like to make the following reply: Even though in the interest of a complete presentation of the entire problem the introduction of the factors influencing the elastic behavior and the inertia resistance in the differential equation appears desirable, practically the total picture would be unnecessarily complicated. The calculation of shear deflection according to Timoshenko should probably be regarded as more exact but also much more complicated. It is doubtful whether the gain in accuracy is commensurate with the increased time expended.

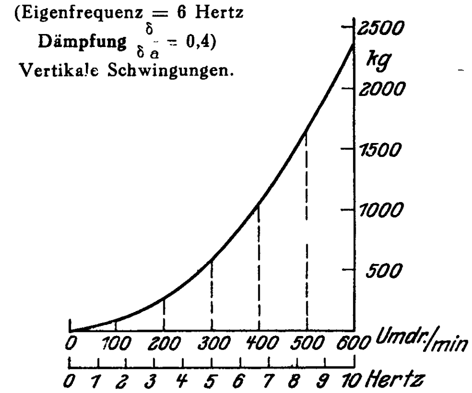
In conclusion I would like to emphasize that my work was conceived from practical considerations and is intended for direct application to practice. Even though many special questions still require special research I nevertheless hope that my paper will contribute to facilitating the method of computing the natural frequency in practice and give an incentive for further studies.

I thank the gentlemen taking part in the discussion for the friendly interest which they have shown in my work and the STG that they have given me the opportunity to present my paper in their meeting.

Schwingungsmessungen auf „Bremse“ 11. Juni 1931.
Auswertungsergebnisse aus den Kurven des Askania-Schwingungsmessers.



Reihenfolge der Eigenfrequenzen. Die Frequenzen oberhalb der III. Eigenfrequenz zeigen die gleiche Gesetzmäßigkeit wie beim homogenen elastischen Balken.



Zentrifugalkräfte der Erregermaschine. (Nach Späth-Losenhausen.)

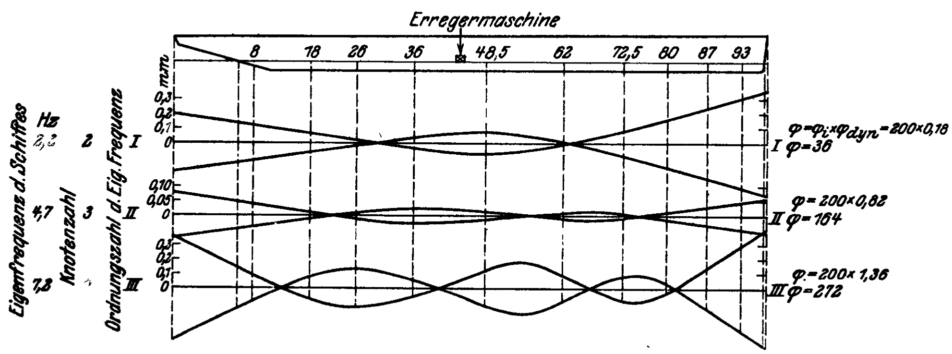
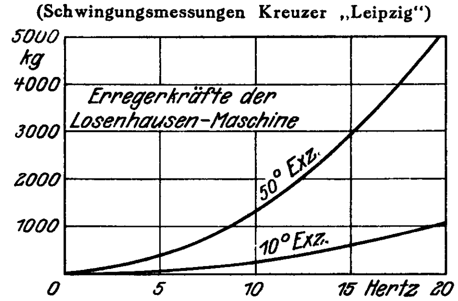


Bild 1.
Si = indiz. Vergaser, *Sdyn* = dynamischer Vergaser.
φ = Gesamt-Vergaser des Messers.

(Schwingungsmessungen Kreuzer „Leipzig“)



Auf Blatt 2 und 3 sind zum besseren Vergleich der verschiedenen Frequenzen nicht die absoluten Amplituden, sondern die auf die Einheit der Kraft (100 kg) umgerechneten aufgetragen.

Bild 2 a. Biegungsschwingungsformen.

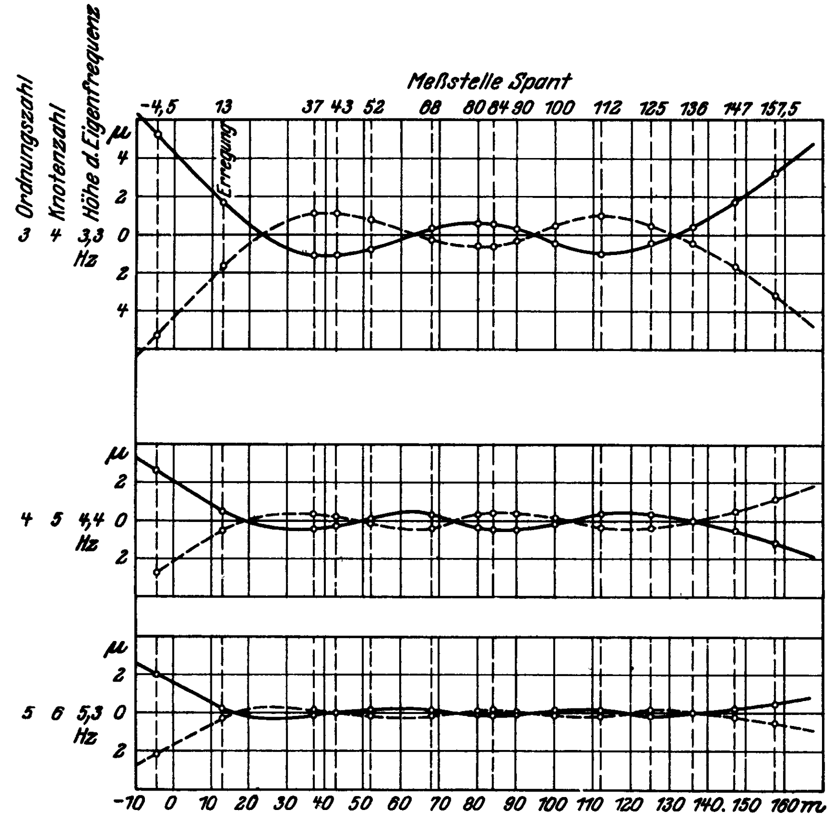


Bild 2. Vertikale Schwingungsformen bei 100 kg Erregerkraft.

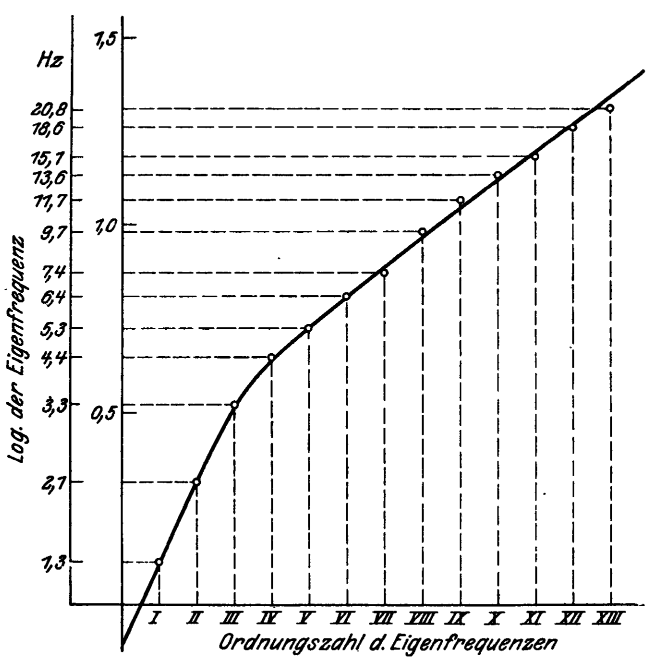
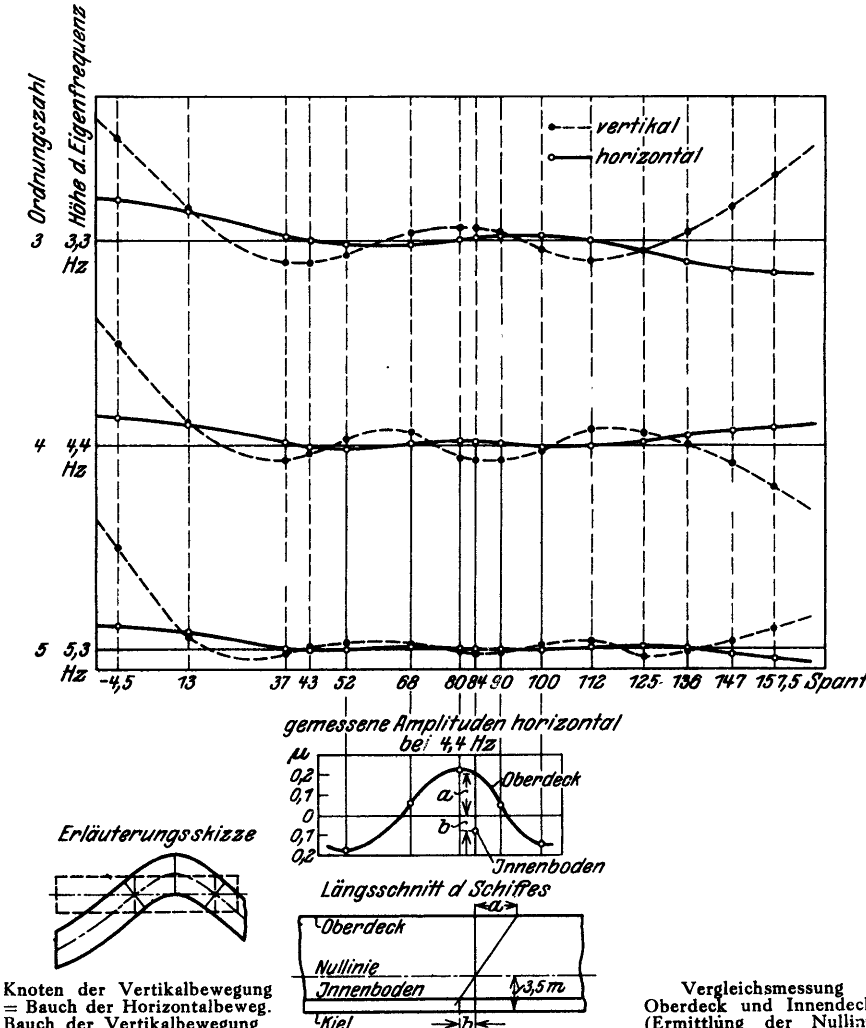


Bild 3. Kreuzer „Leipzig'.
Reihenfolge der Eigenfrequenzen.

Horizontal-Schwingung längsschiffs auf Oberdeck im Vergleich zu den Vertikal-Schwingungen.
(Amplituden-Maßstab entspricht dem auf Blatt 4 und ist für horizontale und vertikale Schwingungen der gleiche.)



Knoten der Vertikalbewegung = Bauch der Horizontalbeweg.
Bauch der Vertikalbewegung = Knoten d. Horizontalbeweg.

Vergleichsmessung Oberdeck und Innendeck. (Ermittlung der Nullinie für Spant 84.)

Bild 4.